

Delenie: n počet deliacich bodov, $a = x_0$, $b = x_n$, $y_i = f(x_i)$, $i = 0, 1, \dots, n$.

obdĺžniková metóda:
$$\int_a^b f(x) dx \approx \frac{b-a}{n} [y_0 + y_1 + y_2 + y_3 + \dots + y_{n-3} + y_{n-2} + y_{n-1}]$$

 resp.
$$\int_a^b f(x) dx \approx \frac{b-a}{n} [y_1 + y_2 + y_3 + \dots + y_{n-3} + y_{n-2} + y_{n-1} + y_n]$$

Chyba (nepresnosť) metódy: $R_n^O \leq \varepsilon_1 \frac{(b-a)^2}{n}$, pričom $\forall x \in \langle a; b \rangle : |f'(x)| \leq \varepsilon_1$

Lichobežníková metóda:
$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [y_0 + 2y_1 + 2y_2 + 2y_3 + \dots + 2y_{n-2} + 2y_{n-1} + y_n]$$

Chyba (nepresnosť) metódy: $R_n^L \leq \varepsilon_2 \frac{(b-a)^3}{12n^2}$, pričom $\forall x \in \langle a; b \rangle : |f''(x)| \leq \varepsilon_2$

Simpsonova metóda: [n je párne]
$$\int_a^b f(x) dx \approx \frac{b-a}{3n} [y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 2y_{n-2} + 4y_{n-1} + y_n]$$

Chyba (nepresnosť) metódy: $R_n^S \leq \varepsilon_4 \frac{(b-a)^4}{180n^4}$, pričom $\forall x \in \langle a; b \rangle : |f''''(x)| \leq \varepsilon_4$

$$\int_1^2 \frac{dx}{x} = [\ln x]_1^2 = \ln 2 - \ln 1 = \ln 2 = \mathbf{0,69314718}.$$

$f(x) = x^{-1}$, $x \in \langle 1; 2 \rangle$, $D_{10} = \{x_i\}_{i=0}^{10} = \{1; 1,1; 1,2; 1,3; \dots; 1,9; 2\}$, $y_i = f(x_i) = \frac{1}{x_i}$, $i = 1, 2, \dots, 10$

$f'(x) = -x^{-2}$, $|f'(x)| = \frac{1}{x^2} \leq 1 = \varepsilon_1$, $x \in \langle 1; 2 \rangle \implies R_{10}^O \leq 1 \cdot \frac{(2-1)^2}{10} = \frac{1}{10} = 0,1$

$f''(x) = 2x^{-3}$, $|f''(x)| = \frac{2}{x^3} \leq 2 = \varepsilon_2$, $x \in \langle 1; 2 \rangle \implies R_{10}^L \leq 2 \cdot \frac{(2-1)^3}{12 \cdot 10^2} = \frac{1}{600} \approx 0,00167$

$f'''(x) = -6x^{-4}$, $f''''(x) = 24x^{-5}$, $|f''''(x)| = \frac{24}{x^5} \leq 24 = \varepsilon_4$, $x \in \langle 1; 2 \rangle \implies R_{10}^S \leq 24 \cdot \frac{(2-1)^4}{180 \cdot 10^4} = \frac{2}{150000} \approx 0,00013$

$y_i = 1/x_i$	obdĺžniková metóda	obdĺžniková metóda	lichobežníková metóda	simpsonova metóda
$x_0 = 1,0$	$y_0 = 1,0$	—	$y_0 = 1,0$	$y_0 = 1,0$
$x_1 = 1,1$	$y_1 = 0,90909091$	$y_1 = 0,90909091$	$2y_1 = 1,81818182$	$4y_1 = 3,63636364$
$x_2 = 1,2$	$y_2 = 0,83333333$	$y_2 = 0,83333333$	$2y_2 = 1,66666666$	$2y_2 = 1,66666666$
$x_3 = 1,3$	$y_3 = 0,76923077$	$y_3 = 0,76923077$	$2y_3 = 1,53846154$	$4y_3 = 3,07692308$
$x_4 = 1,4$	$y_4 = 0,71428571$	$y_4 = 0,71428571$	$2y_4 = 1,42857142$	$2y_4 = 1,42857142$
$x_5 = 1,5$	$y_5 = 0,66666667$	$y_5 = 0,66666667$	$2y_5 = 1,33333334$	$4y_5 = 2,66666668$
$x_6 = 1,6$	$y_6 = 0,625$	$y_6 = 0,625$	$2y_6 = 1,25$	$2y_6 = 1,25$
$x_7 = 1,7$	$y_7 = 0,58823529$	$y_7 = 0,58823529$	$2y_7 = 1,17647058$	$4y_7 = 2,35294116$
$x_8 = 1,8$	$y_8 = 0,55555556$	$y_8 = 0,55555556$	$2y_8 = 1,11111112$	$2y_8 = 1,11111112$
$x_9 = 1,9$	$y_9 = 0,52631579$	$y_9 = 0,52631579$	$2y_9 = 1,05263158$	$4y_9 = 2,10526316$
$x_{10} = 2,0$	—	$y_{10} = 0,5$	$y_{10} = 0,5$	$y_{10} = 0,5$

hodnoty numerického výpočtu

$\int_1^2 \frac{dx}{x} =$	$\Sigma = 7,18771403$	$\Sigma = 6,68771403$	$\Sigma = 13,87542806$	$\Sigma = 20,79450692$
	$\frac{\Sigma}{10} = 0,71877140$	$\frac{\Sigma}{10} = 0,66877140$	$\frac{\Sigma}{20} = 0,69377140$	$\frac{\Sigma}{30} = 0,69315023$

chyby numerického výpočtu

teoretická	0,1	0,1	0,00167	0,000013
skutočná	0,0256242	0,0243758	0,00062422	0,00000305