Matematická analýza 1

2018/2019

2. Číselné postupnosti

Obsah

Zoznam riešených limít – príklady 01–80

01. $\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$ **02.** $\lim\limits_{n\to\infty} n\left[\sqrt[n]{3}-\sqrt[n]{2}\right]$ **03.** $\lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n}$ **04.** $\lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3}$ **05.** $\lim\limits_{n\to\infty} \left[\sqrt{n+1}-n\right]$ **06.** $\lim\limits_{n\to\infty} \left[\sqrt{n+1}-\sqrt{n}\right]$ **07.** $\lim\limits_{n\to\infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}}$

08. $\lim\limits_{n\to\infty} \frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$ **09.** $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{2n}$ **10.** $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{n^2+1}$ **11.** $\lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$ **12.** $\lim\limits_{n\to\infty} \left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$ **13.** $\lim\limits_{n\to\infty} \sqrt[n]{3^n-2^n}$

14. $\lim\limits_{n\to\infty} \sqrt[n]{2^n+1}$ **15.** $\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$ **16.** $\lim\limits_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n]{3}\right]$ **17.** $\lim\limits_{n\to\infty} \left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$ **18.** $\lim\limits_{n\to\infty} \frac{n+3^n}{n-3^n}$ **19.** $\lim\limits_{n\to\infty} \frac{n+3^{-n}}{n-3^{-n}}$ **20.** $\lim\limits_{n\to\infty} \left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$

21. $\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$ **22.** $\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$ **23.** $\lim\limits_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$ **24.** $\lim\limits_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$ **25.** $\lim\limits_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$ **26.** $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$

27. $\lim\limits_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$ **28.** $\lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$ **29.** $\lim\limits_{n\to\infty} n\left[\ln n-\ln(n+2)\right]$ **30.** $\lim\limits_{n\to\infty} n\left[\ln(n+3)-\ln n\right]$

31. $\lim\limits_{n\to\infty} \left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]$ **32.** $\lim\limits_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}}$ **33.** $\lim\limits_{n\to\infty} \left[\sqrt{n^2-n+1}-\sqrt{n^2-3n+2}\right]$ **34.** $\lim\limits_{n\to\infty} \left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$ **35.** $\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$

36. $\lim\limits_{n\to\infty} \left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$ **37.** $\lim\limits_{n\to\infty} \left[\sqrt{n^2+4n+1}-n+1\right]$ **38.** $\lim\limits_{n\to\infty} \left[\sqrt{n^2+n+1}-\sqrt{n-1}\right]$ **39.** $\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$ **40.** $\lim\limits_{n\to\infty} \left[\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right]$

41. $\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$ **42.** $\lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$ **43.** $\lim\limits_{n\to\infty} \left[\sqrt[3]{n^3+1}-n+1\right]$ **44.** $\lim\limits_{n\to\infty} \left[\sqrt[3]{n^3+1}-n+1\right]$ **45.** $\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n+6}$ **46.** $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$

47. $\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$ **48.** $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$ **49.** $\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$ **50.** $\lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$ **51.** $\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$ **52.** $\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$ **53.** $\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$

54. $\lim\limits_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$ **55.** $\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$ **56.** $\lim\limits_{n\to\infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$ **57.** $\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$ **58.** $\lim\limits_{n\to\infty} \frac{2^n\, n!}{n^n}$ **59.** $\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$ **60.** $\lim\limits_{n\to\infty} \frac{3^n\, n!}{n^n}$

61. $\lim\limits_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$ **62.** $\lim\limits_{n\to\infty} \frac{8^n\, n!}{(3n)^n}$ **63.** $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$ **64.** $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$ **65.** $\lim\limits_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$ **66.** $\lim\limits_{n\to\infty} \frac{(\frac{1}{3})^n}{(\frac{1}{5})^n-(\frac{1}{4})^n}$

67. $\lim\limits_{n\to\infty} \frac{(\frac{1}{4})^n}{(\frac{1}{5})^n-(\frac{1}{3})^n}$ **68.** $\lim\limits_{n\to\infty} \frac{(\frac{1}{5})^n}{(\frac{1}{4})^n-(\frac{1}{3})^n}$ **69.** $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$ **70.** $\lim\limits_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}$ **71.** $\lim\limits_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$ **72.** $\lim\limits_{n\to\infty} \left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$

73. $\lim\limits_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$ **74.** $\lim\limits_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$ **75.** $\lim\limits_{n\to\infty} \frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right]$ **76.** $\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$

77. $\lim\limits_{n\to\infty} a_n$, ak $\{a_n\}_{n=1}^{\infty}=\left\{\sqrt{2},\sqrt{\sqrt{2}},\sqrt{\sqrt{\sqrt{2}}},\ldots\right\}$ **78.** $\lim\limits_{n\to\infty} a_n$, ak $\{a_n\}_{n=1}^{\infty}=\left\{\sqrt{2},\sqrt{2+\sqrt{2}},\sqrt{2+\sqrt{2+\sqrt{2}}},\ldots\right\}$

79. $\lim\limits_{n\to\infty} a_n$, ak rekurentne $a_1=2$, $a_{n+1}=\sqrt{2a_n+3}$, $n\in N$ **80.** $32{,}17\overline{71}$ vyjadrite ako zlomok

Číselné postupnosti – Postupnosti reálnych čísel

Číselna postupnosť f je funkcia (zobrazenie) $N \to R$, t. j. $D(f) = N$.

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$$\{a_n\}_{n=1}^{\infty} = \{2n - 1\}_{n=1}^{\infty} = \{1,3,5,7, \ldots\}$$

$$\{a_n\}_{n=1}^{\infty} = \{n^2\}_{n=1}^{\infty} = \{1,4,9,16,25, \ldots\}$$

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Explictné zadanie: $a_n = 2n - 1,\ n \in N.$

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Rekurentné zadanie: $a_1 = 1,\ a_n = a_{n-1} + 2$ pre $n \in N - \{0\}.$

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Explictné zadanie: $a_n = n^2,\ n \in N.$

Rekurentné zadanie: $a_1 = 1,\ a_{n+1} = a_n + 2n + 1$ pre $n \in N.$

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Postupnosť $\{a_n\}_{n=1}^{\infty}$ sa nazýva:

Ohraničená zhora, ak $\exists M \in R$ také, že $\forall n \in N$ platí $a_n \leq M$.

Neohraničená zhora, ak nie je ohraničená zhora.

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Ohraničená, ak je ohraničená zhora a ohraničená zdola,

Neohraničená zhora, ak nie je ohraničená zhora.

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Ohraničená, ak je ohraničená zhora a ohraničená zdola,

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t. j. ak $\exists m, M \in R$ také, že $\forall n \in N$ platí $m \leq a_n \leq M$,

resp. ak $\exists M \in R$ také, že $\forall n \in N$ platí $|a_n| \leq M$.

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Rastúca, ak $\forall n \in N$ platí $a_n < a_{n+1}$.

Neklesajúca, ak $\forall n \in N$ platí $a_n \leq a_{n+1}$.

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$\{k_n\}_{n=1}^{\infty}$ je rastúca postupnosť prirodzených čísel, t. j. indexov.

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$\{a_{2n}\}_{n=1}^{\infty} = \{a_2, a_4, a_6, a_8, \ldots\} = \{4n-1\}_{n=1}^{\infty} = \{3, 7, 11, 15, 19, \ldots\}$
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Podpostupnosti

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$\{a_{k_n}\}_{n=1}^{\infty}$ sa nazýva podpostupnosť (vybraná postupnosť) z $\{a_n\}_{n=1}^{\infty}$.

Podpostupnosti $\{a_n\}_{n=1}^{\infty} = \{2n-1\}_{n=1}^{\infty}$ sú napríklad:

$\{a_{2n}\}_{n=1}^{\infty} = \{a_2, a_4, a_6, a_8, \ldots\} = \{4n-1\}_{n=1}^{\infty} = \{3, 7, 11, 15, 19, \ldots\}$
pre $\{k_n\}_{n=1}^{\infty} = \{2n\}_{n=1}^{\infty} = \{2, 4, 6, 8, \ldots\}$ (párne členy),

$\{a_{n^2}\}_{n=1}^{\infty} = \{a_1, a_4, a_9, a_{16}, \ldots\} = \{2n^2-1\}_{n=1}^{\infty} = \{1, 7, 17, 31, \ldots\}$
pre $\{k_n\}_{n=1}^{\infty} = \{n^2\}_{n=1}^{\infty} = \{1, 4, 9, 16, \ldots\}$,

$\{a_n\}_{n=1}^{\infty} = \{2n-1\}_{n=1}^{\infty}$ pre $\{k_n\}_{n=1}^{\infty} = \{n\}_{n=1}^{\infty} = \{1, 2, 3, 4, \ldots\}$,

$\{a_n\}_{n=11}^{\infty} = \{2n-1\}_{n=11}^{\infty}$, $\{101, 109, 235, 637, 639, 641, \ldots\}$, ...

Konvergencia a divergencia postupnosti

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$$\{(-1)^{n+1}n\}_{n=1}^{\infty} = \{1,-2,3,-4,5,-6,7,-8,\ldots\}, \quad E = \{\pm\infty\}.$$

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$\lim\limits_{n\to\infty} a_n =$
- $a \in R$ — vlastná limita — $\{a_n\}_{n=1}^{\infty} \longmapsto a$
- $\pm\infty$ — nevlastná limita — $\{a_n\}_{n=1}^{\infty} \longmapsto \pm\infty$
- $\nexists$

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$a \in R$	vlastná limita	$\{a_n\}_{n=1}^{\infty} \longmapsto a$ postupnosť $\{a_n\}_{n=1}^{\infty}$ konverguje k číslu a
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Každá monotónna $\{a_n\}_{n=1}^{\infty}$ má limitu, t. j. existuje $\lim\limits_{n\to\infty} a_n = a \in R^*$.

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$$\{a_n\}_{n=1}^{\infty} \begin{cases} \text{neklesajúca} \\ \\ \text{nerastúca} \end{cases}$$

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$\{a_n\}_{n=1}^{\infty}$
- neklesajúca
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- neklesajúca
  - zhora neohraničená $\quad \{a_n\}_{n=1}^{\infty} \longmapsto +\infty$
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Limity postupností

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$\{a_n\}_{n=1}^{\infty}$ neklesajúca, zhora ohraničená; nerastúca, zdola ohraničená $\Rightarrow$ $\{a_n\}_{n=1}^{\infty} \longmapsto a \in R$

$$\{a_n\}_{n=1}^{\infty} \begin{cases} \text{neklesajúca} & \text{zhora ohraničená} \\ \text{nerastúca} & \text{zdola ohraničená} \end{cases} \quad \{a_n\}_{n=1}^{\infty} \longmapsto a \in R$$

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Dôležité limity — naspamäť! $a, b \in R,\ a > 0$

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$$\lim_{n\to\infty} \sqrt[n]{a} = 1, \quad \lim_{n\to\infty} \sqrt[n]{n} = 1,$$

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Dôležité limity — naspamäť! $a, b \in R,\ a > 0$

$$\lim_{n\to\infty} \sqrt[n]{a} = 1, \quad \lim_{n\to\infty} \sqrt[n]{n} = 1, \quad \lim_{n\to\infty} \left(1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}\right) = \mathrm{e},$$

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$$\lim_{n\to\infty} \sqrt[n]{n!} = \infty, \quad \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e, \quad \lim_{n\to\infty} \left(1 + \frac{b}{n}\right)^n = e^b,$$

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Limity postupností

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  - zdola ohraničená: $\{a_n\}_{n=1}^{\infty} \longmapsto a \in R$
  - zdola neohraničená: $\{a_n\}_{n=1}^{\infty} \longmapsto -\infty$

Dôležité limity — naspamäť! $a, b \in R,\ a > 0$

$$\lim_{n\to\infty} \sqrt[n]{a} = 1, \quad \lim_{n\to\infty} \sqrt[n]{n} = 1, \quad \lim_{n\to\infty} \left(1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}\right) = \mathrm{e},$$

$$\lim_{n\to\infty} \sqrt[n]{n!} = \infty, \quad \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = \mathrm{e}, \quad \lim_{n\to\infty} \left(1 + \frac{b}{n}\right)^n = \mathrm{e}^b,$$

$$\lim_{n\to\infty} \frac{n!}{n^n} = 0, \quad \lim_{n\to\infty} n\left(\sqrt[n]{\mathrm{e}} - 1\right) = 1, \quad \lim_{n\to\infty} n\left(\sqrt[n]{a} - 1\right) = \ln a.$$

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$\{a_n\}_{n=1}^{\infty}$, $a_n > 0$ pre všetky $n \in N$

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$\Longrightarrow \quad \lim\limits_{n\to\infty} a_n = 0$ pre $\varrho < 1$,

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Ak $\varrho = 1$, potom o hodnote $\lim\limits_{n\to\infty} a_n$ nevieme rozhodnúť.

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Limity postupností

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\displaystyle\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{a}{(\sqrt[n]{n})^k} = \frac{a}{1^k} = a > 1
\end{array}\right\}$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty.$$

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Limity postupností

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty \qquad a > 1,\ k > 1$$

$$\left.\begin{array}{l} \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{a^{n+1}}{(n+1)^k}}{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{(\frac{n+1}{n})^k} = \lim_{n\to\infty} \frac{a}{(1+\frac{1}{n})^k} = \frac{a}{(1+0)^k} = a > 1 \\ \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{a}{(\sqrt[n]{n})^k} = \frac{a}{1^k} = a > 1 \end{array}\right\}$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty.$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^k}{a^n} \qquad a > 1,\ k > 1$$

$$= \lim_{n\to\infty} \frac{1}{\frac{a^n}{n^k}}$$

Limity postupností

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty \qquad a>1,\ k>1$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{a^{n+1}}{(n+1)^k}}{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{(\frac{n+1}{n})^k} = \lim_{n\to\infty} \frac{a}{(1+\frac{1}{n})^k} = \frac{a}{(1+0)^k} = a > 1$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{a}{(\sqrt[n]{n})^k} = \frac{a}{1^k} = a > 1$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty.$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^k}{a^n} \qquad a>1,\ k>1$$

$$= \lim_{n\to\infty} \frac{1}{\frac{a^n}{n^k}} = \frac{1}{\lim\limits_{n\to\infty} \frac{a^n}{n^k}}$$

Limity postupností

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty$ $\quad a > 1,\ k > 1$

$$\left.\begin{array}{l} \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{a^{n+1}}{(n+1)^k}}{\frac{a^n}{n^k}} = \lim\limits_{n\to\infty} \frac{a}{(\frac{n+1}{n})^k} = \lim\limits_{n\to\infty} \frac{a}{(1+\frac{1}{n})^k} = \frac{a}{(1+0)^k} = a > 1 \\ \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{a^n}{n^k}} = \lim\limits_{n\to\infty} \frac{a}{\sqrt[n]{n^k}} = \lim\limits_{n\to\infty} \frac{a}{(\sqrt[n]{n})^k} = \frac{a}{1^k} = a > 1 \end{array}\right\}$$

$$\Rightarrow \quad \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty.$$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n^k}{a^n}$ $\quad a > 1,\ k > 1$

$$= \lim\limits_{n\to\infty} \frac{1}{\frac{a^n}{n^k}} = \frac{1}{\lim\limits_{n\to\infty} \frac{a^n}{n^k}} = \left[\begin{array}{c} \text{Príklad II} \\ \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty \end{array}\right]$$

Limity postupností

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty$ $\qquad a>1,\ k>1$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{a^{n+1}}{(n+1)^k}}{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{(\frac{n+1}{n})^k} = \lim_{n\to\infty} \frac{a}{(1+\frac{1}{n})^k} = \frac{a}{(1+0)^k} = a > 1$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{a}{(\sqrt[n]{n})^k} = \frac{a}{1^k} = a > 1$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty.$$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n^k}{a^n}$ $\qquad a>1,\ k>1$

$$= \lim_{n\to\infty} \frac{1}{\frac{a^n}{n^k}} = \frac{1}{\lim\limits_{n\to\infty} \frac{a^n}{n^k}} = \left[\begin{array}{c}\text{Príklad II}\\ \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty\end{array}\right] = \frac{1}{\infty}$$

Limity postupností

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty$ $\qquad a > 1,\ k > 1$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{a^{n+1}}{(n+1)^k}}{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{(\frac{n+1}{n})^k} = \lim_{n\to\infty} \frac{a}{(1+\frac{1}{n})^k} = \frac{a}{(1+0)^k} = a > 1$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{a}{(\sqrt[n]{n})^k} = \frac{a}{1^k} = a > 1$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty.$$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n^k}{a^n} = 0$ $\qquad a > 1,\ k > 1$

$$= \lim_{n\to\infty} \frac{1}{\frac{a^n}{n^k}} = \frac{1}{\lim\limits_{n\to\infty} \frac{a^n}{n^k}} = \left[\begin{array}{c} \text{Príklad II} \\ \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty \end{array} \right] = \frac{1}{\infty} = 0.$$

Limity postupností

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty$ $\quad a > 1,\ k > 1$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{a^{n+1}}{(n+1)^k}}{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{(\frac{n+1}{n})^k} = \lim_{n\to\infty} \frac{a}{(1+\frac{1}{n})^k} = \frac{a}{(1+0)^k} = a > 1$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{a^n}{n^k}} = \lim_{n\to\infty} \frac{a}{\sqrt[n]{n^k}} = \lim_{n\to\infty} \frac{a}{(\sqrt[n]{n})^k} = \frac{a}{1^k} = a > 1$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{a^n}{n^k} = \infty.$$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n^k}{a^n} = 0$ $\quad a > 1,\ k > 1$

$$= \lim_{n\to\infty} \frac{1}{\frac{a^n}{n^k}} = \frac{1}{\lim\limits_{n\to\infty} \frac{a^n}{n^k}} = \left[\begin{array}{c} \text{Príklad II} \\ \lim\limits_{n\to\infty} \frac{a^n}{n^k} = \infty \end{array} \right] = \frac{1}{\infty} = 0.$$

Taktiež môžeme použiť postup ako v predchádzajúcom príklade $\lim\limits_{n\to\infty} \frac{a^n}{n^k}$.

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.

Každá úsečka (hrana)

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.

Každá úsečka (hrana) sa rozdelí na tri rovnaké úsečky.

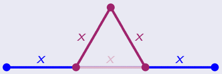
Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.

Každá úsečka (hrana) sa rozdelí na tri rovnaké úsečky. Stredná z úsečiek

Limity postupností

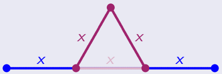
Vypočítajte dĺžku d obvodu Kochovej vločky.



Každá úsečka (hrana) sa rozdelí na tri rovnaké úsečky.
Stredná z úsečiek sa zväčší na dvojnásobok
(z úsečky vzniknú dve strany rovnostranného trojuholníka).

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.



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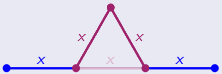
Stredná z úsečiek sa zväčší na dvojnásobok
(z úsečky vzniknú dve strany rovnostranného trojuholníka).
Pôvodná úsečka sa zväčší o tretinu — bude $\frac{4}{3}$-krát dlhšia.

Limity postupností

Vypočítajte dĺžku *d* obvodu Kochovej vločky.

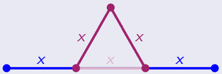


Každá úsečka (hrana) sa rozdelí na tri rovnaké úsečky.
Stredná z úsečiek sa zväčší na dvojnásobok
(z úsečky vzniknú dve strany rovnostranného trojuholníka).
Pôvodná úsečka sa zväčší o tretinu — bude $\frac{4}{3}$-krát dlhšia.

Takže obvod Kochovej vločky *d* sa po každom konštrukčnom kroku zväčší $\frac{4}{3}$-krát.

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.



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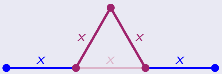
Pôvodná úsečka sa zväčší o tretinu — bude $\frac{4}{3}$-krát dlhšia.

Takže obvod Kochovej vločky d sa po každom konštrukčnom kroku zväčší $\frac{4}{3}$-krát.
Po n-tom kroku bude mať dĺžku $(\frac{4}{3})^n d_\triangle$, kde $d_\triangle$ je obvod prvého trojuholníka.

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.



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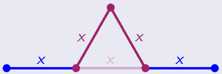
$\Rightarrow \; d = \lim\limits_{n\to\infty} (\frac{4}{3})^n d_\triangle$

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.



Každá úsečka (hrana) sa rozdelí na tri rovnaké úsečky. Stredná z úsečiek sa zväčší na dvojnásobok (z úsečky vzniknú dve strany rovnostranného trojuholníka). Pôvodná úsečka sa zväčší o tretinu — bude $\frac{4}{3}$-krát dlhšia.

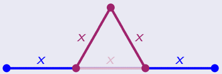
Takže obvod Kochovej vločky d sa po každom konštrukčnom kroku zväčší $\frac{4}{3}$-krát.

Po n-tom kroku bude mať dĺžku $(\frac{4}{3})^n d_\triangle$, kde $d_\triangle$ je obvod prvého trojuholníka.

$\Rightarrow \; d = \lim\limits_{n\to\infty} (\frac{4}{3})^n d_\triangle = \Big[$ Geometrická postupnosť: $\lim\limits_{n\to\infty} (\frac{4}{3})^n = \infty \Big]$

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky.



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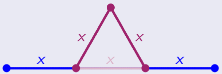
Takže obvod Kochovej vločky d sa po každom konštrukčnom kroku zväčší $\frac{4}{3}$-krát.

Po n-tom kroku bude mať dĺžku $(\frac{4}{3})^n d_\triangle$, kde $d_\triangle$ je obvod prvého trojuholníka.

$$\Rightarrow \ d = \lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n d_\triangle = \left[\text{Geometrická postupnosť: } \lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n = \infty\right] = \infty \cdot d_\triangle$$

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky. Má nekonečnú dĺžku.



Každá úsečka (hrana) sa rozdelí na tri rovnaké úsečky.
Stredná z úsečiek sa zväčší na dvojnásobok
(z úsečky vzniknú dve strany rovnostranného trojuholníka).
Pôvodná úsečka sa zväčší o tretinu — bude $\frac{4}{3}$-krát dlhšia.

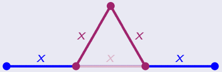
Takže obvod Kochovej vločky d sa po každom konštrukčnom kroku zväčší $\frac{4}{3}$-krát.

Po n-tom kroku bude mať dĺžku $\left(\frac{4}{3}\right)^n d_\triangle$, kde $d_\triangle$ je obvod prvého trojuholníka.

$$\Rightarrow\ d = \lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n d_\triangle = \left[\text{Geometrická postupnosť: } \lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n = \infty\right] = \infty \cdot d_\triangle = \infty.$$

Limity postupností

Vypočítajte dĺžku d obvodu Kochovej vločky. Má nekonečnú dĺžku.

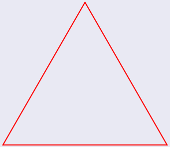


Každá úsečka (hrana) sa rozdelí na tri rovnaké úsečky. Stredná z úsečiek sa zväčší na dvojnásobok (z úsečky vzniknú dve strany rovnostranného trojuholníka). Pôvodná úsečka sa zväčší o tretinu — bude $\frac{4}{3}$-krát dlhšia.

Takže obvod Kochovej vločky d sa po každom konštrukčnom kroku zväčší $\frac{4}{3}$-krát.

Po n-tom kroku bude mať dĺžku $(\frac{4}{3})^n d_\triangle$, kde $d_\triangle$ je obvod prvého trojuholníka.

$$\Rightarrow \ d = \lim_{n\to\infty} (\tfrac{4}{3})^n d_\triangle = \left[\text{Geometrická postupnosť: } \lim_{n\to\infty} (\tfrac{4}{3})^n = \infty\right] = \infty \cdot d_\triangle = \infty.$$



Riešené limity – 01, 02

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{3}-\sqrt[n]{2}\right]$

Riešené limity – 01, 02

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$

$$= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{3} - \sqrt[n]{2}\right]$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right]$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right]\sqrt[n]{2}$$

Riešené limity – 01, 02

$\lim_{n\to\infty} \frac{n^2+n}{n^2-2}$

$$= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}}$$

$\lim_{n\to\infty} n\left[\sqrt[n]{3} - \sqrt[n]{2}\right]$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right] = \lim_{n\to\infty} \left[n\left(\sqrt[n]{3} - 1\right) - n\left(\sqrt[n]{2} - 1\right)\right]$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right]\sqrt[n]{2} = \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right] \cdot \lim_{n\to\infty} \sqrt[n]{2}$$

Riešené limity – 01, 02

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$

$$= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}$$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{3} - \sqrt[n]{2}\right]$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right] = \lim_{n\to\infty} \left[n\left(\sqrt[n]{3} - 1\right) - n\left(\sqrt[n]{2} - 1\right)\right]$$

$$= \lim_{n\to\infty} n\left(\sqrt[n]{3} - 1\right) - \lim_{n\to\infty} n\left(\sqrt[n]{2} - 1\right)$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right]\sqrt[n]{2} = \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right] \cdot \lim_{n\to\infty} \sqrt[n]{2} = \ln\frac{3}{2} \cdot 1$$

Riešené limity – 01, 02

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2}$

$$= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{1+0}{1-0}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{1+0}{1-0}$$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{3} - \sqrt[n]{2}\right] = \ln 3 - ln\, 2 = \ln \frac{3}{2}$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right] = \lim_{n\to\infty} \left[n\left(\sqrt[n]{3} - 1\right) - n\left(\sqrt[n]{2} - 1\right)\right]$$

$$= \lim_{n\to\infty} n\left(\sqrt[n]{3} - 1\right) - \lim_{n\to\infty} n\left(\sqrt[n]{2} - 1\right) = \ln 3 - ln\, 2 = \ln \frac{3}{2}.$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right]\sqrt[n]{2} = \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right] \cdot \lim_{n\to\infty} \sqrt[n]{2} = \ln \frac{3}{2} \cdot 1 = \ln \frac{3}{2}.$$

Riešené limity – 01, 02

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^2-2} = 1$

$$= \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2(1-\frac{2}{n^2})} = \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = 1 \cdot \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{1+0}{1-0} = 1.$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^2-2} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1-\frac{2}{n^2}} = \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{1+0}{1-0} = 1.$$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{3} - \sqrt[n]{2}\right] = \ln 3 - ln\, 2 = \ln \frac{3}{2}$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1 + 1 - \sqrt[n]{2}\right] = \lim_{n\to\infty} \left[n\left(\sqrt[n]{3} - 1\right) - n\left(\sqrt[n]{2} - 1\right)\right]$$

$$= \lim_{n\to\infty} n\left(\sqrt[n]{3} - 1\right) - \lim_{n\to\infty} n\left(\sqrt[n]{2} - 1\right) = \ln 3 - ln\, 2 = \ln \frac{3}{2}.$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right]\sqrt[n]{2} = \lim_{n\to\infty} n\left[\sqrt[n]{\frac{3}{2}} - 1\right] \cdot \lim_{n\to\infty} \sqrt[n]{2} = \ln \frac{3}{2} \cdot 1 = \ln \frac{3}{2}.$$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n}$$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3}$$

$$\lim_{n\to\infty} \left[\sqrt{n+1}-n\right]$$

Riešené limity – 03, 04, 05

$\lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n}$

$= \lim\limits_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$

$\lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3}$

$= \lim\limits_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}}$

$\lim\limits_{n\to\infty} \left[\sqrt{n+1} - n\right]$

$= \lim\limits_{n\to\infty} \sqrt{n}\left[\sqrt{1+\frac{1}{n}} - \sqrt{n}\right]$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n}$$

$$= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}}$$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}}$$

$$\lim_{n\to\infty} \left[\sqrt{n+1} - n\right]$$

$$= \lim_{n\to\infty} \sqrt{n}\left[\sqrt{1+\frac{1}{n}} - \sqrt{n}\right] = \infty\left[\sqrt{1+\frac{1}{\infty}} - \infty\right]$$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n}$$

$$= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}}$$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}}$$

$$\lim_{n\to\infty} \left[\sqrt{n+1}-n\right]$$

$$= \lim_{n\to\infty} \sqrt{n}\left[\sqrt{1+\frac{1}{n}}-\sqrt{n}\right] = \infty\left[\sqrt{1+\frac{1}{\infty}}-\infty\right] = \infty(\sqrt{1+0}-\infty)$$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n}$$

$$= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0}$$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0}$$

$$\lim_{n\to\infty} \left[\sqrt{n+1}-n\right]$$

$$= \lim_{n\to\infty} \sqrt{n}\left[\sqrt{1+\frac{1}{n}} - \sqrt{n}\right] = \infty\left[\sqrt{1+\frac{1}{\infty}}-\infty\right] = \infty(\sqrt{1+0}-\infty)$$

$$= \infty(1-\infty)$$

Riešené limity – 03, 04, 05

$$\lim_{n\to\infty} \frac{n^3-2}{n^2+n} = \infty$$

$$= \lim_{n\to\infty} \frac{n^3-2}{n^2+n} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n-\frac{2}{n^2}}{1+\frac{1}{n}} = \frac{\infty-\frac{2}{\infty}}{1+\frac{1}{\infty}} = \frac{\infty-0}{1+0} = \infty.$$

$$\lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} = 0$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^4-n^3} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}} = \frac{\frac{1}{\infty}+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{0+0}{1-0} = 0.$$

$$\lim_{n\to\infty} \left[\sqrt{n+1} - n\right] = -\infty$$

$$= \lim_{n\to\infty} \sqrt{n}\left[\sqrt{1+\frac{1}{n}} - \sqrt{n}\right] = \infty\left[\sqrt{1+\frac{1}{\infty}}-\infty\right] = \infty(\sqrt{1+0}-\infty)$$
$$= \infty(1-\infty) = -\infty.$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty} \left[\sqrt{n+1} - \sqrt{n}\right]$$

$$\lim_{n\to\infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

$$\lim_{n\to\infty} \frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

Riešené limity – 06, 07, 08

$\lim\limits_{n\to\infty} \left[\sqrt{n+1} - \sqrt{n}\right]$

$$= \lim_{n\to\infty} \left[\sqrt{n+1} - \sqrt{n}\right] \cdot \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$$

$\lim\limits_{n\to\infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}}$

$$= \lim_{n\to\infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}} \cdot \frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$\lim\limits_{n\to\infty} \frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$

$$= \lim_{n\to\infty} \frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}} \cdot \frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}}$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

$$=\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}=\lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}}$$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

$$=\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}}=\lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}}$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

$$=\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}=\lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3}$$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

$$=\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}}=\lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3}$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$=\frac{1}{\infty+\infty}$$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}$$

$$=\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}=\lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3}=\frac{0+1}{2\cdot 0+3}$$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}$$

$$=\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}}=\lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3}$$

$$=\frac{0+1}{-2\cdot 0-3}$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$$
$$=\frac{1}{\infty+\infty}=\frac{1}{\infty}$$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}=\frac{1}{3}$$

$$=\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}=\lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3}=\frac{0+1}{2\cdot0+3}=\frac{1}{3}.$$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}=-\frac{1}{3}$$

$$=\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}}=\lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3}$$
$$=\frac{0+1}{-2\cdot0-3}=-\frac{1}{3}.$$

Riešené limity – 06, 07, 08

$$\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]=0$$

$$=\lim_{n\to\infty}\left[\sqrt{n+1}-\sqrt{n}\right]\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}}=\lim_{n\to\infty}\frac{1}{\sqrt{n+1}+\sqrt{n}}$$
$$=\frac{1}{\infty+\infty}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}=\frac{1}{3}$$

$$=\lim_{n\to\infty}\frac{2^n+3^n}{2^{n+1}+3^{n+1}}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}=\lim_{n\to\infty}\frac{\frac{2^n}{3^n}+\frac{3^n}{3^n}}{\frac{2^{n+1}}{3^n}+\frac{3^{n+1}}{3^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{2(\frac{2}{3})^n+3}=\frac{0+1}{2\cdot0+3}=\frac{1}{3}.$$

$$\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}=-\frac{1}{3}$$

$$=\lim_{n\to\infty}\frac{(-2)^n+(-3)^n}{(-2)^{n+1}+(-3)^{n+1}}\cdot\frac{\frac{1}{(-3)^n}}{\frac{1}{(-3)^n}}=\lim_{n\to\infty}\frac{\frac{(-2)^n}{(-3)^n}+\frac{(-3)^n}{(-3)^n}}{\frac{(-2)^{n+1}}{(-3)^n}+\frac{(-3)^{n+1}}{(-3)^n}}=\lim_{n\to\infty}\frac{(\frac{2}{3})^n+1}{-2(\frac{2}{3})^n-3}$$
$$=\frac{0+1}{-2\cdot0-3}=-\frac{1}{3}.$$

Riešené limity – 09, 10, 11

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$$

Riešené limity – 09, 10, 11

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{2}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{n^2+1}{n}}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{\sqrt{n}+1}{n}}$$

Riešené limity – 09, 10, 11

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{2}=\left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{2}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{n^2+1}{n}}=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{n+\frac{1}{n}}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{\sqrt{n}+1}{n}}=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{\sqrt{n}}+\frac{1}{n}}$$

Riešené limity – 09, 10, 11

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}=e^2$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^2=\left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^2=e^2.$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{n^2+1}{n}}=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{n+\frac{1}{n}}=\left[\begin{array}{l}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e\\ \lim_{n\to\infty}\left(n+\frac{1}{n}\right)=\infty+0=\infty\end{array}\right]$$

$$=e^{\infty}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{\sqrt{n}+1}{n}}=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{\sqrt{n}}+\frac{1}{n}}=\left[\begin{array}{l}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e\\ \lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}+\frac{1}{n}\right)=0+0=0\end{array}\right]$$

$$=e^{0}$$

Riešené limity – 09, 10, 11

$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{2n} = e^2$

$$= \lim_{n\to\infty} \left[\left(1 + \frac{1}{n}\right)^n\right]^2 = \left[\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n\right]^2 = e^2.$$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n^2+1}$

$$= \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^n\right]^{\frac{n^2+1}{n}} = \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^n\right]^{n+\frac{1}{n}} = \left[\begin{array}{l} \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \\ \lim_{n\to\infty} \left(n+\frac{1}{n}\right) = \infty+0=\infty \end{array}\right]$$
$$= e^{\infty}$$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{\sqrt{n}+1}$

$$= \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^n\right]^{\frac{\sqrt{n}+1}{n}} = \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^n\right]^{\frac{1}{\sqrt{n}}+\frac{1}{n}} = \left[\begin{array}{l} \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \\ \lim_{n\to\infty} \left(\frac{1}{\sqrt{n}}+\frac{1}{n}\right) = 0+0=0 \end{array}\right]$$
$$= e^{0}$$

Riešené limity – 09, 10, 11

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}=e^2$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^2=\left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^2=e^2.$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n^2+1}=\infty$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{n^2+1}{n}}=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{n+\frac{1}{n}}=\left[\begin{array}{l}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e\\ \lim_{n\to\infty}\left(n+\frac{1}{n}\right)=\infty+0=\infty\end{array}\right]$$
$$=e^{\infty}=\infty.$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\sqrt{n}+1}=1$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{\sqrt{n}+1}{n}}=\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{\sqrt{n}}+\frac{1}{n}}=\left[\begin{array}{l}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e\\ \lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}+\frac{1}{n}\right)=0+0=0\end{array}\right]$$
$$=e^{0}=1.$$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}$$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1 \geq \left(\frac{2}{3}\right)^n$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{matrix}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{matrix}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$=\lim_{n\to\infty}\frac{n}{2n+4}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1\geq\left(\frac{2}{3}\right)^n \Rightarrow -\left(\frac{2}{3}\right)^1\leq-\left(\frac{2}{3}\right)^n$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$=\lim_{n\to\infty}\frac{n}{2n+4}=\lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1\geq\left(\frac{2}{3}\right)^n \Rightarrow -\left(\frac{2}{3}\right)^1\leq-\left(\frac{2}{3}\right)^n \Rightarrow \frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$=\lim_{n\to\infty}\frac{n}{2n+4}=\lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{2+\frac{4}{n}}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1\geq\left(\frac{2}{3}\right)^n \Rightarrow -\left(\frac{2}{3}\right)^1\leq-\left(\frac{2}{3}\right)^n \Rightarrow \frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow \forall n\in N$ platí: $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{matrix}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{matrix}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$=\lim_{n\to\infty}\frac{n}{2n+4}=\lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{2+\frac{4}{n}}=\frac{1}{2+\frac{4}{\infty}}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1\geq\left(\frac{2}{3}\right)^n \Rightarrow -\left(\frac{2}{3}\right)^1\leq-\left(\frac{2}{3}\right)^n \Rightarrow \frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow \forall n\in N$ platí: $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

$\Rightarrow 1=\lim_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim_{n\to\infty}\sqrt[n]{1}=1$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$=\lim_{n\to\infty}\frac{n}{2n+4}=\lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{2+\frac{4}{n}}=\frac{1}{2+\frac{4}{\infty}}=\frac{1}{2+0}$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1\geq\left(\frac{2}{3}\right)^n \;\Rightarrow\; -\left(\frac{2}{3}\right)^1\leq-\left(\frac{2}{3}\right)^n \;\Rightarrow\; \frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow \forall n\in N$ platí: $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

$\Rightarrow 1=\lim_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim_{n\to\infty}\sqrt[n]{1}=1 \;\Rightarrow\; \lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=1.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]=\frac{1}{2}$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$=\lim_{n\to\infty}\frac{n}{2n+4}=\lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{2+\frac{4}{n}}=\frac{1}{2+\frac{4}{\infty}}=\frac{1}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\cdot 1$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1\geq\left(\frac{2}{3}\right)^n \Rightarrow -\left(\frac{2}{3}\right)^1\leq-\left(\frac{2}{3}\right)^n \Rightarrow \frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow \forall n\in N$ platí: $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

$\Rightarrow 1=\lim_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim_{n\to\infty}\sqrt[n]{1}=1 \Rightarrow \lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=1.$

Riešené limity – 12, 13

$$\lim_{n\to\infty}\left[\frac{n}{2}-\frac{1+2+3+\cdots+n}{n+2}\right]=\frac{1}{2}$$

$$=\left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+\cdots+n=\frac{(1+n)n}{2}\end{array}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{\frac{n(n+1)}{2}}{n+2}\right]=\lim_{n\to\infty}\left[\frac{n}{2}-\frac{n(n+1)}{2(n+2)}\right]=\lim_{n\to\infty}\frac{n(n+2)-n(n+1)}{2(n+2)}$$

$$=\lim_{n\to\infty}\frac{n}{2n+4}=\lim_{n\to\infty}\frac{n}{2n+4}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1}{2+\frac{4}{n}}=\frac{1}{2+\frac{4}{\infty}}=\frac{1}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\sqrt[n]{3^n-2^n}=3$$

$$=\lim_{n\to\infty}\sqrt[n]{3^n\left(1-\frac{2^n}{3^n}\right)}=\lim_{n\to\infty}3\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=3\cdot 1=3.$$

$\forall n\in N$ platí: $\left(\frac{2}{3}\right)^1\geq\left(\frac{2}{3}\right)^n \Rightarrow -\left(\frac{2}{3}\right)^1\leq-\left(\frac{2}{3}\right)^n \Rightarrow \frac{1}{3}=1-\frac{2}{3}\leq 1-\left(\frac{2}{3}\right)^n\leq 1.$

$\Rightarrow \forall n\in N$ platí: $\sqrt[n]{\frac{1}{3}}\leq\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\sqrt[n]{1}.$

$\Rightarrow 1=\lim_{n\to\infty}\sqrt[n]{\frac{1}{3}}\leq\lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}\leq\lim_{n\to\infty}\sqrt[n]{1}=1 \Rightarrow \lim_{n\to\infty}\sqrt[n]{1-\left(\frac{2}{3}\right)^n}=1.$

Riešené limity – 14, 15, 16

$\lim\limits_{n\to\infty} \sqrt[n]{2^n+1}$

$\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n]{3}\right]$

Riešené limity – 14, 15, 16

$$\lim_{n\to\infty} \sqrt[n]{2^n+1}$$

$\forall n \in N$ platí: $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n$

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2} - \sqrt[n]{3}\right]$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right]$$

Riešené limity – 14, 15, 16

$$\lim_{n\to\infty} \sqrt[n]{2^n+1}$$

$\forall n \in N$ platí: $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n \Rightarrow 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1}$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2} - \sqrt[n]{3}\right]$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right] = \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1\right] - \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1\right]$$

Riešené limity – 14, 15, 16

$\lim\limits_{n\to\infty} \sqrt[n]{2^n+1}$

$\forall n\in N$ platí: $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n \Rightarrow 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$\Rightarrow 2 = \lim\limits_{n\to\infty} 2 \leq \lim\limits_{n\to\infty} \sqrt[n]{2^n+1} \leq \lim\limits_{n\to\infty} 2\cdot\sqrt[n]{2} = 2\cdot 1 = 2$

$\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$

$$= \lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1}$$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{2} - \sqrt[n]{3}\right]$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right] = \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1\right] - \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1\right]$$

$$= \left[\begin{array}{c}\lim\limits_{n\to\infty} n(\sqrt[n]{a}-1) = \ln a \\ \text{pre všetky } a>0\end{array}\right]$$

Riešené limity – 14, 15, 16

$$\lim_{n\to\infty} \sqrt[n]{2^n+1} = 2$$

$\forall n \in N$ platí: $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n \Rightarrow 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$$\Rightarrow 2 = \lim_{n\to\infty} 2 \leq \lim_{n\to\infty} \sqrt[n]{2^n+1} \leq \lim_{n\to\infty} 2\cdot\sqrt[n]{2} = 2\cdot 1 = 2 \Rightarrow \lim_{n\to\infty} \sqrt[n]{2^n+1} = 2.$$

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$$

$$= \lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1} = \frac{\frac{1}{\infty}-1}{\frac{1}{\infty}+1}$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2} - \sqrt[n]{3}\right]$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right] = \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1\right] - \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1\right]$$

$$= \left[\begin{matrix}\lim_{n\to\infty} n(\sqrt[n]{a}-1) = \ln a \\ \text{pre všetky } a > 0\end{matrix}\right] = \ln 2 - \ln 3$$

Riešené limity – 14, 15, 16

$\lim\limits_{n\to\infty} \sqrt[n]{2^n+1} = 2$

$\forall n\in N$ platí: $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n \Rightarrow 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$\Rightarrow 2 = \lim\limits_{n\to\infty} 2 \leq \lim\limits_{n\to\infty} \sqrt[n]{2^n+1} \leq \lim\limits_{n\to\infty} 2\cdot\sqrt[n]{2} = 2\cdot 1 = 2 \Rightarrow \lim\limits_{n\to\infty} \sqrt[n]{2^n+1} = 2.$

$\lim\limits_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}}$

$$= \lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1} = \frac{\frac{1}{\infty}-1}{\frac{1}{\infty}+1} = \frac{0-1}{0+1}$$

$\lim\limits_{n\to\infty} n\left[\sqrt[n]{2} - \sqrt[n]{3}\right] = \ln\frac{2}{3}$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right] = \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1\right] - \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1\right]$$

$$= \left[\begin{array}{c} \lim\limits_{n\to\infty} n(\sqrt[n]{a}-1) = \ln a \\ \text{pre všetky } a > 0 \end{array}\right] = \ln 2 - \ln 3 = \ln\frac{2}{3}.$$

Riešené limity – 14, 15, 16

$$\lim_{n\to\infty} \sqrt[n]{2^n+1} = 2$$

$\forall n \in N$ platí: $2^n < 2^n+1 < 2^n+2^n = 2\cdot 2^n \Rightarrow 2 = \sqrt[n]{2^n} \leq \sqrt[n]{2^n+1} \leq \sqrt[n]{2\cdot 2^n} = 2\cdot\sqrt[n]{2}.$

$$\Rightarrow 2 = \lim_{n\to\infty} 2 \leq \lim_{n\to\infty} \sqrt[n]{2^n+1} \leq \lim_{n\to\infty} 2\cdot\sqrt[n]{2} = 2\cdot 1 = 2 \Rightarrow \lim_{n\to\infty} \sqrt[n]{2^n+1} = 2.$$

$$\lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} = -1$$

$$= \lim_{n\to\infty} \frac{1-\sqrt{n}}{1+\sqrt{n}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}}-1}{\frac{1}{\sqrt{n}}+1} = \frac{\frac{1}{\sqrt{\infty}}-1}{\frac{1}{\sqrt{\infty}}+1} = \frac{\frac{1}{\infty}-1}{\frac{1}{\infty}+1} = \frac{0-1}{0+1} = -1.$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2} - \sqrt[n]{3}\right] = \ln\frac{2}{3}$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1 - \sqrt[n]{3} + 1\right] = \lim_{n\to\infty} n\left[\sqrt[n]{2} - 1\right] - \lim_{n\to\infty} n\left[\sqrt[n]{3} - 1\right]$$

$$= \left[\begin{array}{c} \lim_{n\to\infty} n(\sqrt[n]{a}-1) = \ln a \\ \text{pre všetky } a > 0 \end{array}\right] = \ln 2 - \ln 3 = \ln\frac{2}{3}.$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty} \left[\frac{n^2}{n+2} - \frac{n^2}{n+3} \right]$$

$$\lim_{n\to\infty} \frac{n+3^n}{n-3^n}$$

$$\lim_{n\to\infty} \frac{n+3^{-n}}{n-3^{-n}}$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$$

$$=\lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n}{3^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0\right]$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$$

$$=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$$

$$=\lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}=\lim_{n\to\infty}\frac{n^2(n+3-n-2)}{n^2+2n+3n+6}$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n}{3^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0\right]=\lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$$

$$=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}}$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$$

$$=\lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}=\lim_{n\to\infty}\frac{n^2(n+3-n-2)}{n^2+2n+3n+6}=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n}{3^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0\right]=\lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$=\lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$$

$$=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}}=\frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}}$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$$

$$=\lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}=\lim_{n\to\infty}\frac{n^2(n+3-n-2)}{n^2+2n+3n+6}=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}$$

$$=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n}{3^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0\right]=\lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$=\lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}=\frac{0+1}{0-1}$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$$

$$=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{1}{n\cdot3^n}}{1-\frac{1}{n\cdot3^n}}=\frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}}=\frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$$

$$=\lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}=\lim_{n\to\infty}\frac{n^2(n+3-n-2)}{n^2+2n+3n+6}=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}$$

$$=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}=\lim_{n\to\infty}\frac{1}{1+\frac{5}{n}+\frac{6}{n^2}}$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}=-1$$

$$=\left[\text{Označme } a_n=\tfrac{n}{3^n} \text{ pre všetky } n\in N \;\middle|\; \left.\begin{aligned}&\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\tfrac{n}{3^n}}=\lim_{n\to\infty}\tfrac{\sqrt[n]{n}}{3}=\tfrac{1}{3}<1\\&\lim_{n\to\infty}\tfrac{a_{n+1}}{a_n}=\lim_{n\to\infty}\tfrac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim_{n\to\infty}\tfrac{n+1}{3n}=\tfrac{1}{3}<1\end{aligned}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\tfrac{n}{3^n}=0\right]=\lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$=\lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}=\frac{0+1}{0-1}=-1.$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}$$

$$=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}}=\frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}}=\frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}=\frac{1+0}{1-0}$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$$

$$=\lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}=\lim_{n\to\infty}\frac{n^2(n+3-n-2)}{n^2+2n+3n+6}=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}$$

$$=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}=\lim_{n\to\infty}\frac{1}{1+\frac{5}{n}+\frac{6}{n^2}}=\frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}}$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}=-1$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n}{3^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim\limits_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0\right]=\lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$=\lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}=\frac{0+1}{0-1}=-1.$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}=1$$

$$=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}}=\frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}}=\frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}=\frac{1+0}{1-0}=1.$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]$$

$$= \lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)} = \lim_{n\to\infty}\frac{n^2(n+3-n-2)}{n^2+2n+3n+6} = \lim_{n\to\infty}\frac{n^2}{n^2+5n+6}$$

$$= \lim_{n\to\infty}\frac{n^2}{n^2+5n+6}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty}\frac{1}{1+\frac{5}{n}+\frac{6}{n^2}} = \frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}} = \frac{1}{1+0+0}$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n} = -1$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n}{3^n} & \left.\begin{array}{l}\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1\\ \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1\end{array}\right\}\Rightarrow \lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0 \\ \text{pre všetky } n\in N & \end{array}\right] = \lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$= \lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1} = \frac{0+1}{0-1} = -1.$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}} = 1$$

$$= \lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}} = \frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}} = \frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}} = \frac{1+0}{1-0} = 1.$$

Riešené limity – 17, 18, 19

$$\lim_{n\to\infty}\left[\frac{n^2}{n+2}-\frac{n^2}{n+3}\right]=1$$

$$=\lim_{n\to\infty}\frac{n^2(n+3)-n^2(n+2)}{(n+2)(n+3)}=\lim_{n\to\infty}\frac{n^2(n+3-n-2)}{n^2+2n+3n+6}=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}$$

$$=\lim_{n\to\infty}\frac{n^2}{n^2+5n+6}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}=\lim_{n\to\infty}\frac{1}{1+\frac{5}{n}+\frac{6}{n^2}}=\frac{1}{1+\frac{5}{\infty}+\frac{6}{\infty}}=\frac{1}{1+0+0}=1.$$

$$\lim_{n\to\infty}\frac{n+3^n}{n-3^n}=-1$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n}{3^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n}{3^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n}}{3}=\frac{1}{3}<1 \\ \text{pre všetky } n\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}}=\lim_{n\to\infty}\frac{n+1}{3n}=\frac{1}{3}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{3^n}=0\right]=\lim_{n\to\infty}\frac{n+3^n}{n-3^n}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$=\lim_{n\to\infty}\frac{\frac{n}{3^n}+1}{\frac{n}{3^n}-1}=\frac{0+1}{0-1}=-1.$$

$$\lim_{n\to\infty}\frac{n+3^{-n}}{n-3^{-n}}=1$$

$$=\lim_{n\to\infty}\frac{n+\frac{1}{3^n}}{n-\frac{1}{3^n}}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{1}{n\cdot 3^n}}{1-\frac{1}{n\cdot 3^n}}=\frac{1+\frac{1}{\infty\cdot\infty}}{1-\frac{1}{\infty\cdot\infty}}=\frac{1+\frac{1}{\infty}}{1-\frac{1}{\infty}}=\frac{1+0}{1-0}=1.$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{array}{c}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{array}\right]$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{array}{c}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{matrix}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\left[\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right]}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{array}{c}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\left[\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right]}$$

$$=\frac{2-0}{\infty\left[\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right]}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{array}{c}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\left[\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right]}$$

$$=\frac{2-0}{\infty\left[\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right]}=\frac{2}{\infty[1+1\cdot1+1]}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{array}{c}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\left[\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right]}$$

$$=\frac{2-0}{\infty\left[\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right]}=\frac{2}{\infty[1+1\cdot1+1]}=\frac{2}{\infty\cdot3}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{array}{c}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\left[\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right]}$$

$$=\frac{2-0}{\infty\left[\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right]}=\frac{2}{\infty[1+1\cdot1+1]}=\frac{2}{\infty\cdot3}=\frac{2}{\infty}$$

Riešené limity – 20

$$\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]=0$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^2+3n+1}-\sqrt[3]{n^2+n+2}\right]\cdot\frac{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n^2+3n+1)-(n^2+n+2)}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt[3]{(n^2+3n+1)^2}+\sqrt[3]{n^2+3n+1}\,\sqrt[3]{n^2+n+2}+\sqrt[3]{(n^2+n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{n(2-\frac{1}{n})}{\sqrt[3]{n^2\cdot n^2}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\left[\begin{array}{c}\frac{n}{\sqrt[3]{n^2\cdot n^2}}=\frac{n}{\sqrt[3]{n^3\cdot n}}=\frac{1}{\sqrt[3]{n}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt[3]{n}\left[\sqrt[3]{(1+\frac{3}{n}+\frac{1}{n^2})^2}+\sqrt[3]{1+\frac{3}{n}+\frac{1}{n^2}}\,\sqrt[3]{1+\frac{1}{n}+\frac{2}{n^2}}+\sqrt[3]{(1+\frac{1}{n}+\frac{2}{n^2})^2}\right]}$$

$$=\frac{2-\frac{1}{\infty}}{\sqrt[3]{\infty}\left[\sqrt[3]{(1+\frac{3}{\infty}+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{3}{\infty}+\frac{1}{\infty}}\,\sqrt[3]{1+\frac{1}{\infty}+\frac{2}{\infty}}+\sqrt[3]{(1+\frac{1}{\infty}+\frac{2}{\infty})^2}\right]}$$

$$=\frac{2-0}{\infty\left[\sqrt[3]{(1+0+0)^2}+\sqrt[3]{1+0+0}\,\sqrt[3]{1+0+0}+\sqrt[3]{(1+0+0)^2}\right]}=\frac{2}{\infty[1+1\cdot1+1]}=\frac{2}{\infty\cdot3}=\frac{2}{\infty}=0.$$

Riešené limity – 21, 22

$$\lim_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$$

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$$

Riešené limity – 21, 22

$$\lim_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$$

$$= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}}$$

$$= \left[\lim_{n\to\infty} \frac{8n+1}{4n-3} \right]$$

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6$$

$$= \lim_{n\to\infty} \left(\frac{2n-1}{2n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right)^6$$

Riešené limity – 21, 22

$$\lim_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$$

$$= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}}$$

$$= \left[\lim_{n\to\infty} \frac{8n+1}{4n-3} = \lim_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} \right]$$

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^6$$

$$= \lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6$$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$

$$= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}}$$

$$= \left[\lim\limits_{n\to\infty} \frac{8n+1}{4n-3} = \lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} \right]$$

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$

$$= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6$$

$$= \lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6$$

Riešené limity – 21, 22

$$\lim_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$$

$$= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}}$$

$$= \left[\lim_{n\to\infty} \frac{8n+1}{4n-3} = \lim_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} \right]$$

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6 = \left(1-\frac{4}{\infty}\right)^6$$

$$= \lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6$$

Riešené limity – 21, 22

$$\lim_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$$

$$= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}}$$

$$= \left[\lim_{n\to\infty} \frac{8n+1}{4n-3} = \lim_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0} \right]$$

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6$$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6 = \left(1-\frac{4}{\infty}\right)^6 = (1-0)^6$$
$$= 1^6$$

$$= \lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6 = 1^6$$

Riešené limity – 21, 22

$$\lim_{n\to\infty} 4^{\frac{8n+1}{4n-3}}$$

$$= 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty}\frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty}\frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}} = 4^2$$

$$= \left[\lim_{n\to\infty}\frac{8n+1}{4n-3} = \lim_{n\to\infty}\frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0} = 2\right] = 4^2$$

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^6 = 1$$

$$= \lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^6 = \lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6 = \left(1-\frac{4}{\infty}\right)^6 = (1-0)^6$$

$$= 1^6 = 1.$$

$$= \lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim_{n\to\infty}\left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6 = 1^6 = 1.$$

Riešené limity – 21, 22

$\lim\limits_{n\to\infty} 4^{\frac{8n+1}{4n-3}} = 16$

$$= 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}} = 4^{\lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}} = 4^{\lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}}} = 4^{\frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}}} = 4^{\frac{8+0}{4-0}} = 4^2 = 16.$$

$$= \left[\lim\limits_{n\to\infty} \frac{8n+1}{4n-3} = \lim\limits_{n\to\infty} \frac{8n+1}{4n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{8+\frac{1}{n}}{4-\frac{3}{n}} = \frac{8+\frac{1}{\infty}}{4-\frac{3}{\infty}} = \frac{8+0}{4-0} = 2\right] = 4^2 = 16.$$

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^6 = 1$

$$= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^6 = \lim\limits_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^6 = \left(1-\frac{4}{2\cdot\infty+3}\right)^6 = \left(1-\frac{4}{\infty}\right)^6 = (1-0)^6$$
$$= 1^6 = 1.$$

$$= \lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}\right)^6 = \lim\limits_{n\to\infty} \left(\frac{2-\frac{1}{n}}{2+\frac{3}{n}}\right)^6 = \left(\frac{2-\frac{1}{\infty}}{2+\frac{3}{\infty}}\right)^6 = \left(\frac{2-0}{2+0}\right)^6 = 1^6 = 1.$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+3+\cdots+n = \frac{(1+n)n}{2} \end{array} \right]$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

$$= \left[\begin{array}{l} \text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2 \end{array} \right]$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+3+\cdots+n = \frac{(1+n)n}{2}\end{array}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

$$= \left[\begin{array}{l}\text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2}\\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2\end{array}\right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+3+\cdots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

$$= \left[\begin{array}{l} \text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2 \end{array} \right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{matrix} \text{Aritmetický rad} \\ 1+2+3+\cdots+n = \frac{(1+n)n}{2} \end{matrix} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{matrix} n^2 = n\cdot n = \sqrt{n^4} \\ \text{pre všetky } n\in N \end{matrix} \right]$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

$$= \left[\begin{matrix} \text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2 \end{matrix} \right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+3+\cdots+n = \frac{(1+n)n}{2}\end{array}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} n^2 = n\cdot n = \sqrt{n^4}\\ \text{pre všetky } n\in N\end{array}\right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

$$= \left[\begin{array}{ll}\text{Aritmetické rady} & 1+2+3+\cdots+n = \frac{(1+n)n}{2}\\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2 & \end{array}\right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+3+\cdots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} n^2 = n\cdot n = \sqrt{n^4} \\ \text{pre všetky } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

$$= \left[\begin{array}{l} \text{Aritmetické rady} \qquad\qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2 \end{array} \right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{matrix} \text{Aritmetický rad} \\ 1+2+3+\cdots+n = \frac{(1+n)n}{2} \end{matrix} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{matrix} n^2 = n\cdot n = \sqrt{n^4} \\ \text{pre všetky } n\in N \end{matrix} \right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$$

$$= \left[\begin{matrix} \text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2 \end{matrix} \right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0}$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{array}{c}\text{Aritmetický rad}\\ 1+2+3+\cdots+n = \frac{(1+n)n}{2}\end{array}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} n^2 = n\cdot n = \sqrt{n^4} \\ \text{pre všetky } n\in N\end{array}\right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

$$= \left[\begin{array}{l}\text{Aritmetické rady} \qquad\qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2\end{array}\right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{matrix}\text{Aritmetický rad}\\ 1+2+3+\cdots+n = \frac{(1+n)n}{2}\end{matrix}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{matrix} n^2 = n\cdot n = \sqrt{n^4} \\ \text{pre všetky } n\in N\end{matrix}\right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

$$= \left[\begin{matrix}\text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2\end{matrix}\right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$$

$$= \left[\begin{array}{c} \text{Aritmetický rad} \\ 1+2+3+\cdots+n = \frac{(1+n)n}{2} \end{array} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} n^2 = n\cdot n = \sqrt{n^4} \\ \text{pre všetky } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}} = \frac{1+0}{2\cdot 3}$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

$$= \left[\begin{array}{l} \text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot 1-1)+(2\cdot 2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2 \end{array} \right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Riešené limity – 23, 24

$$\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}} = \frac{1}{6}$$

$$= \left[\begin{array}{c}\text{Aritmetický rad} \\ 1+2+3+\cdots+n = \frac{(1+n)n}{2}\end{array}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{\sqrt{9n^4+1}} = \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} = \left[\begin{array}{l} n^2 = n\cdot n = \sqrt{n^4} \\ \text{pre všetky } n\in N\end{array}\right]$$

$$= \lim_{n\to\infty} \frac{n(n+1)}{2\sqrt{9n^4+1}} \cdot \frac{\frac{1}{n\cdot n}}{\frac{1}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{n+1}{n}}{2\sqrt{\frac{9n^4+1}{n^4}}} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}} = \frac{1+\frac{1}{\infty}}{2\sqrt{9+\frac{1}{\infty}}}$$

$$= \frac{1+0}{2\sqrt{9+0}} = \frac{1+0}{2\cdot 3} = \frac{1}{6}.$$

$$\lim_{n\to\infty} \frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n} = 2$$

$$= \left[\begin{array}{l}\text{Aritmetické rady} \qquad 1+2+3+\cdots+n = \frac{(1+n)n}{2} \\ 1+3+5+\cdots+(2n-1) = (2\cdot1-1)+(2\cdot2-1)+\cdots+(2n-1) = \frac{(1+2n-1)n}{2} = n^2\end{array}\right] = \lim_{n\to\infty} \frac{n^2}{\frac{(1+n)n}{2}}$$

$$= \lim_{n\to\infty} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{2n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}} = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1+0} = 2.$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \right]$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1}$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$$

$$= \frac{1}{6} \lim_{n\to\infty} \left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$$

$$= \frac{1}{6} \lim_{n\to\infty} (1+\tfrac{1}{n})(2+\tfrac{1}{n}) = \tfrac{1}{6}(1+\tfrac{1}{\infty})(2+\tfrac{1}{\infty})$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$$

$$= -\sqrt{\infty} - \sqrt{\infty}$$

Riešené limity – 25, 26

$\lim\limits_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$$

$$= \frac{1}{6}\lim_{n\to\infty}(1+\tfrac{1}{n})(2+\tfrac{1}{n}) = \frac{1}{6}(1+\tfrac{1}{\infty})(2+\tfrac{1}{\infty}) = \frac{1}{6}(1+0)(2+0)$$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$$

$$= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3}$$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \tfrac{n(n+1)(2n+1)}{6} \right] = \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$$

$$= \frac{1}{6} \lim_{n\to\infty} (1+\tfrac{1}{n})(2+\tfrac{1}{n}) = \tfrac{1}{6}(1+\tfrac{1}{\infty})(2+\tfrac{1}{\infty}) = \tfrac{1}{6}(1+0)(2+0) = \tfrac{1}{6}\cdot 1\cdot 2$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} = -\infty$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} \cdot \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$$

$$= -\sqrt{\infty} - \sqrt{\infty} = -\infty - \infty = -\infty.$$

Riešené limity – 25, 26

$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3} = \frac{1}{3}$$

$$= \left[1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}\right] = \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{n^3} = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n\cdot n}$$

$$= \frac{1}{6}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) = \frac{1}{6}\left(1+\frac{1}{\infty}\right)\left(2+\frac{1}{\infty}\right) = \frac{1}{6}(1+0)(2+0) = \frac{1}{6}\cdot 1\cdot 2 = \frac{1}{3}.$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}} = -\infty$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n+1}-\sqrt{n^2+n+2}}\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{(n^2+n+1)-(n^2+n+2)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{-1} = \frac{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}{-1}$$

$$= -\sqrt{\infty}-\sqrt{\infty} = -\infty-\infty = -\infty.$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

$$= \left[1^2+3^2+5^2+\cdots+(2n-1)^2 = \tfrac{n(2n+1)(2n-1)}{3} = \tfrac{n(4n^2-1)}{3} \right]$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

$$= \left[1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3} \right] = \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

$$= \left[1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3}\right] = \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2}$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

$$= \left[1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3} \right] = \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2}$$

$$= \lim_{n\to\infty} \left(\frac{4}{3} - \frac{1}{3n^2} \right)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n\to\infty} \sqrt{n^2} \left[\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}} \right]$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

$$= \left[1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3} \right] = \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2}$$

$$= \lim_{n\to\infty} \left(\frac{4}{3} - \frac{1}{3n^2} \right) = \frac{4}{3} - \frac{1}{\infty}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n\to\infty} \sqrt{n^2} \left[\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}} \right]$$

$$= \left[\lim_{n\to\infty} \sqrt{n^2} = \lim_{n\to\infty} n = \infty \;\middle|\; \lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0 \;\middle|\; \lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0 \right]$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}$$

$$= \left[1^2+3^2+5^2+\cdots+(2n-1)^2 = \tfrac{n(2n+1)(2n-1)}{3} = \tfrac{n(4n^2-1)}{3}\right] = \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2}$$

$$= \lim_{n\to\infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n\to\infty} \sqrt{n^2}\left[\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}}\right]$$

$$= \left[\lim_{n\to\infty} \sqrt{n^2} = \lim_{n\to\infty} n = \infty \,\middle|\, \lim_{n\to\infty} \tfrac{1}{n} = \tfrac{1}{\infty} = 0 \,\middle|\, \lim_{n\to\infty} \tfrac{1}{n^2} = \tfrac{1}{\infty^2} = \tfrac{1}{\infty} = 0\right]$$

$$= \frac{1}{2} \cdot \infty \left(\sqrt{1-0+0} + \sqrt{1-0-0}\right)$$

Riešené limity – 27, 28

$$\lim_{n\to\infty}\frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3}=\frac{4}{3}$$

$$=\left[1^2+3^2+5^2+\cdots+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}=\frac{n(4n^2-1)}{3}\right]=\lim_{n\to\infty}\frac{\frac{n(4n^2-1)}{3}}{n^3}=\lim_{n\to\infty}\frac{4n^2-1}{3n^2}$$

$$=\lim_{n\to\infty}\left(\frac{4}{3}-\frac{1}{3n^2}\right)=\frac{4}{3}-\frac{1}{\infty}=\frac{4}{3}-0=\frac{4}{3}.$$

$$\lim_{n\to\infty}\frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}}\cdot\frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}=\lim_{n\to\infty}\frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2}=\frac{1}{2}\lim_{n\to\infty}\sqrt{n^2}\left[\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{1}{n}-\frac{1}{n^2}}\right]$$

$$=\left[\lim_{n\to\infty}\sqrt{n^2}=\lim_{n\to\infty}n=\infty\;\middle|\;\lim_{n\to\infty}\frac{1}{n}=\frac{1}{\infty}=0\;\middle|\;\lim_{n\to\infty}\frac{1}{n^2}=\frac{1}{\infty^2}=\frac{1}{\infty}=0\right]$$

$$=\frac{1}{2}\cdot\infty\left(\sqrt{1-0+0}+\sqrt{1-0-0}\right)=\frac{1}{2}\cdot\infty\cdot 2$$

Riešené limity – 27, 28

$$\lim_{n\to\infty} \frac{1^2+3^2+5^2+\cdots+(2n-1)^2}{n^3} = \frac{4}{3}$$

$$= \left[1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3} = \frac{n(4n^2-1)}{3} \right] = \lim_{n\to\infty} \frac{\frac{n(4n^2-1)}{3}}{n^3} = \lim_{n\to\infty} \frac{4n^2-1}{3n^2}$$

$$= \lim_{n\to\infty} \left(\frac{4}{3} - \frac{1}{3n^2}\right) = \frac{4}{3} - \frac{1}{\infty} = \frac{4}{3} - 0 = \frac{4}{3}.$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} = \infty$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{n^2-n+1}-\sqrt{n^2-n-1}} \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}} = \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{(n^2-n+1)-(n^2-n-1)}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2-n+1}+\sqrt{n^2-n-1}}{2} = \frac{1}{2} \lim_{n\to\infty} \sqrt{n^2} \left[\sqrt{1-\frac{1}{n}+\frac{1}{n^2}} + \sqrt{1-\frac{1}{n}-\frac{1}{n^2}} \right]$$

$$= \left[\lim_{n\to\infty} \sqrt{n^2} = \lim_{n\to\infty} n = \infty \;\middle|\; \lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0 \;\middle|\; \lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{\infty^2} = \frac{1}{\infty} = 0 \right]$$

$$= \frac{1}{2} \cdot \infty \left(\sqrt{1-0+0} + \sqrt{1-0-0}\right) = \frac{1}{2} \cdot \infty \cdot 2 = \infty.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2}$$

$$= \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\tfrac{n}{n+2} = \lim_{n\to\infty} \ln\left(\tfrac{n}{n+2}\right)^n$$

$$= \lim_{n\to\infty} \ln\left(\tfrac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\tfrac{n+2-2}{n+2}\right)^n$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$$

$$= \lim_{n\to\infty} \ln\left[\left(1+\frac{-2}{n+2}\right)^{n+2}\right]^{\frac{n}{n+2}}$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \ln\left[\left(1+\frac{2}{n}\right)^n\right]^{-1}$$

$$= \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$$

$$= \lim_{n\to\infty} \ln\left[\left(1+\frac{-2}{n+2}\right)^{n+2}\right]^{\frac{n}{n+2}} = \ln\left(e^{-2}\right)^1$$

$$\lim_{n\to\infty}\left(1+\frac{-2}{n+2}\right)^{n+2} = \left[\begin{array}{c|c} \text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty \end{array}\right] = \lim_{m\to\infty}\left(1+\frac{-2}{m}\right)^m = \left[\begin{array}{c} \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R \end{array}\right] = e^{-2},$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\Big(\frac{n}{n+2}\Big)^n = \lim_{n\to\infty} \ln\Big(\frac{n+2}{n}\Big)^{-n} = \lim_{n\to\infty} \ln\Big(1+\frac{2}{n}\Big)^{-n}$$

$$= \lim_{n\to\infty} \ln\Big[\Big(1+\frac{2}{n}\Big)^n\Big]^{-1} = \left[\begin{matrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right]$$

$$= \lim_{n\to\infty} \ln\Big(\frac{n}{n+2}\Big)^n = \lim_{n\to\infty} \ln\Big(\frac{n+2-2}{n+2}\Big)^n = \lim_{n\to\infty} \ln\Big(1+\frac{-2}{n+2}\Big)^n$$

$$= \lim_{n\to\infty} \ln\Big[\Big(1+\frac{-2}{n+2}\Big)^{n+2}\Big]^{\frac{n}{n+2}} = \ln\big(e^{-2}\big)^1$$

$$\lim_{n\to\infty}\frac{n}{n+2} = \lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{n+2}{n}} = \lim_{n\to\infty}\frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2}{n}\right)^{-n} = \lim_{n\to\infty} \ln\left(1+\frac{2}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \ln\left[\left(1+\frac{2}{n}\right)^n\right]^{-1} = \left[\begin{matrix}\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right] = \ln\left(e^2\right)^{-1}$$

$$= \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty} \ln\left(1+\frac{-2}{n+2}\right)^n$$

$$= \lim_{n\to\infty} \ln\left[\left(1+\frac{-2}{n+2}\right)^{n+2}\right]^{\frac{n}{n+2}} = \ln\left(e^{-2}\right)^1$$

$$\lim_{n\to\infty}\left(1+\frac{-2}{n+2}\right)^{n+2} = \left[\begin{matrix}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{matrix}\right] = \lim_{m\to\infty}\left(1+\frac{-2}{m}\right)^m = \left[\begin{matrix}\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right] = e^{-2},$$

$$\lim_{n\to\infty}\frac{n}{n+2} = \lim_{n\to\infty}\frac{n}{n+2}\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\ln\Big(\frac{n}{n+2}\Big)^n = \lim_{n\to\infty}\ln\Big(\frac{n+2}{n}\Big)^{-n} = \lim_{n\to\infty}\ln\Big(1+\frac{2}{n}\Big)^{-n}$$

$$= \lim_{n\to\infty}\ln\Big[\Big(1+\frac{2}{n}\Big)^n\Big]^{-1} = \left[\begin{array}{c}\lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = \ln\big(e^2\big)^{-1} = \ln e^{-2}$$

$$= \lim_{n\to\infty}\ln\Big(\frac{n}{n+2}\Big)^n = \lim_{n\to\infty}\ln\Big(\frac{n+2-2}{n+2}\Big)^n = \lim_{n\to\infty}\ln\Big(1+\frac{-2}{n+2}\Big)^n$$

$$= \lim_{n\to\infty}\ln\Big[\Big(1+\frac{-2}{n+2}\Big)^{n+2}\Big]^{\frac{n}{n+2}} = \ln\big(e^{-2}\big)^1 = \ln e^{-2}$$

$$\lim_{n\to\infty}\Big(1+\frac{-2}{n+2}\Big)^{n+2} = \left[\begin{array}{c|c}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\Big(1+\frac{-2}{m}\Big)^m = \left[\begin{array}{c}\lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = e^{-2},$$

$$\lim_{n\to\infty}\frac{n}{n+2} = \lim_{n\to\infty}\frac{n}{n+2}\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big] = -2$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty} \ln\Big(\frac{n}{n+2}\Big)^n = \lim_{n\to\infty} \ln\Big(\frac{n+2}{n}\Big)^{-n} = \lim_{n\to\infty} \ln\Big(1+\frac{2}{n}\Big)^{-n}$$

$$= \lim_{n\to\infty} \ln\Big[\Big(1+\frac{2}{n}\Big)^n\Big]^{-1} = \left[\begin{array}{c}\lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = \ln\left(e^2\right)^{-1} = \ln e^{-2} = -2.$$

$$= \lim_{n\to\infty} \ln\Big(\frac{n}{n+2}\Big)^n = \lim_{n\to\infty} \ln\Big(\frac{n+2-2}{n+2}\Big)^n = \lim_{n\to\infty} \ln\Big(1+\frac{-2}{n+2}\Big)^n$$

$$= \lim_{n\to\infty} \ln\Big[\Big(1+\frac{-2}{n+2}\Big)^{n+2}\Big]^{\frac{n}{n+2}} = \ln\left(e^{-2}\right)^1 = \ln e^{-2} = -2.$$

$$\lim_{n\to\infty}\Big(1+\frac{-2}{n+2}\Big)^{n+2} = \left[\begin{array}{c|c}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\Big(1+\frac{-2}{m}\Big)^m = \left[\begin{array}{c}\lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = e^{-2},$$

$$\lim_{n\to\infty}\frac{n}{n+2} = \lim_{n\to\infty}\frac{n}{n+2}\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{1+\frac{2}{n}} = \frac{1}{1+\frac{2}{\infty}} = \frac{1}{1+0} = 1.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2}$$

$$= \lim_{n\to\infty} \ln\left(\frac{n}{n+2}\right)^n$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\frac{n+2}{n}\Big]$$

$$= \lim_{n\to\infty} \ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty} \ln\big(\tfrac{n+2-2}{n+2}\big)^n$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\tfrac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\tfrac{n+2}{n}\Big] = -\lim_{n\to\infty} n\cdot\ln\tfrac{n+2}{n}$$

$$= \lim_{n\to\infty}\ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(\tfrac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^n$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\frac{n+2}{n}\Big] = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$$

$$= -\lim_{n\to\infty}\ln\Big(1+\frac{2}{n}\Big)^n$$

$$= \lim_{n\to\infty}\ln\Big(\frac{n}{n+2}\Big)^n = \lim_{n\to\infty}\ln\Big(\frac{n+2-2}{n+2}\Big)^n = \lim_{n\to\infty}\ln\Big(1+\frac{-2}{n+2}\Big)^n$$

$$= \lim_{n\to\infty}\ln\Big(1+\frac{-2}{n+2}\Big)^{n+2-2}$$

Riešené limity – 29

$$\lim_{n\to\infty} n\left[\ln n - \ln(n+2)\right]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\left[-n\cdot\ln\frac{n+2}{n}\right] = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$$

$$= -\lim_{n\to\infty}\ln\left(1+\frac{2}{n}\right)^n = \left[\begin{array}{c}\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right]$$

$$= \lim_{n\to\infty}\ln\left(\frac{n}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(\frac{n+2-2}{n+2}\right)^n = \lim_{n\to\infty}\ln\left(1+\frac{-2}{n+2}\right)^n$$

$$= \lim_{n\to\infty}\ln\left(1+\frac{-2}{n+2}\right)^{n+2-2} = \lim_{n\to\infty}\ln\left[\left(1+\frac{-2}{n+2}\right)^{n+2}\left(1+\frac{-2}{n+2}\right)^{-2}\right]$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\frac{n+2}{n}\Big] = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$$

$$= -\lim_{n\to\infty}\ln\big(1+\tfrac{2}{n}\big)^n = \begin{bmatrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{bmatrix} = -\ln e^2$$

$$= \lim_{n\to\infty}\ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(\tfrac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^n$$

$$= \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big[\big(1+\tfrac{-2}{n+2}\big)^{n+2}\big(1+\tfrac{-2}{n+2}\big)^{-2}\Big]$$

$$= \ln\big[e^{-2}\cdot 1\big]$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{n+2} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\big(1+\tfrac{-2}{m}\big)^m = \begin{bmatrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{bmatrix} = e^{-2},$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\frac{n+2}{n}\Big] = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$$

$$= -\lim_{n\to\infty}\ln\big(1+\tfrac{2}{n}\big)^n = \left[\begin{array}{c}\lim\limits_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a\\ \text{pre všetky } a\in R\end{array}\right] = -\ln e^2 = -2.$$

$$= \lim_{n\to\infty}\ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(\tfrac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^n$$

$$= \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big[\big(1+\tfrac{-2}{n+2}\big)^{n+2}\big(1+\tfrac{-2}{n+2}\big)^{-2}\Big]$$

$$= \ln\Big[e^{-2}\cdot 1\Big]$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{-2} = \big(1+\tfrac{-2}{\infty}\big)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\frac{n+2}{n}\Big] = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$$

$$= -\lim_{n\to\infty}\ln\big(1+\tfrac{2}{n}\big)^n = \left[\begin{matrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right] = -\ln e^2 = -2.$$

$$= \lim_{n\to\infty}\ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(\tfrac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^n$$

$$= \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big[\big(1+\tfrac{-2}{n+2}\big)^{n+2}\big(1+\tfrac{-2}{n+2}\big)^{-2}\Big]$$

$$= \ln\big[e^{-2}\cdot 1\big]$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{n+2} = \left[\begin{matrix}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{matrix}\right] = \lim_{m\to\infty}\big(1+\tfrac{-2}{m}\big)^m = \left[\begin{matrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right] = e^{-2},$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{-2} = \big(1+\tfrac{-2}{\infty}\big)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big]$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\frac{n+2}{n}\Big] = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$$

$$= -\lim_{n\to\infty}\ln\big(1+\tfrac{2}{n}\big)^n = \begin{bmatrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a\\ \text{pre všetky } a\in R\end{bmatrix} = -\ln e^2 = -2.$$

$$= \lim_{n\to\infty}\ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(\tfrac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^n$$

$$= \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big[\big(1+\tfrac{-2}{n+2}\big)^{n+2}\big(1+\tfrac{-2}{n+2}\big)^{-2}\Big]$$

$$= \ln\big[e^{-2}\cdot 1\big] = \ln e^{-2}$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{n+2} = \begin{bmatrix}\text{Subst.} & n\to\infty\\ m=n+2 & m\to\infty\end{bmatrix} = \lim_{m\to\infty}\big(1+\tfrac{-2}{m}\big)^m = \begin{bmatrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a\\ \text{pre všetky } a\in R\end{bmatrix} = e^{-2},$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{-2} = \big(1+\tfrac{-2}{\infty}\big)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$$

Riešené limity – 29

$$\lim_{n\to\infty} n\Big[\ln n - \ln(n+2)\Big] = -2$$

$$= \lim_{n\to\infty} n\cdot\ln\frac{n}{n+2} = \lim_{n\to\infty}\Big[-n\cdot\ln\frac{n+2}{n}\Big] = -\lim_{n\to\infty} n\cdot\ln\frac{n+2}{n}$$

$$= -\lim_{n\to\infty}\ln\big(1+\tfrac{2}{n}\big)^n = \left[\begin{matrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right] = -\ln e^2 = -2.$$

$$= \lim_{n\to\infty}\ln\big(\tfrac{n}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(\tfrac{n+2-2}{n+2}\big)^n = \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^n$$

$$= \lim_{n\to\infty}\ln\big(1+\tfrac{-2}{n+2}\big)^{n+2-2} = \lim_{n\to\infty}\ln\Big[\big(1+\tfrac{-2}{n+2}\big)^{n+2}\big(1+\tfrac{-2}{n+2}\big)^{-2}\Big]$$

$$= \ln\big[e^{-2}\cdot 1\big] = \ln e^{-2} = -2.$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{n+2} = \left[\begin{matrix}\text{Subst.} & n\to\infty \\ m=n+2 & m\to\infty\end{matrix}\right] = \lim_{m\to\infty}\big(1+\tfrac{-2}{m}\big)^m = \left[\begin{matrix}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right] = e^{-2},$$

$$\lim_{n\to\infty}\big(1+\tfrac{-2}{n+2}\big)^{-2} = \big(1+\tfrac{-2}{\infty}\big)^{-2} = (1+0)^{-2} = 1^{-2} = 1.$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}$$

$$\lim_{n\to\infty}\Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right)$$

$$\lim_{n\to\infty}\Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]$$

$$= \lim_{n\to\infty}\Big[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\Big[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n$$

$$\lim_{n\to\infty} \Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]$$

$$= \lim_{n\to\infty} \Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty} \Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right]$$

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right]$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = \ln e^3$$

$$\lim_{n\to\infty} \Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]$$

$$= \lim_{n\to\infty} \Big[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}$$

$$= \lim_{n\to\infty} \Big[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\left[\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right]}$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n} = 3$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = \ln e^3 = 3.$$

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right]$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}}$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\left[\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right]}$$

$$= \left[\lim_{n\to\infty}\sqrt{n^2} = \lim_{n\to\infty} n = \infty \;\middle|\; \lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0 \;\middle|\; \lim_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a\in R\right]$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n} = 3$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{matrix}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{matrix}\right] = \ln e^3 = 3.$$

$$\lim_{n\to\infty}\Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]$$

$$= \lim_{n\to\infty}\Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}} = \frac{-1}{\infty}$$

$$= \lim_{n\to\infty}\Big[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\Big]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\left[\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right]}$$

$$= \left[\lim_{n\to\infty}\sqrt{n^2} = \lim_{n\to\infty} n = \infty \;\middle|\; \lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0 \;\middle|\; \lim_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a\in R\right] = \frac{-1}{\infty[\sqrt{1-0+0}+\sqrt{1-0+0}]}$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3) - \ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n} = 3$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = \ln e^3 = 3.$$

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right] = 0$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}} = \frac{-1}{\infty} = 0.$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1} - \sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\left[\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right]}$$

$$= \left[\lim_{n\to\infty}\sqrt{n^2} = \lim_{n\to\infty} n = \infty \;\middle|\; \lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0 \;\middle|\; \lim_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a\in R\right] = \frac{-1}{\infty[\sqrt{1-0+0}+\sqrt{1-0+0}]}$$

$$= \frac{-1}{\infty\cdot 2}$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3)-\ln n\Big]=\lim_{n\to\infty} n\cdot\ln\frac{n+3}{n}=3$$

$$=\lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right)=\lim_{n\to\infty}\ln\left(1+\frac{3}{n}\right)^n=\left[\begin{matrix}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a\\ \text{pre všetky } a\in R\end{matrix}\right]=\ln e^3=3.$$

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]=0$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}=\lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$=\lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}=\frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}}=\frac{-1}{\sqrt{\infty}+\sqrt{\infty}}=\frac{-1}{\infty}=0.$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}=\lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$=\lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}=\lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\left[\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right]}$$

$$=\left[\lim_{n\to\infty}\sqrt{n^2}=\lim_{n\to\infty}n=\infty\;\middle|\;\lim_{n\to\infty}\frac{1}{n}=\frac{1}{\infty}=0\;\middle|\;\lim_{n\to\infty}\frac{a}{n^2}=\frac{a}{\infty^2}=\frac{a}{\infty}=0\text{ pre } a\in R\right]=\frac{-1}{\infty[\sqrt{1-0+0}+\sqrt{1-0+0}]}$$

$$=\frac{-1}{\infty\cdot 2}=\frac{-1}{\infty}$$

Riešené limity – 30, 31

$$\lim_{n\to\infty} n\Big[\ln(n+3)-\ln n\Big] = \lim_{n\to\infty} n\cdot\ln\frac{n+3}{n} = 3$$

$$= \lim_{n\to\infty} n\cdot\ln\left(1+\frac{3}{n}\right) = \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R\end{array}\right] = \ln e^3 = 3.$$

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right] = 0$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \frac{-1}{\sqrt{\infty^2+\infty+1}+\sqrt{\infty^2+\infty+2}} = \frac{-1}{\sqrt{\infty}+\sqrt{\infty}} = \frac{-1}{\infty} = 0.$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n^2+n+2}\right]\cdot\frac{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{(n^2+n+1)-(n^2+n+2)}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}}$$

$$= \lim_{n\to\infty}\frac{-1}{\sqrt{n^2+n+1}+\sqrt{n^2+n+2}} = \lim_{n\to\infty}\frac{-1}{\sqrt{n^2}\left[\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{2}{n^2}}\right]}$$

$$= \left[\lim_{n\to\infty}\sqrt{n^2} = \lim_{n\to\infty} n = \infty \;\middle|\; \lim_{n\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0 \;\middle|\; \lim_{n\to\infty}\frac{a}{n^2} = \frac{a}{\infty^2} = \frac{a}{\infty} = 0 \text{ pre } a\in R\right] = \frac{-1}{\infty\left[\sqrt{1-0+0}+\sqrt{1-0+0}\right]}$$

$$= \frac{-1}{\infty\cdot 2} = \frac{-1}{\infty} = 0.$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n}{2^n} & \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{n+1}{2n} = \lim\limits_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n}{2^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n} = \lim\limits_{n\to\infty}\sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty}\frac{\sqrt[n]{n}}{2} = \frac{1}{2}<1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty}\frac{n+1}{2n} = \lim\limits_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0}$$

$$\lim_{n\to\infty}\left[\sqrt{n^2-n+1}-\sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty}\left[\sqrt{n^2-n+1}-\sqrt{n^2-3n+2}\right]\cdot\frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty}\frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n}{2^n} & \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{n+1}{2n} = \lim\limits_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0}$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n}{2^n} & \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \text{pre všetky } n\in N & \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \lim_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty}$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{l} n = \sqrt{n^2} \\ \text{pre } n\in N \end{array} \right]$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n}{2^n} & \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{n+1}{2n} = \lim\limits_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{l} n=\sqrt{n^2} \\ \text{pre } n\in N \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$= \left[\begin{array}{l} \text{Označme } a_n = \frac{n}{2^n} \\ \text{pre všetky } n\in N \end{array} \left| \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{n+1}{2n} = \lim\limits_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{l} n = \sqrt{n^2} \\ \text{pre } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}} = \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$= \left[\begin{array}{l} \text{Označme } a_n = \frac{n}{2^n} \\ \text{pre všetky } n\in N \end{array} \middle| \begin{array}{l} \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \lim_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{l} n = \sqrt{n^2} \\ \text{pre } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}} = \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n}{2^n} & \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim\limits_{n\to\infty} \frac{n+1}{2n} = \lim\limits_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right]$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{l} n=\sqrt{n^2} \\ \text{pre } n\in N \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}} = \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}} = \frac{2-0}{\sqrt{1-0+0}+\sqrt{1-0+0}}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty}\frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}}=\lim_{n\to\infty}\frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}}=0$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n}{2^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n}{2^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n}}{2}=\frac{1}{2}<1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}}=\lim\limits_{n\to\infty}\frac{n+1}{2n}=\lim\limits_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2n}\right)=\frac{1}{2}+\frac{1}{\infty}=\frac{1}{2}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n}{2^n}=0\right]$$

$$=\frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0}=\frac{8+0+0}{\infty+2-0}=\frac{8}{\infty}=0.$$

$$\lim_{n\to\infty}\left[\sqrt{n^2-n+1}-\sqrt{n^2-3n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2-n+1}-\sqrt{n^2-3n+2}\right]\cdot\frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$=\lim_{n\to\infty}\frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}=\lim_{n\to\infty}\frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}=\left[\begin{array}{l}n=\sqrt{n^2}\\ \text{pre } n\in N\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}\cdot\frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}}=\lim_{n\to\infty}\frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}}$$

$$=\frac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}}=\frac{2-0}{\sqrt{1-0+0}+\sqrt{1-0+0}}=\frac{2}{1+1}$$

Riešené limity – 32, 33

$$\lim_{n\to\infty} \frac{8+3^{-n}+4\cdot 5^{-n}}{3n+2-n\cdot 2^{-n}} = \lim_{n\to\infty} \frac{8+\frac{1}{3^n}+\frac{4}{5^n}}{3n+2-\frac{n}{2^n}} = 0$$

$$= \left[\begin{array}{l} \text{Označme } a_n = \frac{n}{2^n} \\ \text{pre všetky } n\in N \end{array} \middle| \begin{array}{l} \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{2} = \frac{1}{2} < 1 \\ \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \lim_{n\to\infty} \frac{n+1}{2n} = \lim_{n\to\infty} \left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}+\frac{1}{\infty} = \frac{1}{2} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} = 0 \right]$$

$$= \frac{8+\frac{1}{\infty}+\frac{4}{\infty}}{3\cdot\infty+2-0} = \frac{8+0+0}{\infty+2-0} = \frac{8}{\infty} = 0.$$

$$\lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] = 1$$

$$= \lim_{n\to\infty} \left[\sqrt{n^2-n+1} - \sqrt{n^2-3n+2}\right] \cdot \frac{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}}$$

$$= \lim_{n\to\infty} \frac{(n^2-n+1)-(n^2-3n+2)}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} = \left[\begin{array}{l} n = \sqrt{n^2} \\ \text{pre } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} \frac{2n-1}{\sqrt{n^2-n+1}+\sqrt{n^2-3n+2}} \cdot \frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2}}} = \lim_{n\to\infty} \frac{2-\frac{1}{n}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}+\sqrt{1-\frac{3}{n}+\frac{2}{n^2}}}$$

$$= \frac{2-\frac{1}{\infty}}{\sqrt{1-\frac{1}{\infty}+\frac{1}{\infty}}+\sqrt{1-\frac{3}{\infty}+\frac{2}{\infty}}} = \frac{2-0}{\sqrt{1-0+0}+\sqrt{1-0+0}} = \frac{2}{1+1} = 1.$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)}$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}=\lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)}=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}=\lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)}=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$

$$=\frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1}$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}=\lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}}=\frac{-1}{\infty+\infty\cdot\infty+\infty}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)}=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$

$$=\frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1}=\infty+\infty\cdot\infty+\infty$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]$$

$$= \lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$= \lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}} = \lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$= \frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}} = \frac{-1}{\infty+\infty\cdot\infty+\infty} = \frac{-1}{\infty}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}} = \infty$$

$$= \lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$

$$= \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)} = \lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$

$$= \frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1} = \infty+\infty\cdot\infty+\infty = \infty.$$

Riešené limity – 34, 35

$$\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]=0$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n+1}-\sqrt[3]{n+2}\right]\cdot\frac{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\lim_{n\to\infty}\frac{(n+1)-(n+2)}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}=\lim_{n\to\infty}\frac{-1}{\sqrt[3]{(n+1)^2}+\sqrt[3]{n+1}\,\sqrt[3]{n+2}+\sqrt[3]{(n+2)^2}}$$

$$=\frac{-1}{\sqrt[3]{(\infty+1)^2}+\sqrt[3]{\infty+1}\,\sqrt[3]{\infty+2}+\sqrt[3]{(\infty+2)^2}}=\frac{-1}{\infty+\infty\cdot\infty+\infty}=\frac{-1}{\infty}=0.$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}=\infty$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n-1}-\sqrt[3]{n-2}}\cdot\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{(n-1)-(n-2)}=\lim_{n\to\infty}\frac{\sqrt[3]{(n-1)^2}+\sqrt[3]{n-1}\,\sqrt[3]{n-2}+\sqrt[3]{(n-2)^2}}{1}$$

$$=\frac{\sqrt[3]{(\infty-1)^2}+\sqrt[3]{\infty-1}\,\sqrt[3]{\infty-2}+\sqrt[3]{(\infty-2)^2}}{1}=\infty+\infty\cdot\infty+\infty=\infty.$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\,\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\,\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{array}{c}\lim_{n\to\infty}(n^4\pm1)^k=\infty\\ \text{pre } k\in N\end{array}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{array}{c}\lim\limits_{n\to\infty}(n^4\pm1)^k=\infty\\ \text{pre } k\in N\end{array}\right]$$

$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\lim_{n\to\infty}(n^4\pm1)^k=\infty \text{ pre } k\in N\right]$$

$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}=\frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}=\lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{matrix}\lim\limits_{n\to\infty}(n^4\pm1)^k=\infty\\ \text{pre } k\in N\end{matrix}\right]$$

$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}=\frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}=\frac{1}{\infty}$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}=\lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

$$=\frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{array}{c}\lim_{n\to\infty}(n^4\pm1)^k=\infty\\ \text{pre } k\in N\end{array}\right]$$

$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}=\frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}=\frac{1}{\infty}=0.$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}=\lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

$$=\frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})}=\frac{1}{(\infty+\infty)\cdot(\infty+\infty)}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{array}{c}\lim_{n\to\infty}(n^4\pm1)^k=\infty\\ \text{pre } k\in N\end{array}\right]$$

$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}=\frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}=\frac{1}{\infty}=0.$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}=\lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

$$=\frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})}=\frac{1}{(\infty+\infty)\cdot(\infty+\infty)}=\frac{1}{\infty\cdot\infty}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{array}{c}\lim_{n\to\infty}(n^4\pm1)^k=\infty\\ \text{pre } k\in N\end{array}\right]$$

$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}=\frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}=\frac{1}{\infty}=0.$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}=\lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

$$=\frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})}=\frac{1}{(\infty+\infty)\cdot(\infty+\infty)}=\frac{1}{\infty\cdot\infty}=\frac{1}{\infty}$$

Riešené limity – 36

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]=0$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[4]{(n^4-1)^3}+\sqrt[4]{(n^4-1)^2}\sqrt[4]{n^4+1}+\sqrt[4]{n^4-1}\sqrt[4]{(n^4+1)^2}+\sqrt[4]{(n^4+1)^3}}=\left[\begin{array}{c}\lim_{n\to\infty}(n^4\pm1)^k=\infty\\ \text{pre } k\in N\end{array}\right]$$

$$=\frac{1}{\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}\sqrt[4]{\infty}+\sqrt[4]{\infty}}=\frac{1}{\infty+\infty\cdot\infty+\infty\cdot\infty+\infty}=\frac{1}{\infty}=0.$$

$$=\lim_{n\to\infty}\left[\sqrt[4]{n^4-1}-\sqrt[4]{n^4+1}\right]\cdot\frac{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt[4]{(n^4-1)^2}-\sqrt[4]{(n^4+1)^2}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}=\lim_{n\to\infty}\frac{\sqrt{n^4-1}-\sqrt{n^4+1}}{\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1}}\cdot\frac{\sqrt{n^4-1}+\sqrt{n^4+1}}{\sqrt{n^4-1}+\sqrt{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{(n^4-1)-(n^4+1)}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}=\lim_{n\to\infty}\frac{1}{(\sqrt[4]{n^4-1}+\sqrt[4]{n^4+1})\cdot(\sqrt{n^4-1}+\sqrt{n^4+1})}$$

$$=\frac{1}{(\sqrt[4]{\infty-1}+\sqrt[4]{\infty+1})\cdot(\sqrt{\infty-1}+\sqrt{\infty+1})}=\frac{1}{(\infty+\infty)\cdot(\infty+\infty)}=\frac{1}{\infty\cdot\infty}=\frac{1}{\infty}=0.$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{array}{c} n=\sqrt{n^2} \\ \text{pre } n\in N\end{array}\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{array}{c} n=\sqrt{n^2} \\ \text{pre } n\in N\end{array}\right]$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=1+\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{array}{c}n=\sqrt{n^2}\\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}}=\frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{array}{c}n=\sqrt{n^2}\\ \text{pre } n\in N\end{array}\right]=1+\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}=1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{array}{c}n=\sqrt{n^2}\\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}}=\frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}}=\frac{6}{\sqrt{1+0+0}+1-0}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{array}{c}n=\sqrt{n^2}\\ \text{pre } n\in N\end{array}\right]=1+\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}=1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1}=1+\frac{4+0}{\sqrt{1+0+0}+1}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}}=\frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}}=\frac{6}{\sqrt{1+0+0}+1-0}=\frac{6}{2}$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=1+\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}=1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1}=1+\frac{4+0}{\sqrt{1+0+0}+1}$$

$$=1+\frac{4}{2}$$

Riešené limity – 37

$$\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n+1\right]=3$$

$$=\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-(n-1)\right]\cdot\frac{\sqrt{n^2+4n+1}+(n-1)}{\sqrt{n^2+4n+1}+(n-1)}=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n-1)^2}{\sqrt{n^2+4n+1}+n-1}$$

$$=\lim_{n\to\infty}\frac{(n^2+4n+1)-(n^2-2n+1)}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}=\lim_{n\to\infty}\frac{6n}{\sqrt{n^2+4n+1}+n-1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{6}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1-\frac{1}{n}}=\frac{6}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1-\frac{1}{\infty}}=\frac{6}{\sqrt{1+0+0}+1-0}=\frac{6}{2}=3.$$

$$=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt{n^2+4n+1}-n\right]\cdot\frac{\sqrt{n^2+4n+1}+n}{\sqrt{n^2+4n+1}+n}$$

$$=1+\lim_{n\to\infty}\frac{(n^2+4n+1)-n^2}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}=1+\lim_{n\to\infty}\frac{4n+1}{\sqrt{n^2+4n+1}+n}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=1+\lim_{n\to\infty}\frac{4+\frac{1}{n}}{\sqrt{1+\frac{4}{n}+\frac{1}{n^2}}+1}=1+\frac{4+\frac{1}{\infty}}{\sqrt{1+\frac{4}{\infty}+\frac{1}{\infty}}+1}=1+\frac{4+0}{\sqrt{1+0+0}+1}$$

$$=1+\frac{4}{2}=3.$$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n-1}\right]$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right]$$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n-1}\right]$$

$$=\lim_{n\to\infty}\sqrt{n}\left[\sqrt{n+1+\tfrac{1}{n}}-\sqrt{1-\tfrac{1}{n}}\right]$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)}$$

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right]$$

$$=\lim_{n\to\infty}\sqrt[4]{n}\left[\sqrt[4]{n^3+\tfrac{1}{n}}-\sqrt[4]{1+\tfrac{1}{n}}\right]$$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n-1}\right]$$

$$=\lim_{n\to\infty}\sqrt{n}\left[\sqrt{n+1+\tfrac{1}{n}}-\sqrt{1-\tfrac{1}{n}}\right]=\infty\left[\sqrt{\infty+1+\tfrac{1}{\infty}}-\sqrt{1-\tfrac{1}{\infty}}\right]$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)}=\frac{1}{\infty\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)}$$

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right]$$

$$=\lim_{n\to\infty}\sqrt[4]{n}\left[\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right]=\infty\left[\sqrt[4]{\infty+\frac{1}{\infty}}-\sqrt[4]{1+\frac{1}{\infty}}\right]$$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n-1}\right]$$

$$=\lim_{n\to\infty}\sqrt{n}\left[\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right]=\infty\left[\sqrt{\infty+1+\frac{1}{\infty}}-\sqrt{1-\frac{1}{\infty}}\right]$$

$$=\infty\left(\sqrt{\infty+1+0}-\sqrt{1-0}\right)$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)}=\frac{1}{\infty\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)}=\frac{1}{\infty(\sqrt[3]{\infty+1+0}-\sqrt[3]{1-0})}$$

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right]$$

$$=\lim_{n\to\infty}\sqrt[4]{n}\left[\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right]=\infty\left[\sqrt[4]{\infty+\frac{1}{\infty}}-\sqrt[4]{1+\frac{1}{\infty}}\right]$$

$$=\infty\left(\sqrt[4]{\infty+0}-\sqrt[4]{1+0}\right)$$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n-1}\right]$$

$$=\lim_{n\to\infty}\sqrt{n}\left[\sqrt{n+1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right]=\infty\left[\sqrt{\infty+1+\frac{1}{\infty}}-\sqrt{1-\frac{1}{\infty}}\right]$$
$$=\infty\left(\sqrt{\infty+1+0}-\sqrt{1-0}\right)=\infty(\infty-1)$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)}=\frac{1}{\infty\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)}=\frac{1}{\infty(\sqrt[3]{\infty+1+0}-\sqrt[3]{1-0})}$$
$$=\frac{1}{\infty(\infty-1)}$$

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right]$$

$$=\lim_{n\to\infty}\sqrt[4]{n}\left[\sqrt[4]{n^3+\frac{1}{n}}-\sqrt[4]{1+\frac{1}{n}}\right]=\infty\left[\sqrt[4]{\infty+\frac{1}{\infty}}-\sqrt[4]{1+\frac{1}{\infty}}\right]$$
$$=\infty\left(\sqrt[4]{\infty+0}-\sqrt[4]{1+0}\right)=\infty(\infty-1)$$

Riešené limity – 38, 39, 40

$$\lim_{n\to\infty}\left[\sqrt{n^2+n+1}-\sqrt{n-1}\right]=\infty$$

$$=\lim_{n\to\infty}\sqrt{n}\left[\sqrt{n+1+\tfrac{1}{n}}-\sqrt{1-\tfrac{1}{n}}\right]=\infty\left[\sqrt{\infty+1+\tfrac{1}{\infty}}-\sqrt{1-\tfrac{1}{\infty}}\right]$$
$$=\infty\left(\sqrt{\infty+1+0}-\sqrt{1-0}\right)=\infty(\infty-1)=\infty.$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+n+1}-\sqrt[3]{n-1}}=0$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n}\left(\sqrt{n^2+1+\frac{1}{n}}-\sqrt[3]{1-\frac{1}{n}}\right)}=\frac{1}{\infty\left(\sqrt[3]{\infty+1+\frac{1}{\infty}}-\sqrt[3]{1-\frac{1}{\infty}}\right)}=\frac{1}{\infty(\sqrt[3]{\infty+1+0}-\sqrt[3]{1-0})}$$
$$=\frac{1}{\infty(\infty-1)}=0.$$

$$\lim_{n\to\infty}\left[\sqrt[4]{n^4+1}-\sqrt[4]{n+1}\right]=\infty$$

$$=\lim_{n\to\infty}\sqrt[4]{n}\left[\sqrt[4]{n^3+\tfrac{1}{n}}-\sqrt[4]{1+\tfrac{1}{n}}\right]=\infty\left[\sqrt[4]{\infty+\tfrac{1}{\infty}}-\sqrt[4]{1+\tfrac{1}{\infty}}\right]$$
$$=\infty\left(\sqrt[4]{\infty+0}-\sqrt[4]{1+0}\right)=\infty(\infty-1)=\infty.$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

Riešené limity – 41, 42

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)}$$

Riešené limity – 41, 42

$$\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+1}-n}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[3]{n^3+1}-n}\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\lim_{n\to\infty}\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$\lim_{n\to\infty}\frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)}=\frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)}$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+0}-\sqrt[5]{1+0}\right)}$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N \end{array}\right]$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+0}-\sqrt[5]{1+0}\right)}$$

$$= \frac{1}{\infty(\infty-1)}$$

Riešené limity – 41, 42

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} n^2 \left[\sqrt[3]{(1+\tfrac{1}{n^3})^2} + \sqrt[3]{1+\tfrac{1}{n^3}} + 1 \right]$$

$\lim\limits_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+0}-\sqrt[5]{1+0}\right)}$$

$$= \frac{1}{\infty(\infty-1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{matrix} n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N \end{matrix}\right]$$

$$= \lim_{n\to\infty} n^2\left[\sqrt[3]{\left(1+\tfrac{1}{n^3}\right)^2} + \sqrt[3]{1+\tfrac{1}{n^3}} + 1\right] = \infty\left[\sqrt[3]{\left(1+\tfrac{1}{\infty}\right)^2} + \sqrt[3]{1+\tfrac{1}{\infty}} + 1\right]$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}}-\sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}}-\sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+0}-\sqrt[5]{1+0}\right)}$$

$$= \frac{1}{\infty(\infty-1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} n^2\left[\sqrt[3]{(1+\tfrac{1}{n^3})^2} + \sqrt[3]{1+\tfrac{1}{n^3}} + 1\right] = \infty\left[\sqrt[3]{(1+\tfrac{1}{\infty})^2} + \sqrt[3]{1+\tfrac{1}{\infty}} + 1\right]$$

$$= \infty\left(\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1\right)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+0} - \sqrt[5]{1+0}\right)}$$

$$= \frac{1}{\infty(\infty-1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} n^2 \left[\sqrt[3]{(1+\tfrac{1}{n^3})^2} + \sqrt[3]{1+\tfrac{1}{n^3}} + 1 \right] = \infty \left[\sqrt[3]{(1+\tfrac{1}{\infty})^2} + \sqrt[3]{1+\tfrac{1}{\infty}} + 1 \right]$$

$$= \infty\left(\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1 \right) = \infty(1+1+1)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt[5]{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+0} - \sqrt[5]{1+0}\right)}$$

$$= \frac{1}{\infty(\infty-1)} = 0.$$

Riešené limity – 41, 42

$$\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} = \infty$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[3]{n^3+1}-n} \cdot \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2} = \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{(n^3+1)-n^3}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{1} = \left[\begin{array}{c} n^2 = n\cdot n = \sqrt[3]{n^3}\cdot n = \sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N \end{array} \right]$$

$$= \lim_{n\to\infty} n^2\left[\sqrt[3]{(1+\tfrac{1}{n^3})^2} + \sqrt[3]{1+\tfrac{1}{n^3}} + 1\right] = \infty\left[\sqrt[3]{(1+\tfrac{1}{\infty})^2} + \sqrt[3]{1+\tfrac{1}{\infty}} + 1\right]$$

$$= \infty\left(\sqrt[3]{(1+0)^2} + \sqrt[3]{1+0} + 1\right) = \infty(1+1+1) = \infty.$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[5]{n^5+1}-\sqrt[5]{n^4+1}} = 0$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[5]{n^4}\left(\sqrt{n+\frac{1}{n^4}} - \sqrt[5]{1+\frac{1}{n^4}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+\frac{1}{\infty}} - \sqrt[5]{1+\frac{1}{\infty}}\right)} = \frac{1}{\infty\left(\sqrt[5]{\infty+0} - \sqrt[5]{1+0}\right)}$$

$$= \frac{1}{\infty(\infty-1)} = 0.$$

Riešené limity – 43

$$\lim_{n\to\infty} \left[\sqrt[3]{n^3+1} - n + 1\right]$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$=\left[\begin{array}{c} n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N\end{array}\right]$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$=\left[\begin{matrix}n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$=\left[\begin{array}{c} n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$=\frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$=\left[\begin{array}{c}n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$=\frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2}=\frac{3-0+0}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}(1-0)+(1-0)^2}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$=\left[\begin{array}{c} n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3} \\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$=\frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2}=\frac{3-0+0}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}(1-0)+(1-0)^2}$$

$$=\frac{3}{1+1\cdot 1+1}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$=\left[\begin{array}{c}n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$=\frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2}=\frac{3-0+0}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}(1-0)+(1-0)^2}$$

$$=\frac{3}{1+1\cdot1+1}=\frac{3}{3}$$

Riešené limity – 43

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]=1$$

$$=\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-(n-1)\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{(n^3+1)-(n-1)^3}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{(n^3+1)-(n^3-3n^2+3n-1)}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}$$

$$=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}=\lim_{n\to\infty}\frac{3n^2-3n+2}{\sqrt[3]{(n^3+1)^2}+\sqrt[3]{n^3+1}(n-1)+(n-1)^2}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$=\left[\begin{array}{c}n^2=n\cdot n=\sqrt[3]{n^3}\cdot n=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{3-\frac{3}{n}+\frac{2}{n^2}}{\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}(1-\frac{1}{n})+(1-\frac{1}{n})^2}$$

$$=\frac{3-\frac{3}{\infty}+\frac{2}{\infty}}{\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}(1-\frac{1}{\infty})+(1-\frac{1}{\infty})^2}=\frac{3-0+0}{\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}(1-0)+(1-0)^2}$$

$$=\frac{3}{1+1\cdot1+1}=\frac{3}{3}=1.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}$$

Riešené limity – 43, 44

$\lim\limits_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{matrix}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{matrix}\right]$$

$\lim\limits_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{array}{c}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]$$

$$=1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0=\infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{array}{c}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]$$

$$=1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}=1+\frac{1}{\infty\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0=\infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{array}{c}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]$$

$$=1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}=1+\frac{1}{\infty\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$=1+\frac{1}{\infty\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0=\infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{array}{c}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]$$

$$=1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}=1+\frac{1}{\infty\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$=1+\frac{1}{\infty\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)}=1+\frac{1}{\infty(1+1+1)}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0=\infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{array}{c}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]$$

$$=1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}=1+\frac{1}{\infty\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$=1+\frac{1}{\infty\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)}=1+\frac{1}{\infty(1+1+1)}=1+\frac{1}{\infty}$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0=\infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{matrix}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{matrix}\right]$$

$$=1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}=1+\frac{1}{\infty\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$=1+\frac{1}{\infty\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)}=1+\frac{1}{\infty(1+1+1)}=1+\frac{1}{\infty}=1+0$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0=\infty.$$

Riešené limity – 43, 44

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n+1\right]=1$$

$$=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]=1+\lim_{n\to\infty}\left[\sqrt[3]{n^3+1}-n\right]\cdot\frac{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}$$

$$=1+\lim_{n\to\infty}\frac{(n^3+1)-n^3}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=1+\lim_{n\to\infty}\frac{1}{\sqrt[3]{(n^3+1)^2}+n\sqrt[3]{n^3+1}+n^2}=\left[\begin{array}{c}n^2=n\sqrt[3]{n^3}=\sqrt[3]{n^3\cdot n^3}\\ \text{pre všetky } n\in N\end{array}\right]$$

$$=1+\lim_{n\to\infty}\frac{1}{n^2\left(\sqrt[3]{(1+\frac{1}{n^3})^2}+\sqrt[3]{1+\frac{1}{n^3}}+1\right)}=1+\frac{1}{\infty\left(\sqrt[3]{(1+\frac{1}{\infty})^2}+\sqrt[3]{1+\frac{1}{\infty}}+1\right)}$$

$$=1+\frac{1}{\infty\left(\sqrt[3]{(1+0)^2}+\sqrt[3]{1+0}+1\right)}=1+\frac{1}{\infty(1+1+1)}=1+\frac{1}{\infty}=1+0=1.$$

$$\lim_{n\to\infty}\frac{(n+4)!-(n+2)!}{(n+3)!}=\infty$$

$$=\lim_{n\to\infty}\left[\frac{(n+4)!}{(n+3)!}-\frac{(n+2)!}{(n+3)!}\right]=\lim_{n\to\infty}\left[\frac{(n+4)(n+3)!}{(n+3)!}-\frac{(n+2)!}{(n+3)(n+2)!}\right]=\lim_{n\to\infty}\left[(n+4)-\frac{1}{n+3}\right]$$

$$=(\infty+4)-\frac{1}{\infty+3}=\infty-0=\infty.$$

Riešené limity – 45, 46

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n+6}$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$$

Riešené limity – 45, 46

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6}$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

Riešené limity – 45, 46

$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n+6}$

$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{n+6}$

$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}$

$= \left[\text{Označme } a_n=\frac{n^k}{5^n} \text{ pre } n\in N,\ k\in N \;\middle|\; \begin{array}{l} \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1 \\ \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+\frac{1}{\infty})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^k}{5^n} = 0 \right]$

$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}} \cdot \frac{\frac{1}{5^n}}{\frac{1}{5^n}}$

Riešené limity – 45, 46

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n+6}$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{n+6} = \lim_{n\to\infty} \left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n+6}{2n+3}}$$

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{5^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}$$

Riešené limity – 45, 46

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1\\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}=\frac{0+0-0+2}{0-0+3\cdot 5}$$

Riešené limity – 45, 46

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\frac{n+6}{2n+3}=\lim_{n\to\infty}\frac{n+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{6}{n}}{2+\frac{3}{n}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}=\frac{2}{15}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1 \\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5}=\frac{0+0-0+2}{0-0+3\cdot5}=\frac{2}{15}.$$

Riešené limity – 45, 46

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{n+6} = \lim_{n\to\infty} \left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n+6}{2n+3}}$$

$$= \left[\mathrm{e}^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1-\frac{4}{m}\right)^{m} = \left[\begin{array}{c} \lim\limits_{n\to\infty} \left(1-\frac{a}{n}\right)^{n} = \lim\limits_{n\to\infty} \left(1+\frac{-a}{n}\right)^{n} = \mathrm{e}^{-a} \\ \text{pre všetky } a\in R \end{array}\right] = \mathrm{e}^{-4},$$

$$\lim_{n\to\infty} \frac{n+6}{2n+3} = \lim_{n\to\infty} \frac{n+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1+\frac{6}{n}}{2+\frac{3}{n}} = \frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{1+0}{2+0} = \frac{1}{2}.$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}} = \frac{2}{15}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{5^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n} = \lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5} = \frac{(1+\frac{1}{\infty})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1 \end{array}\right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n^k}{5^n} = 0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{0+0-0+2}{0-0+3\cdot 5} = \frac{2}{15}.$$

Riešené limity – 45, 46

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n+6}=e^{-2}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{\frac{1}{2}}=e^{-2}.$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n+6}{2n+3}=\lim_{n\to\infty}\frac{n+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{6}{n}}{2+\frac{3}{n}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}=\frac{2}{15}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \left.\begin{array}{l}\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1\\ \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\\ \text{pre } n\in N,\ k\in N & \end{array}\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}+2}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}=\frac{0+0-0+2}{0-0+3\cdot 5}=\frac{2}{15}.$$

Riešené limity – 47, 48

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$$

$$=\left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{5^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1 \end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1 \\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$$

$$=\left[\begin{array}{c|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{\infty}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1\\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5}=\frac{0+0-0}{0-0+3\cdot5}$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{\infty}$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n+3}=\lim_{n\to\infty}\frac{n^6+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2+\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}=\frac{0+0-0}{0-0+3\cdot 5}=\frac{0}{15}$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$
$$=\left[e^{-4}\right]^{\infty}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n+3}=\lim_{n\to\infty}\frac{n^6+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2+\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}=0$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \left.\begin{array}{l}\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1\\ \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\\ \text{pre } n\in N,\ k\in N & \end{array}\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5}=\frac{0+0-0}{0-0+3\cdot5}=\frac{0}{15}=0.$$

Riešené limity – 47, 48

$\lim\limits_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$
$$=\left[e^{-4}\right]^{\infty}=e^{-\infty}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{matrix}\text{Subst.}\\ m=2n+3\end{matrix}\middle|\begin{matrix}n\to\infty\\ m\to\infty\end{matrix}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{matrix}\lim\limits_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim\limits_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{matrix}\right]=e^{-4},$$
$$\lim_{n\to\infty}\frac{n^6+6}{2n+3}=\lim_{n\to\infty}\frac{n^6+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2+\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$\lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}=0$

$$=\left[\begin{matrix}\text{Označme } a_n=\frac{n^k}{5^n}\\ \text{pre } n\in N,\ k\in N\end{matrix}\middle|\left.\begin{matrix}\lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1\\ \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{matrix}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$
$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5}=\frac{0+0-0}{0-0+3\cdot5}=\frac{0}{15}=0.$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{\infty}=e^{-\infty}=\frac{1}{e^{\infty}}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim\limits_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n+3}=\lim_{n\to\infty}\frac{n^6+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2+\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}=0$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1\\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5}=\frac{0+0-0}{0-0+3\cdot 5}=\frac{0}{15}=0.$$

Riešené limity – 47, 48

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{n^6+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{n^6+6} = \lim_{n\to\infty} \left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$
$$= \left[e^{-4}\right]^{\infty} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty}$$

$$\lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{2n+3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1-\frac{4}{m}\right)^{m} = \left[\begin{array}{c} \lim\limits_{n\to\infty} \left(1-\frac{a}{n}\right)^{n} = \lim\limits_{n\to\infty} \left(1+\frac{-a}{n}\right)^{n} = e^{-a} \\ \text{pre všetky } a\in R \end{array}\right] = e^{-4},$$

$$\lim_{n\to\infty} \frac{n^6+6}{2n+3} = \lim_{n\to\infty} \frac{n^6+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^5+\frac{6}{n}}{2+\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}} = 0$$

$$= \left[\begin{array}{c|l} \text{Označme } a_n=\frac{n^k}{5^n} & \left.\begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{5} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{5} = \frac{1^k}{5} = \frac{1}{5} < 1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{5n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{5} = \frac{(1+\frac{1}{\infty})^k}{5} = \frac{(1+0)^k}{5} = \frac{1}{5} < 1 \end{array}\right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n^k}{5^n} = 0 \\ \text{pre } n\in N,\ k\in N & \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot 5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}} = \lim_{n\to\infty} \frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot 5} = \frac{0+0-0}{0-0+3\cdot 5} = \frac{0}{15} = 0.$$

Riešené limity – 47, 48

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{n^6+6}=0$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{n^6+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{\infty}=e^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n+3}=\lim_{n\to\infty}\frac{n^6+6}{2n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2+\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}=0$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{5^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{5}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{5}=\frac{1^k}{5}=\frac{1}{5}<1\\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{5^{n+1}}}{\frac{n^k}{5^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{5n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{5}=\frac{(1+\frac{1}{\infty})^k}{5}=\frac{(1+0)^k}{5}=\frac{1}{5}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{5^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2}{2n^2-n^3+3\cdot5^{n+1}}\cdot\frac{\frac{1}{5^n}}{\frac{1}{5^n}}=\lim_{n\to\infty}\frac{\frac{2n^3}{5^n}+\frac{3n^4}{5^n}-\frac{n^2}{5^n}}{\frac{2n^2}{5^n}-\frac{n^3}{5^n}+3\cdot5}=\frac{0+0-0}{0-0+3\cdot5}=\frac{0}{15}=0.$$

Riešené limity – 49, 50

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

Riešené limity – 49, 50

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6}$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

$\forall n \in N$ platí $n+5 \leq 5n+1$ pretože $4 \leq 4n$

Riešené limity – 49, 50

$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

$= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6}$

$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$

$\forall n \in N$ platí $n+5 \leq 5n+1$ pretože $4 \leq 4n$ a tiež platí $5n+1 < 5n+25 = 5(n+5)$.

Riešené limity – 49, 50

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left[\left(1+\frac{2}{2n-3}\right)^{2n-3}\right]^{\frac{n^6+6}{2n-3}}$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

$\forall n \in N$ platí $n+5 \leq 5n+1$ pretože $4 \leq 4n$ a tiež platí $5n+1 < 5n+25 = 5(n+5)$.

$\Rightarrow \forall n \in N$ platí: $1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5$

Riešené limity – 49, 50

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

$$= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left[\left(1+\frac{2}{2n-3}\right)^{2n-3}\right]^{\frac{n^6+6}{2n-3}}$$
$$= \left[e^2\right]^{\infty}$$

$$\lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{2n-3} = \begin{bmatrix} \text{Subst.} & n\to\infty \\ m=2n-3 & m\to\infty \end{bmatrix} = \lim_{m\to\infty} \left(1+\frac{2}{m}\right)^m = \begin{bmatrix} \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R \end{bmatrix} = e^2,$$

$\lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$

$\forall n\in N$ platí $n+5 \leq 5n+1$ pretože $4 \leq 4n$ a tiež platí $5n+1 < 5n+25 = 5(n+5)$.

$\Rightarrow \forall n\in N$ platí: $1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5,$ t. j. $1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$

Riešené limity – 49, 50

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left[\left(1+\frac{2}{2n-3}\right)^{2n-3}\right]^{\frac{n^6+6}{2n-3}}$$
$$= \left[e^2\right]^{\infty}$$

$$\lim_{n\to\infty} \frac{n^6+6}{2n-3} = \lim_{n\to\infty} \frac{n^6+6}{2n-3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^5+\frac{6}{n}}{2-\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}} = \frac{\infty+0}{2-0} = \infty.$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}}$$

$\forall n\in N$ platí $n+5 \leq 5n+1$ pretože $4 \leq 4n$ a tiež platí $5n+1 < 5n+25 = 5(n+5)$.

$\Rightarrow \forall n\in N$ platí: $1 = \frac{n+5}{n+5} \leq \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5,$ t. j. $1 = \sqrt[n]{1} \leq \sqrt[n]{\frac{5n+1}{n+5}} \leq \sqrt[n]{5}.$

$\Rightarrow 1 = \lim_{n\to\infty} 1 \leq \lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} \leq \lim_{n\to\infty} \sqrt[n]{5} = 1$

Riešené limity – 49, 50

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n-3}\right)^{n^6+6}$

$$= \lim_{n\to\infty} \left(\frac{2n-3+2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{n^6+6} = \lim_{n\to\infty} \left[\left(1+\frac{2}{2n-3}\right)^{2n-3}\right]^{\frac{n^6+6}{2n-3}}$$
$$= \left[e^2\right]^{\infty}$$

$$\lim_{n\to\infty} \left(1+\frac{2}{2n-3}\right)^{2n-3} = \left[\begin{array}{c|c} \text{Subst.} & n\to\infty \\ m=2n-3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{2}{m}\right)^m = \left[\begin{array}{c} \lim\limits_{n\to\infty} \left(1+\frac{a}{n}\right)^n = e^a \\ \text{pre všetky } a\in R \end{array}\right] = e^2,$$

$$\lim_{n\to\infty} \frac{n^6+6}{2n-3} = \lim_{n\to\infty} \frac{n^6+6}{2n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^5+\frac{6}{n}}{2-\frac{3}{n}} = \frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}} = \frac{\infty+0}{2-0} = \infty.$$

$\lim\limits_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1$

$\forall n\in N$ platí $n+5 \le 5n+1$ pretože $4 \le 4n$ a tiež platí $5n+1 < 5n+25 = 5(n+5)$.

$\Rightarrow \forall n\in N$ platí: $1 = \frac{n+5}{n+5} \le \frac{5n+1}{n+5} < \frac{5(n+5)}{n+5} = 5$, t. j. $1 = \sqrt[n]{1} \le \sqrt[n]{\frac{5n+1}{n+5}} \le \sqrt[n]{5}$.

$$\Rightarrow 1 = \lim_{n\to\infty} 1 \le \lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} \le \lim_{n\to\infty} \sqrt[n]{5} = 1 \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{5n+1}{n+5}} = 1.$$

Riešené limity – 49, 50

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n-3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n-3+2}{2n-3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1+\frac{2}{2n-3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1+\frac{2}{2n-3}\right)^{2n-3}\right]^{\frac{n^6+6}{2n-3}}$$

$$=\left[e^2\right]^{\infty}=e^{\infty}$$

$$\lim_{n\to\infty}\left(1+\frac{2}{2n-3}\right)^{2n-3}=\left[\begin{matrix}\text{Subst.} & n\to\infty\\ m=2n-3 & m\to\infty\end{matrix}\right]=\lim_{m\to\infty}\left(1+\frac{2}{m}\right)^{m}=\left[\begin{matrix}\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=e^{a}\\ \text{pre všetky } a\in R\end{matrix}\right]=e^2,$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n-3}=\lim_{n\to\infty}\frac{n^6+6}{2n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2-\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}=1$$

$\forall n\in N$ platí $n+5\leq 5n+1$ pretože $4\leq 4n$ a tiež platí $5n+1<5n+25=5(n+5)$.

$\Rightarrow \forall n\in N$ platí: $1=\frac{n+5}{n+5}\leq\frac{5n+1}{n+5}<\frac{5(n+5)}{n+5}=5$, t. j. $1=\sqrt[n]{1}\leq\sqrt[n]{\frac{5n+1}{n+5}}\leq\sqrt[n]{5}$.

$\Rightarrow 1=\lim_{n\to\infty}1\leq\lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}\leq\lim_{n\to\infty}\sqrt[n]{5}=1 \Rightarrow \lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}=1.$

Riešené limity – 49, 50

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n-3}\right)^{n^6+6}=\infty$$

$$=\lim_{n\to\infty}\left(\frac{2n-3+2}{2n-3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1+\frac{2}{2n-3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1+\frac{2}{2n-3}\right)^{2n-3}\right]^{\frac{n^6+6}{2n-3}}$$

$$=\left[e^2\right]^{\infty}=e^{\infty}=\infty.$$

$$\lim_{n\to\infty}\left(1+\frac{2}{2n-3}\right)^{2n-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{2}{m}\right)^m=\left[\begin{array}{c}\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a\\ \text{pre všetky } a\in R\end{array}\right]=e^2,$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n-3}=\lim_{n\to\infty}\frac{n^6+6}{2n-3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{n^5+\frac{6}{n}}{2-\frac{3}{n}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}=1$$

$\forall n\in N$ platí $n+5\leq 5n+1$ pretože $4\leq 4n$ a tiež platí $5n+1<5n+25=5(n+5)$.

$\Rightarrow \forall n\in N$ platí: $1=\frac{n+5}{n+5}\leq\frac{5n+1}{n+5}<\frac{5(n+5)}{n+5}=5$, t. j. $1=\sqrt[n]{1}\leq\sqrt[n]{\frac{5n+1}{n+5}}\leq\sqrt[n]{5}$.

$\Rightarrow 1=\lim_{n\to\infty}1\leq\lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}\leq\lim_{n\to\infty}\sqrt[n]{5}=1 \Rightarrow \lim_{n\to\infty}\sqrt[n]{\frac{5n+1}{n+5}}=1.$

Riešené limity – 51, 52

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$$

$$\lim_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$$

Riešené limity – 51, 52

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6}$$

$$\lim_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$$

$$= \lim_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$= \lim_{n\to\infty} \frac{n^3\left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}$$

Riešené limity – 51, 52

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n^6+3}\right)^{n+6}$$

$$\lim_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$$

$$= \lim_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}}$$

$$= \lim_{n\to\infty} \frac{n^3(\frac{2}{n}+3+\frac{5}{n^3})}{n^4(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \lim_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{n(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})}$$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

$$= \lim_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n^6+3}\right)^{n+6} = \lim_{n\to\infty} \left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n+6}{2n^6+3}}$$

$\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

$$= \lim_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}}$$

$$= \lim_{n\to\infty} \frac{n^3\left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{n\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot\left(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}\right)}$$

Riešené limity – 51, 52

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n+6}{2n^6+3}}$$

$$=\left[e^{-4}\right]^0$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^m=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{1-\infty-0+0}$$

$$=\lim_{n\to\infty}\frac{n^3\left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{n\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot\left(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}\right)}$$

$$=\frac{0+3+0}{\infty\cdot(0-1-0+0)}$$

Riešené limity – 51, 52

$\lim\limits_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$

$= \lim\limits_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left(1-\frac{4}{2n^6+3}\right)^{n+6} = \lim\limits_{n\to\infty} \left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n+6}{2n^6+3}}$

$= \left[e^{-4}\right]^{0}$

$\lim\limits_{n\to\infty} \frac{n+6}{2n^6+3} = \lim\limits_{n\to\infty} \frac{n+6}{2n^6+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}} = \frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0.$

$\lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}$

$= \lim\limits_{n\to\infty} \frac{2n^2+3n^3+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}} = \frac{0+3+0}{1-\infty-0+0} = \frac{3}{-\infty}$

$= \lim\limits_{n\to\infty} \frac{n^3(\frac{2}{n}+3+\frac{5}{n^3})}{n^4(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \lim\limits_{n\to\infty} \frac{\frac{2}{n}+3+\frac{5}{n^3}}{n(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4})} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty})}$

$= \frac{0+3+0}{\infty\cdot(0-1-0+0)} = \frac{3}{\infty\cdot(-1)}$

Riešené limity – 51, 52

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n+6}{2n^6+3}}$$
$$=\left[e^{-4}\right]^{0}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$
$$\lim_{n\to\infty}\frac{n+6}{2n^6+3}=\lim_{n\to\infty}\frac{n+6}{2n^6+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}=0$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{1-\infty-0+0}=\frac{3}{-\infty}=0.$$

$$=\lim_{n\to\infty}\frac{n^3\left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{n\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot\left(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}\right)}$$
$$=\frac{0+3+0}{\infty\cdot(0-1-0+0)}=\frac{3}{\infty\cdot(-1)}=0.$$

Riešené limity – 51, 52

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n+6}{2n^6+3}}$$

$$=\left[e^{-4}\right]^{0}=e^{0}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim\limits_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n+6}{2n^6+3}=\lim_{n\to\infty}\frac{n+6}{2n^6+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}=0$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{1-\infty-0+0}=\frac{3}{-\infty}=0.$$

$$=\lim_{n\to\infty}\frac{n^3\left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{n\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot\left(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}\right)}$$

$$=\frac{0+3+0}{\infty\cdot(0-1-0+0)}=\frac{3}{\infty\cdot(-1)}=0.$$

Riešené limity – 51, 52

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n+6}=1$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n+6}{2n^6+3}}$$
$$=\left[e^{-4}\right]^0=e^0=1.$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^m=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$
$$\lim_{n\to\infty}\frac{n+6}{2n^6+3}=\lim_{n\to\infty}\frac{n+6}{2n^6+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}=\lim_{n\to\infty}\frac{1+\frac{6}{n}}{2n^5+\frac{3}{n}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}=0$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^3+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{1-n-\frac{1}{n}+\frac{2}{n^3}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{1-\infty-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{1-\infty-0+0}=\frac{3}{-\infty}=0.$$

$$=\lim_{n\to\infty}\frac{n^3\left(\frac{2}{n}+3+\frac{5}{n^3}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\lim_{n\to\infty}\frac{\frac{2}{n}+3+\frac{5}{n^3}}{n\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\infty\cdot\left(\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}\right)}$$
$$=\frac{0+3+0}{\infty\cdot(0-1-0+0)}=\frac{3}{\infty\cdot(-1)}=0.$$

Riešené limity – 53, 54

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

$$\lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6}$$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

$$=\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{n^3\left(\frac{2}{n}-3+\frac{5}{n^3}\right)}$$

Riešené limity – 53, 54

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n^6+3}\right)^{n^6+6}$$

$$\lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

$$= \lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}}$$

$$= \lim_{n\to\infty} \frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})} = \lim_{n\to\infty} \frac{n(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{\frac{2}{n}-3+\frac{5}{n^3}}$$

Riešené limity – 53, 54

$$\lim_{n\to\infty} \left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n^6+3}\right)^{n^6+6} = \lim_{n\to\infty} \left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n^6+6}{2n^6+3}}$$

$$\lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

$$= \lim_{n\to\infty} \frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}}$$

$$= \lim_{n\to\infty} \frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})} = \lim_{n\to\infty} \frac{n(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{\frac{2}{n}-3+\frac{5}{n^3}} = \frac{\infty\cdot(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty})}{\frac{2}{\infty}-3+\frac{5}{\infty}}$$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n^6+6}{2n^6+3}}$$

$$=\left[e^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^m=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

$$=\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{1-\infty+0+0}{0-3+0}$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{n^3\left(\frac{2}{n}-3+\frac{5}{n^3}\right)}=\lim_{n\to\infty}\frac{n\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{\infty\cdot\left(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty}\right)}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{\infty\cdot(0-1+0+0)}{0-3+0}$$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\Big(\frac{2n^6-1}{2n^6+3}\Big)^{n^6+6}$$

$$=\lim_{n\to\infty}\Big(\frac{2n^6+3-4}{2n^6+3}\Big)^{n^6+6}=\lim_{n\to\infty}\Big(1-\frac{4}{2n^6+3}\Big)^{n^6+6}=\lim_{n\to\infty}\Big[\Big(1-\frac{4}{2n^6+3}\Big)^{2n^6+3}\Big]^{\frac{n^6+6}{2n^6+3}}$$
$$=\big[e^{-4}\big]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n^6+3}=\lim_{n\to\infty}\frac{n^6+6}{2n^6+3}\cdot\frac{\frac{1}{n^6}}{\frac{1}{n^6}}=\lim_{n\to\infty}\frac{1+\frac{6}{n^6}}{2+\frac{3}{n^6}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}$$

$$=\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{1-\infty+0+0}{0-3+0}=\frac{-\infty}{-3}$$

$$=\lim_{n\to\infty}\frac{n^4(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{n^3(\frac{2}{n}-3+\frac{5}{n^3})}=\lim_{n\to\infty}\frac{n(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4})}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{\infty\cdot(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty})}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{\infty\cdot(0-1+0+0)}{0-3+0}$$
$$=\frac{\infty\cdot(-1)}{-3}$$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n^6+6}{2n^6+3}}$$
$$=\left[e^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n^6+6}{2n^6+3}=\lim_{n\to\infty}\frac{n^6+6}{2n^6+3}\cdot\frac{\frac{1}{n^6}}{\frac{1}{n^6}}=\lim_{n\to\infty}\frac{1+\frac{6}{n^6}}{2+\frac{3}{n^6}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}=\infty$$

$$=\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{1-\infty+0+0}{0-3+0}=\frac{-\infty}{-3}=\infty.$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{n^3\left(\frac{2}{n}-3+\frac{5}{n^3}\right)}=\lim_{n\to\infty}\frac{n\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{\infty\cdot\left(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty}\right)}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{\infty\cdot(0-1+0+0)}{0-3+0}$$
$$=\frac{\infty\cdot(-1)}{-3}=\infty.$$

Riešené limity – 53, 54

$$\lim_{n\to\infty}\left(\frac{2n^6-1}{2n^6+3}\right)^{n^6+6}=\mathrm{e}^{-2}$$

$$=\lim_{n\to\infty}\left(\frac{2n^6+3-4}{2n^6+3}\right)^{n^6+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{n^6+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}\right]^{\frac{n^6+6}{2n^6+3}}$$
$$=\left[\mathrm{e}^{-4}\right]^{\frac{1}{2}}=\mathrm{e}^{-2}.$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n^6+3}\right)^{2n^6+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n^6+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^m=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^n=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^n=\mathrm{e}^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=\mathrm{e}^{-4},$$
$$\lim_{n\to\infty}\frac{n^6+6}{2n^6+3}=\lim_{n\to\infty}\frac{n^6+6}{2n^6+3}\cdot\frac{\frac{1}{n^6}}{\frac{1}{n^6}}=\lim_{n\to\infty}\frac{1+\frac{6}{n^6}}{2+\frac{3}{n^6}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}=\infty$$

$$=\lim_{n\to\infty}\frac{n^3-n^4+2n^2+2}{2n^2-3n^3+5}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{1-n+\frac{2}{n}+\frac{2}{n^3}}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{1-\infty+\frac{2}{\infty}+\frac{2}{\infty}}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{1-\infty+0+0}{0-3+0}=\frac{-\infty}{-3}=\infty.$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{n^3\left(\frac{2}{n}-3+\frac{5}{n^3}\right)}=\lim_{n\to\infty}\frac{n\left(\frac{1}{n}-1+\frac{2}{n^2}+\frac{2}{n^4}\right)}{\frac{2}{n}-3+\frac{5}{n^3}}=\frac{\infty\cdot\left(\frac{1}{\infty}-1+\frac{2}{\infty}+\frac{2}{\infty}\right)}{\frac{2}{\infty}-3+\frac{5}{\infty}}=\frac{\infty\cdot(0-1+0+0)}{0-3+0}$$
$$=\frac{\infty\cdot(-1)}{-3}=\infty.$$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}}$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}$$

Riešené limity – 55, 56

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty} \left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$= \lim_{n\to\infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}$$

$$= \lim_{n\to\infty} \frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim_{n\to\infty} \frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}$$

Riešené limity – 55, 56

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty}\left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}}$$

$$\lim_{n\to\infty} \frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$= \lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}$$

$$= \lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)} = \lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}} = \frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}$$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}}$$

$$=\left[\mathrm{e}^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=\mathrm{e}^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=\mathrm{e}^{-4},$$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}$$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}}$$

$$=\left[\mathrm{e}^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}=\frac{3}{-1}$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}=\frac{3}{-1}$$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}}$$

$$=\left[\mathrm{e}^{-4}\right]^{\frac{1}{2}}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim\limits_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=\mathrm{e}^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=\mathrm{e}^{-4},$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}=-3$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}=\frac{3}{-1}=-3.$$

$$=\lim_{n\to\infty}\frac{n^4\left(\frac{2}{n^2}+3+\frac{5}{n^4}\right)}{n^4\left(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\right)}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}=\frac{3}{-1}=-3.$$

Riešené limity – 55, 56

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\Big)^{\sqrt[6]{n}+6}=e^{-2}$$

$$=\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\Big)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\Big[\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3}\Big]^{\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}}$$

$$=\big[e^{-4}\big]^{\frac{1}{2}}=e^{-2}.$$

$$\lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1-\frac{4}{m}\Big)^m=\left[\begin{array}{c}\lim_{n\to\infty}\big(1-\frac{a}{n}\big)^n=\lim_{n\to\infty}\big(1+\frac{-a}{n}\big)^n=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{1+0}{2+0}=\frac{1}{2}.$$

$$\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}=-3$$

$$=\lim_{n\to\infty}\frac{2n^2+3n^4+5}{n^3-n^4-n^2+2}\cdot\frac{\frac{1}{n^4}}{\frac{1}{n^4}}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}=\frac{3}{-1}=-3.$$

$$=\lim_{n\to\infty}\frac{n^4\big(\frac{2}{n^2}+3+\frac{5}{n^4}\big)}{n^4\big(\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}\big)}=\lim_{n\to\infty}\frac{\frac{2}{n^2}+3+\frac{5}{n^4}}{\frac{1}{n}-1-\frac{1}{n^2}+\frac{2}{n^4}}=\frac{\frac{2}{\infty}+3+\frac{5}{\infty}}{\frac{1}{\infty}-1-\frac{1}{\infty}+\frac{2}{\infty}}=\frac{0+3+0}{0-1-0+0}=\frac{3}{-1}=-3.$$

Riešené limity – 57, 58

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2^n\, n!}{n^n}$$

Riešené limity – 57, 58

$$\lim_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2^n\, n!}{n^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\, n!}{n^n}}$$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}$$

Riešené limity – 57, 58

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$

$$= \lim_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim_{n\to\infty} \left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{2^n\, n!}{n^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\, n!}{n^n}} = \lim_{n\to\infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{2n^n}{(n+1)^n}$$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{0}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{n}$$

Riešené limity – 57, 58

$\lim\limits_{n\to\infty} \left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$

$= \lim\limits_{n\to\infty} \left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty} \left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6} = \lim\limits_{n\to\infty} \left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$
$= \left[e^{-4}\right]^{0}$

$\lim\limits_{n\to\infty} \frac{\sqrt[6]{n}+6}{2n+3} = \lim\limits_{n\to\infty} \frac{\sqrt[6]{n}+6}{2n+3} \cdot \frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}} = \lim\limits_{n\to\infty} \frac{1+\frac{6}{\sqrt[6]{n}}}{2\sqrt[6]{n^5}+\frac{3}{\sqrt[6]{n}}} = \frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}} = \frac{1+0}{\infty+0} = \frac{1}{\infty} = 0.$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{2^n\, n!}{n^n}$

$\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\, n!}{n^n}} = \lim\limits_{n\to\infty} \frac{2(n+1)n^n}{(n+1)^{n+1}} = \lim\limits_{n\to\infty} \frac{2n^n}{(n+1)^n} = 2 \lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^n = 2 \lim\limits_{n\to\infty} \left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$$
$$=\left[e^{-4}\right]^{0}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2\sqrt[6]{n^5}+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=2\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$
$$=2\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}$$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{0}=e^{0}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2\sqrt[6]{n^5}+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=2\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$=2\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}=2\,e^{-1}=\frac{2}{e}$$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}=1$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$$
$$=\left[e^{-4}\right]^{0}=e^{0}=1.$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2\sqrt[6]{n^5}+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=2\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$
$$=2\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}=2\,e^{-1}=\frac{2}{e}\approx\frac{2}{2{,}718}<1$$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}=1$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{0}=e^{0}=1.$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim\limits_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2\sqrt[6]{n^5}+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=2\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$=2\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}=2\,e^{-1}=\frac{2}{e}\approx\frac{2}{2{,}718}<1\qquad\Rightarrow\ \lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}=0.$$

Riešené limity – 57, 58

$$\lim_{n\to\infty}\left(\frac{2n-1}{2n+3}\right)^{\sqrt[6]{n}+6}=1$$

$$=\lim_{n\to\infty}\left(\frac{2n+3-4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{\sqrt[6]{n}+6}=\lim_{n\to\infty}\left[\left(1-\frac{4}{2n+3}\right)^{2n+3}\right]^{\frac{\sqrt[6]{n}+6}{2n+3}}$$

$$=\left[e^{-4}\right]^{0}=e^{0}=1.$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2n+3}\right)^{2n+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2n+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}=\lim_{n\to\infty}\frac{\sqrt[6]{n}+6}{2n+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{1+\frac{6}{\sqrt[6]{n}}}{2\sqrt[6]{n^5}+\frac{3}{\sqrt[6]{n}}}=\frac{1+\frac{6}{\infty}}{2\cdot\infty+\frac{3}{\infty}}=\frac{1+0}{\infty+0}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}=0$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{2^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{2(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{2n^n}{(n+1)^n}=2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=2\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$=2\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}=2\,e^{-1}=\frac{2}{e}\approx\frac{2}{2{,}718}<1\qquad\Rightarrow\quad\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2^n\,n!}{n^n}=0.$$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n\, n!}{n^n}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n\, n!}{n^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\, n!}{n^n}}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n\, n!}{n^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\, n!}{n^n}} = \lim_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{n+6}{2\sqrt[6]{n}+3}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n\, n!}{n^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\, n!}{n^n}} = \lim_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{3n^n}{(n+1)^n}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\Big)^{n+6}$$

$$= \lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\Big)^{n+6} = \lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{n+6}$$

$$= \lim_{n\to\infty}\Bigg[\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3}\Bigg]^{\frac{n+6}{2\sqrt[6]{n}+3}} = \big[e^{-4}\big]^{\infty}$$

$$\lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\Big(1-\frac{4}{m}\Big)^{m} = \left[\begin{array}{c}\lim_{n\to\infty}\big(1-\frac{a}{n}\big)^{n} = \lim_{n\to\infty}\big(1+\frac{-a}{n}\big)^{n} = e^{-a} \\ \text{pre všetky } a\in R\end{array}\right] = e^{-4},$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{3^n\, n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\, n!}{n^n}} = \lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n$$

Riešené limity – 59, 60

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty} \left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{n+6}{2\sqrt[6]{n}+3}} = \left[e^{-4}\right]^{\infty}$$

$$\lim_{n\to\infty} \frac{n+6}{2\sqrt[6]{n}+3} = \lim_{n\to\infty} \frac{n+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}} = \lim_{n\to\infty} \frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n\, n!}{n^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\, n!}{n^n}} = \lim_{n\to\infty} \frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty} \frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = 3\lim_{n\to\infty} \left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\Big)^{n+6}$$

$$=\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\Big)^{n+6}=\lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{n+6}$$

$$=\lim_{n\to\infty}\Big[\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3}\Big]^{\frac{n+6}{2\sqrt[6]{n}+3}}=\big[e^{-4}\big]^{\infty}$$

$$\lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1-\frac{4}{m}\Big)^{m}=\left[\begin{array}{c}\lim\limits_{n\to\infty}\big(1-\frac{a}{n}\big)^{n}=\lim\limits_{n\to\infty}\big(1+\frac{-a}{n}\big)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{3n^n}{(n+1)^n}=3\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n=3\lim_{n\to\infty}\Big[\Big(\frac{n+1}{n}\Big)^n\Big]^{-1}$$

$$=3\lim_{n\to\infty}\Big[\Big(1+\frac{1}{n}\Big)^n\Big]^{-1}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{n+6}{2\sqrt[6]{n}+3}}=\left[e^{-4}\right]^{\infty}=e^{-\infty}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{3n^n}{(n+1)^n}=3\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=3\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$=3\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}=3\,e^{-1}=\frac{3}{e}$$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6} = \lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$= \lim_{n\to\infty}\left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{n+6}{2\sqrt[6]{n}+3}} = \left[e^{-4}\right]^{\infty} = e^{-\infty} = \frac{1}{e^{\infty}}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1-\frac{4}{m}\right)^m = \left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1-\frac{a}{n}\right)^n = \lim\limits_{n\to\infty}\left(1+\frac{-a}{n}\right)^n = e^{-a} \\ \text{pre všetky } a\in R\end{array}\right] = e^{-4},$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3} = \lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}} = \lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}} = \frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}} = \frac{\infty+0}{2+0} = \infty.$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{3^n\, n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\, n!}{n^n}} = \lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}} = \lim_{n\to\infty}\frac{3n^n}{(n+1)^n} = 3\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n = 3\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$= 3\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1} = 3\,e^{-1} = \frac{3}{e} \approx \frac{3}{2{,}718} > 1$$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\right)^{n+6}=\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left[\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}\right]^{\frac{n+6}{2\sqrt[6]{n}+3}}=\left[e^{-4}\right]^{\infty}=e^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}$$

$$\lim_{n\to\infty}\left(1-\frac{4}{2\sqrt[6]{n}+3}\right)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1-\frac{4}{m}\right)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\left(1-\frac{a}{n}\right)^{n}=\lim_{n\to\infty}\left(1+\frac{-a}{n}\right)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n\,n!}{n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{3n^n}{(n+1)^n}=3\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=3\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$=3\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}=3\,e^{-1}=\frac{3}{e}\approx\frac{3}{2{,}718}>1\qquad\Rightarrow\ \lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n\,n!}{n^n}=\infty.$$

Riešené limity – 59, 60

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}+3}\Big)^{n+6}=0$$

$$=\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}+3-4}{2\sqrt[6]{n}+3}\Big)^{n+6}=\lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{n+6}$$

$$=\lim_{n\to\infty}\Big[\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3}\Big]^{\frac{n+6}{2\sqrt[6]{n}+3}}=\big[e^{-4}\big]^{\infty}=e^{-\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0.$$

$$\lim_{n\to\infty}\Big(1-\frac{4}{2\sqrt[6]{n}+3}\Big)^{2\sqrt[6]{n}+3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}+3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1-\frac{4}{m}\Big)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\big(1-\frac{a}{n}\big)^{n}=\lim_{n\to\infty}\big(1+\frac{-a}{n}\big)^{n}=e^{-a}\\ \text{pre všetky } a\in R\end{array}\right]=e^{-4},$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}+3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2+\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2+\frac{3}{\infty}}=\frac{\infty+0}{2+0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n\,n!}{n^n}=\infty$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{3^{n+1}\,(n+1)!}{(n+1)^{n+1}}}{\frac{3^n\,n!}{n^n}}=\lim_{n\to\infty}\frac{3(n+1)n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{3n^n}{(n+1)^n}=3\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n=3\lim_{n\to\infty}\Big[\Big(\frac{n+1}{n}\Big)^n\Big]^{-1}$$

$$=3\lim_{n\to\infty}\Big[\Big(1+\frac{1}{n}\Big)^n\Big]^{-1}=3\,e^{-1}=\frac{3}{e}\approx\frac{3}{2{,}718}>1\qquad\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n\,n!}{n^n}=\infty.$$

Riešené limity – 61, 62

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{8^n\, n!}{(3n)^n} = \lim_{n\to\infty} \frac{8^n\, n!}{3^n n^n}$$

Riešené limity – 61, 62

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{8^n\, n!}{(3n)^n} = \lim_{n\to\infty} \frac{8^n\, n!}{3^n n^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\, n!}{3^n\, n^n}}$$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n\,n!}{3^n\,n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\,n!}{3^n\,n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}$$

Riešené limity – 61, 62

$$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty} \left[\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{8^n\, n!}{(3n)^n} = \lim_{n\to\infty} \frac{8^n\, n!}{3^n n^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\, n!}{3^n\, n^n}} = \lim_{n\to\infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n\to\infty} \frac{8n^n}{3(n+1)^n}$$

Riešené limity – 61, 62

$\lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$

$$= \lim_{n\to\infty} \left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6} = \lim_{n\to\infty} \left[\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$= \left[e^2\right]^{\infty}$$

$$\lim_{n\to\infty} \left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3} = \left[\begin{array}{l|l} \text{Subst.} & n\to\infty \\ m=2\sqrt[6]{n}-3 & m\to\infty \end{array}\right] = \lim_{m\to\infty} \left(1+\frac{2}{m}\right)^{m} = \left[\begin{array}{c} \lim_{n\to\infty} \left(1+\frac{a}{n}\right)^{n} = e^{a} \\ \text{pre všetky } a\in R \end{array}\right] = e^2,$$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{8^n\, n!}{(3n)^n} = \lim_{n\to\infty} \frac{8^n\, n!}{3^n n^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\, n!}{3^n\, n^n}} = \lim_{n\to\infty} \frac{8(n+1)n^n}{3(n+1)^{n+1}} = \lim_{n\to\infty} \frac{8n^n}{3(n+1)^n} = \frac{8}{3} \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\Big)^{n+6}$$

$$=\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big[\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}\Big]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$=\big[e^2\big]^{\infty}$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\,n!}{3^n\,n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n=\frac{8}{3}\lim_{n\to\infty}\Big[\Big(\frac{n+1}{n}\Big)^n\Big]^{-1}$$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$=\left[e^2\right]^{\infty}$$

$$\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{2}{m}\right)^m=\left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a\\ \text{pre všetky } a\in R\end{array}\right]=e^2,$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\,n!}{3^n\,n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=\frac{8}{3}\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$=\frac{8}{3}\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}$$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\Big)^{n+6}$$

$$=\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big[\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}\Big]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$=\big[\mathrm{e}^2\big]^{\infty}=\mathrm{e}^{\infty}$$

$$\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1+\frac{2}{m}\Big)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n=\mathrm{e}^a\\ \text{pre všetky } a\in R\end{array}\right]=\mathrm{e}^2,$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\,n!}{3^n\,n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n=\frac{8}{3}\lim_{n\to\infty}\Big[\Big(\frac{n+1}{n}\Big)^n\Big]^{-1}$$

$$=\frac{8}{3}\lim_{n\to\infty}\Big[\Big(1+\frac{1}{n}\Big)^n\Big]^{-1}=\frac{8}{3}\,\mathrm{e}^{-1}=\frac{8}{3\,\mathrm{e}}$$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\Big)^{n+6}=\infty$$

$$=\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big[\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}\Big]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$=\big[e^2\big]^{\infty}=e^{\infty}=\infty.$$

$$\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1+\frac{2}{m}\Big)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^{n}=e^{a}\\ \text{pre všetky } a\in R\end{array}\right]=e^2,$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\,n!}{3^n\,n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n=\frac{8}{3}\lim_{n\to\infty}\Big[\Big(\frac{n+1}{n}\Big)^n\Big]^{-1}$$

$$=\frac{8}{3}\lim_{n\to\infty}\Big[\Big(1+\frac{1}{n}\Big)^n\Big]^{-1}=\frac{8}{3}e^{-1}=\frac{8}{3e}\approx\frac{8}{3\cdot 2{,}718}=\frac{8}{8{,}154}<1$$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\Big)^{n+6}=\infty$$

$$=\lim_{n\to\infty}\Big(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{n+6}=\lim_{n\to\infty}\Big[\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}\Big]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$=\big[\mathrm{e}^2\big]^{\infty}=\mathrm{e}^{\infty}=\infty.$$

$$\lim_{n\to\infty}\Big(1+\frac{2}{2\sqrt[6]{n}-3}\Big)^{2\sqrt[6]{n}-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1+\frac{2}{m}\Big)^{m}=\left[\begin{array}{c}\lim_{n\to\infty}\big(1+\frac{a}{n}\big)^n=\mathrm{e}^a\\ \text{pre všetky } a\in R\end{array}\right]=\mathrm{e}^2,$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\,n!}{3^n\,n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\Big(\frac{n}{n+1}\Big)^n=\frac{8}{3}\lim_{n\to\infty}\Big[\Big(\frac{n+1}{n}\Big)^n\Big]^{-1}$$

$$=\frac{8}{3}\lim_{n\to\infty}\Big[\Big(1+\frac{1}{n}\Big)^n\Big]^{-1}=\frac{8}{3}\,\mathrm{e}^{-1}=\frac{8}{3\,\mathrm{e}}\approx\frac{8}{3\cdot 2{,}718}=\frac{8}{8{,}154}<1\quad\Rightarrow\quad\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}=0.$$

Riešené limity – 61, 62

$$\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-1}{2\sqrt[6]{n}-3}\right)^{n+6}=\infty$$

$$=\lim_{n\to\infty}\left(\frac{2\sqrt[6]{n}-3+2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{n+6}=\lim_{n\to\infty}\left[\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}\right]^{\frac{n+6}{2\sqrt[6]{n}-3}}$$

$$=\left[e^2\right]^{\infty}=e^{\infty}=\infty.$$

$$\lim_{n\to\infty}\left(1+\frac{2}{2\sqrt[6]{n}-3}\right)^{2\sqrt[6]{n}-3}=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=2\sqrt[6]{n}-3 & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{2}{m}\right)^m=\left[\begin{array}{c}\lim\limits_{n\to\infty}\left(1+\frac{a}{n}\right)^n=e^a\\ \text{pre všetky } a\in R\end{array}\right]=e^2,$$

$$\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}=\lim_{n\to\infty}\frac{n+6}{2\sqrt[6]{n}-3}\cdot\frac{\frac{1}{\sqrt[6]{n}}}{\frac{1}{\sqrt[6]{n}}}=\lim_{n\to\infty}\frac{\sqrt[6]{n^5}+\frac{6}{\sqrt[6]{n}}}{2-\frac{3}{\sqrt[6]{n}}}=\frac{\infty+\frac{6}{\infty}}{2-\frac{3}{\infty}}=\frac{\infty+0}{2-0}=\infty.$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{(3n)^n}=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}=0$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{8^{n+1}\,(n+1)!}{3^{n+1}\,(n+1)^{n+1}}}{\frac{8^n\,n!}{3^n\,n^n}}=\lim_{n\to\infty}\frac{8(n+1)n^n}{3(n+1)^{n+1}}=\lim_{n\to\infty}\frac{8n^n}{3(n+1)^n}=\frac{8}{3}\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=\frac{8}{3}\lim_{n\to\infty}\left[\left(\frac{n+1}{n}\right)^n\right]^{-1}$$

$$=\frac{8}{3}\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^n\right]^{-1}=\frac{8}{3}e^{-1}=\frac{8}{3e}\approx\frac{8}{3\cdot 2{,}718}=\frac{8}{8{,}154}<1\quad\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{8^n\,n!}{3^n n^n}=0.$$

Riešené limity – 63, 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

Riešené limity – 63, 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^3} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty \right]$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{7^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{7}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{7}=\frac{1^k}{7}=\frac{1}{7}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{7n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{7}=\frac{(1+\frac{1}{\infty})^k}{7}=\frac{(1+0)^k}{7}=\frac{1}{7}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{7^n}=0 \right]$$

Riešené limity – 63, 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\text{Označme } a_n=\frac{5^n}{n^3} \text{ pre všetky } n\in N \;\middle|\; \begin{array}{l} \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty \right]$$

$$= \lim_{n\to\infty} \frac{n^4\left(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4}\right)}{n^3\left(\frac{2}{n}-1\right)}$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

$$= \left[\text{Označme } a_n=\frac{n^k}{7^n} \text{ pre } n\in N,\ k\in N \;\middle|\; \begin{array}{l} \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{7}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{7}=\frac{1^k}{7}=\frac{1}{7}<1 \\ \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{7n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{7}=\frac{(1+\frac{1}{\infty})^k}{7}=\frac{(1+0)^k}{7}=\frac{1}{7}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{7^n}=0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}\cdot\frac{\frac{1}{7^n}}{\frac{1}{7^n}}$$

Riešené limity – 63, 64

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{5^n}{n^3} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \text{pre všetky } n\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty\right]$$

$$=\lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)}=\lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{7^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{7^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{7}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{7}=\frac{1^k}{7}=\frac{1}{7}<1 \\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{7n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{7}=\frac{(1+\frac{1}{\infty})^k}{7}=\frac{(1+0)^k}{7}=\frac{1}{7}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{7^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}\cdot\frac{\frac{1}{7^n}}{\frac{1}{7^n}}=\lim_{n\to\infty}\frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6}$$

Riešené limity – 63, 64

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{5^n}{n^3} & \left.\begin{array}{l}\lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1\\ \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1\end{array}\right\}\Rightarrow \lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{5^n}{n^3}=\infty \\ \text{pre všetky } n\in N & \end{array}\right]$$

$$=\lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)}=\lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}=\frac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1}$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{7^n} & \left.\begin{array}{l}\lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{7}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{7}=\frac{1^k}{7}=\frac{1}{7}<1\\ \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{7n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{7}=\frac{(1+\frac{1}{\infty})^k}{7}=\frac{(1+0)^k}{7}=\frac{1}{7}<1\end{array}\right\}\Rightarrow \lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{n^k}{7^n}=0 \\ \text{pre } n\in N,\ k\in N & \end{array}\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}\cdot\frac{\frac{1}{7^n}}{\frac{1}{7^n}}=\lim_{n\to\infty}\frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6}=\left[\begin{array}{l}\text{Geometrická postupnosť}\\ \lim\limits_{n\to\infty}\frac{5^n}{7^n}=\lim\limits_{n\to\infty}(\frac{5}{7})^n=0\end{array}\right]$$

Riešené limity – 63, 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^3} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty \right]$$

$$=\lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)}=\lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}=\frac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1}$$

$$=\frac{\infty(0+3-0-\infty)}{0-1}$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{7^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{7}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{7}=\frac{1^k}{7}=\frac{1}{7}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{7n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{7}=\frac{(1+\frac{1}{\infty})^k}{7}=\frac{(1+0)^k}{7}=\frac{1}{7}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{7^n}=0 \right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}\cdot\frac{\frac{1}{7^n}}{\frac{1}{7^n}}=\lim_{n\to\infty}\frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6}=\left[\begin{array}{l}\text{Geometrická postupnosť} \\ \lim\limits_{n\to\infty}\frac{5^n}{7^n}=\lim\limits_{n\to\infty}(\frac{5}{7})^n=0\end{array}\right]$$

$$=\frac{2\cdot 0+3\cdot 0-0+2\cdot 0}{2\cdot 0-0+6}$$

Riešené limity – 63, 64

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2-2\cdot5^n}{2n^2-n^3}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{5^n}{n^3} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty\right]$$

$$=\lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)}=\lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}=\frac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1}$$

$$=\frac{\infty(0+3-0-\infty)}{0-1}=\frac{-\infty}{-1}$$

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+6\cdot7^n}$$

$$=\left[\begin{array}{l|l}\text{Označme } a_n=\frac{n^k}{7^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{7}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{7}=\frac{1^k}{7}=\frac{1}{7}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{7n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{7}=\frac{(1+\frac{1}{\infty})^k}{7}=\frac{(1+0)^k}{7}=\frac{1}{7}<1\end{array}\right\}\Rightarrow\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{7^n}=0\right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot5^n}{2n^2-n^3+6\cdot7^n}\cdot\frac{\frac{1}{7^n}}{\frac{1}{7^n}}=\lim_{n\to\infty}\frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6}=\left[\begin{array}{l}\text{Geometrická postupnosť} \\ \lim\limits_{n\to\infty}\frac{5^n}{7^n}=\lim\limits_{n\to\infty}\left(\frac{5}{7}\right)^n=0\end{array}\right]$$

$$=\frac{2\cdot0+3\cdot0-0+2\cdot0}{2\cdot0-0+6}=\frac{0}{6}$$

Riešené limity – 63, 64

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2-2\cdot 5^n}{2n^2-n^3} = \infty$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^3} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \text{pre všetky } n\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty \right]$$

$$= \lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}-2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \frac{\infty(\frac{2}{\infty}+3-\frac{1}{\infty}-2\cdot\infty)}{\frac{2}{\infty}-1}$$

$$= \frac{\infty(0+3-0-\infty)}{0-1} = \frac{-\infty}{-1} = \infty.$$

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n} = 0$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{7^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{7^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{7}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{7}=\frac{1^k}{7}=\frac{1}{7}<1 \\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{7^{n+1}}}{\frac{n^k}{7^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{7n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{7}=\frac{(1+\frac{1}{\infty})^k}{7}=\frac{(1+0)^k}{7}=\frac{1}{7}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^k}{7^n}=0 \right]$$

$$= \lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 7^n}\cdot\frac{\frac{1}{7^n}}{\frac{1}{7^n}} = \lim_{n\to\infty}\frac{2\cdot\frac{n^3}{7^n}+3\cdot\frac{n^4}{7^n}-\frac{n^2}{7^n}+2\cdot\frac{5^n}{7^n}}{2\cdot\frac{n^2}{7^n}-\frac{n^3}{7^n}+6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{7^n}=\lim_{n\to\infty}(\frac{5}{7})^n=0 \end{array} \right]$$

$$= \frac{2\cdot 0+3\cdot 0-0+2\cdot 0}{2\cdot 0-0+6} = \frac{0}{6} = 0.$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$= \left[\begin{array}{c} \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}} \\ \text{pre všetky } n \in N \end{array} \right]$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$= \left[\begin{array}{c} \sqrt[3]{n^5} = \sqrt[6]{n^{10}} = \sqrt[12]{n^{20}} = \sqrt[15]{n^{25}} \\ \text{pre všetky } n\in N \end{array} \right] = \lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\left[\begin{array}{c}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\left[\begin{array}{c}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\left[\begin{matrix}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}\\ \text{pre všetky } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(\frac{n^6-2}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(\frac{n+1}{n}\right)^3}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(\frac{n-1}{n}\right)^3}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$= \left[\begin{matrix}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}\\ \text{pre všetky } n\in N\end{matrix}\right] = \lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(\frac{n^6-2}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(\frac{n+1}{n}\right)^3}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(1-\frac{1}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(1-\frac{1}{n}\right)}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\left[\begin{array}{c}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(\frac{n^6-2}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(\frac{n+1}{n}\right)^3}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(\frac{n-1}{n}\right)^3}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(1-\frac{1}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(1-\frac{1}{n}\right)}}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$= \left[\begin{array}{c} \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}} \\ \text{pre všetky } n\in N \end{array} \right] = \lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(\frac{n^6-2}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(\frac{n+1}{n}\right)^3}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(1-\frac{1}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(1-\frac{1}{n}\right)}} = \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N \right]$$

$$= \frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\sqrt[6]{0\cdot(1+0)}}{2\sqrt[3]{1+0}+3\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\left[\begin{array}{c}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(\frac{n^6-2}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(\frac{n+1}{n}\right)^3}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(\frac{n-1}{n}\right)^3}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(1-\frac{1}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(1-\frac{1}{n}\right)}}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$=\frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\sqrt[6]{0\cdot(1+0)}}{2\sqrt[3]{1+0}+3\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}}=\frac{0-0+3\cdot 0}{2\cdot 1+3\cdot 0-0}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\left[\begin{array}{c}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}}\\ \text{pre všetky } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}}\cdot\frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}}=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(\frac{n^6-2}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(\frac{n+1}{n}\right)^3}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(\frac{n-1}{n}\right)^3}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(1-\frac{1}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(1-\frac{1}{n}\right)}}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$=\frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\sqrt[6]{0\cdot(1+0)}}{2\sqrt[3]{1+0}+3\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}}=\frac{0-0+3\cdot 0}{2\cdot 1+3\cdot 0-0}=\frac{0}{2}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = 0$$

$$= \left[\begin{matrix}\sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}}=\sqrt[15]{n^{25}} \\ \text{pre všetky } n\in N\end{matrix}\right] = \lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[15]{(n^6-2)^3}+3\sqrt[6]{(n+1)^3}}{2\sqrt[3]{n^5+1}+3\sqrt[12]{(n^6-1)^3}-\sqrt[6]{(n-1)^3}} \cdot \frac{\frac{1}{\sqrt[3]{n^5}}}{\frac{1}{\sqrt[3]{n^5}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^{25}}}+3\sqrt[6]{\frac{(n+1)^3}{n^{10}}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^{20}}}-\sqrt[6]{\frac{(n-1)^3}{n^{10}}}} = \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{(n^6-2)^3}{n^7\cdot(n^6)^3}}+3\sqrt[6]{\frac{(n+1)^3}{n^7\cdot n^3}}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{(n^6-1)^3}{n^2\cdot(n^6)^3}}-\sqrt[6]{\frac{(n-1)^3}{n^7\cdot n^3}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{n^4-1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(\frac{n^6-2}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(\frac{n+1}{n}\right)^3}}{2\sqrt[3]{\frac{n^5+1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(\frac{n^6-1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(\frac{n-1}{n}\right)^3}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{\frac{1}{n}-\frac{1}{n^5}}-\sqrt[15]{\frac{1}{n^7}\left(1-\frac{1}{n^6}\right)^3}+3\sqrt[6]{\frac{1}{n^7}\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}\left(1-\frac{1}{n^6}\right)^3}-\sqrt[6]{\frac{1}{n^7}\left(1-\frac{1}{n}\right)}} = \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$= \frac{\sqrt[3]{0-0}-\sqrt[15]{0\cdot(1-0)^3}+3\sqrt[6]{0\cdot(1+0)}}{2\sqrt[3]{1+0}+3\sqrt[12]{0\cdot(1-0)^3}-\sqrt[6]{0\cdot(1-0)}} = \frac{0-0+3\cdot 0}{2\cdot 1+3\cdot 0-0} = \frac{0}{2} = 0.$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}}=\left[\begin{array}{c|c|c}\text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}}\\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}}\end{array}\right]$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}}=\left[\begin{array}{c|c|c}\text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}}\\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}}\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}}=\left[\begin{array}{c|c|c}\text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}}\\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}}\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)}=\left[\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}\text{ pre všetky } k\in N\right]$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}} \\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}} \end{array}\right]$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)} = \left[\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}} \text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt[6]{\frac{1}{n^5}}\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\sqrt{1-\frac{1}{n}}\right)}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}} \\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)} = \left[\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}} \text{ pre všetky } k\in N\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt[6]{\frac{1}{n^5}}\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\sqrt{1-\frac{1}{n}}\right)} = \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}}=\left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}} \\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}} \end{array}\right]$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)} = \left[\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}} \text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty} \frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt[6]{\frac{1}{n^5}}\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\sqrt{1-\frac{1}{n}}\right)} = \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$=\frac{\sqrt[3]{1-0}-\sqrt[15]{0}\sqrt[5]{1-0}+3\sqrt[6]{0}\sqrt{1+0}}{\sqrt[3]{n}(2\sqrt[3]{1+0}+3\sqrt[12]{0}\sqrt[4]{1-0}-\sqrt[6]{0}\sqrt{1-0})}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}}=\left[\begin{array}{c|c|c}\text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}}\\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}}\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)}=\left[\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}}\text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt[6]{\frac{1}{n^5}}\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\sqrt{1-\frac{1}{n}}\right)}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0\text{ pre všetky } k\in N\right]$$

$$=\frac{\sqrt[3]{1-0}-\sqrt[15]{0}\sqrt[5]{1-0}+3\sqrt[6]{0}\sqrt{1+0}}{\sqrt[3]{n}(2\sqrt[3]{1+0}+3\sqrt[12]{0}\sqrt[4]{1-0}-\sqrt[6]{0}\sqrt{1-0})}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty(2\cdot1+3\cdot0\cdot1-0\cdot1)}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}} \\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)} = \left[\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}}=\frac{1}{\sqrt[3]{n}} \text{ pre všetky } k\in N\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt[6]{\frac{1}{n^5}}\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\sqrt{1-\frac{1}{n}}\right)} = \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$= \frac{\sqrt[3]{1-0}-\sqrt[15]{0}\sqrt[5]{1-0}+3\sqrt[6]{0}\sqrt{1+0}}{\sqrt[3]{n}(2\sqrt[3]{1+0}+3\sqrt[12]{0}\sqrt[4]{1-0}-\sqrt[6]{0}\sqrt{1-0})} = \frac{1-0\cdot1+3\cdot0\cdot1}{\infty(2\cdot1+3\cdot0\cdot1-0\cdot1)} = \frac{1}{2\cdot\infty}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = 0$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4\left(1-\frac{1}{n^4}\right)}-\sqrt[5]{n^6\left(1-\frac{2}{n^6}\right)}+3\sqrt{n\left(1+\frac{1}{n}\right)}}{2\sqrt[3]{n^5\left(1+\frac{1}{n^5}\right)}+3\sqrt[4]{n^6\left(1-\frac{1}{n^6}\right)}-\sqrt{n\left(1-\frac{1}{n}\right)}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[5]{n^6}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt{n}\sqrt{1+\frac{1}{n}}}{2\sqrt[3]{n^5}\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[4]{n^6}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt{n}\sqrt{1-\frac{1}{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n^4}=\sqrt[6]{n^8}=\sqrt[15]{n^{20}} & \sqrt[5]{n^6}=\sqrt[15]{n^{18}} \\ \sqrt{n}=\sqrt[6]{n^3} & \sqrt[3]{n^5}=\sqrt[6]{n^{10}}=\sqrt[12]{n^{20}} & \sqrt[4]{n^6}=\sqrt[12]{n^{18}} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^4}\left(\sqrt[3]{1-\frac{1}{n^4}}-\frac{\sqrt[15]{n^{18}}}{\sqrt[15]{n^{20}}}\sqrt[5]{1-\frac{2}{n^6}}+3\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^8}}\sqrt{1+\frac{1}{n}}\right)}{\sqrt[3]{n^5}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\frac{\sqrt[12]{n^{18}}}{\sqrt[12]{n^{20}}}\sqrt[4]{1-\frac{1}{n^6}}-\frac{\sqrt[6]{n^3}}{\sqrt[6]{n^{10}}}\sqrt{1-\frac{1}{n}}\right)} = \left[\frac{\sqrt[3]{n^4}}{\sqrt[3]{n^5}} = \frac{1}{\sqrt[3]{n}} \text{ pre všetky } k\in N\right]$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{1-\frac{1}{n^4}}-\sqrt[15]{\frac{1}{n^2}}\sqrt[5]{1-\frac{2}{n^6}}+3\sqrt[6]{\frac{1}{n^5}}\sqrt{1+\frac{1}{n}}}{\sqrt[3]{n}\left(2\sqrt[3]{1+\frac{1}{n^5}}+3\sqrt[12]{\frac{1}{n^2}}\sqrt[4]{1-\frac{1}{n^6}}-\sqrt[6]{\frac{1}{n^7}}\sqrt{1-\frac{1}{n}}\right)} = \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$= \frac{\sqrt[3]{1-0}-\sqrt[15]{0}\sqrt[5]{1-0}+3\sqrt[6]{0}\sqrt{1+0}}{\sqrt[3]{n}(2\sqrt[3]{1+0}+3\sqrt[12]{0}\sqrt[4]{1-0}-\sqrt[6]{0}\sqrt{1-0})} = \frac{1-0\cdot 1+3\cdot 0\cdot 1}{\infty(2\cdot 1+3\cdot 0\cdot 1-0\cdot 1)} = \frac{1}{2\cdot\infty} = 0.$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

Riešené limity – 65

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = \lim_{n\to\infty} \frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]} = \left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}} \text{ pre všetky } k\in N\right]$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}\text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}\text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}} = \lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]} = \left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}} \text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]} = \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty \;\middle|\; \lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0 \text{ pre všetky } k,\ m\in N\right]$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}\text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0\text{ pre všetky } k\in N\right]$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty\;\middle|\;\lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0\text{ pre všetky } k,\ m\in N\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot 0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left[2\cdot(1+0)^{\frac{1}{3}}+3\cdot 0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right]}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}\text{ pre všetky }k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0\text{ pre všetky }k\in N\right]$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty\;\middle|\;\lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0\text{ pre všetky }k,\ m\in N\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left[2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right]}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty\cdot[2\cdot1+3\cdot0\cdot1-0\cdot1]}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}\text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0\text{ pre všetky } k\in N\right]$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty\;\middle|\;\lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0\text{ pre všetky } k,\ m\in N\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left[2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right]}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty\cdot[2\cdot1+3\cdot0\cdot1-0\cdot1]}=\frac{1}{\infty\cdot2}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}\text{ pre všetky }k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0\text{ pre všetky }k\in N\right]$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty\;\middle|\;\lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0\text{ pre všetky }k,\ m\in N\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left[2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right]}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty\cdot[2\cdot1+3\cdot0\cdot1-0\cdot1]}=\frac{1}{\infty\cdot2}=\frac{1}{\infty}$$

Riešené limity – 65

$$\lim_{n\to\infty}\frac{\sqrt[3]{n^4-1}-\sqrt[5]{n^6-2}+3\sqrt{n+1}}{2\sqrt[3]{n^5+1}+3\sqrt[4]{n^6-1}-\sqrt{n-1}}=\lim_{n\to\infty}\frac{(n^4-1)^{\frac{1}{3}}-(n^6-2)^{\frac{1}{5}}+3(n+1)^{\frac{1}{2}}}{2(n^5+1)^{\frac{1}{3}}+3(n^6-1)^{\frac{1}{4}}-(n-1)^{\frac{1}{2}}}=0$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}}(1+\frac{1}{n})^{\frac{1}{2}}}{2n^{\frac{5}{3}}(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}}(1-\frac{1}{n})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{4}{3}}\left[(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{6}{5}-\frac{4}{3}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{1}{2}-\frac{4}{3}}(1+\frac{1}{n})^{\frac{1}{2}}\right]}{n^{\frac{5}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{6}{4}-\frac{5}{3}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{1}{2}-\frac{5}{3}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\frac{n^{\frac{4}{3}}}{n^{\frac{5}{3}}}=n^{\frac{4}{3}-\frac{5}{3}}=n^{-\frac{1}{3}}=\frac{1}{n^{\frac{1}{3}}}\text{ pre všetky }k\in N\right]$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{\frac{18-20}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{\frac{3-8}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{\frac{18-20}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{\frac{3-10}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}$$

$$=\lim_{n\to\infty}\frac{(1-\frac{1}{n^4})^{\frac{1}{3}}-n^{-\frac{2}{15}}(1-\frac{2}{n^6})^{\frac{1}{5}}+3n^{-\frac{5}{6}}(1+\frac{1}{n})^{\frac{1}{2}}}{n^{\frac{1}{3}}\left[2(1+\frac{1}{n^5})^{\frac{1}{3}}+3n^{-\frac{2}{12}}(1-\frac{1}{n^6})^{\frac{1}{4}}-n^{-\frac{7}{6}}(1-\frac{1}{n})^{\frac{1}{2}}\right]}=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0\text{ pre všetky }k\in N\right]$$

$$=\left[\lim_{n\to\infty}n^{\frac{1}{3}}=\lim_{n\to\infty}\sqrt[3]{n}=\infty\;\middle|\;\lim_{n\to\infty}n^{-\frac{k}{m}}=\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0\text{ pre všetky }k,\ m\in N\right]$$

$$=\frac{(1-0)^{\frac{1}{3}}-0\cdot(1-0)^{\frac{1}{5}}+3\cdot0\cdot(1+0)^{\frac{1}{2}}}{\infty\cdot\left[2\cdot(1+0)^{\frac{1}{3}}+3\cdot0\cdot(1-0)^{\frac{1}{4}}-0\cdot(1-0)^{\frac{1}{2}}\right]}=\frac{1-0\cdot1+3\cdot0\cdot1}{\infty\cdot[2\cdot1+3\cdot0\cdot1-0\cdot1]}=\frac{1}{\infty\cdot2}=\frac{1}{\infty}=0.$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{(\frac{1}{3})^n}{(\frac{1}{5})^n-(\frac{1}{4})^n} = \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{4})^n}{(\frac{1}{5})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{5})^n}{(\frac{1}{4})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\left(\frac{1}{3}\right)^n}{\left(\frac{1}{5}\right)^n-\left(\frac{1}{4}\right)^n} = \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n}$$

$$\lim_{n\to\infty} \frac{\left(\frac{1}{4}\right)^n}{\left(\frac{1}{5}\right)^n-\left(\frac{1}{3}\right)^n} = \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n}$$

$$\lim_{n\to\infty} \frac{\left(\frac{1}{5}\right)^n}{\left(\frac{1}{4}\right)^n-\left(\frac{1}{3}\right)^n} = \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n}$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{(\frac{1}{3})^n}{(\frac{1}{5})^n-(\frac{1}{4})^n} = \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{4})^n}{(\frac{1}{5})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{5})^n}{(\frac{1}{4})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1}$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{\left(\frac{1}{3}\right)^n}{\left(\frac{1}{5}\right)^n-\left(\frac{1}{4}\right)^n} = \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4}{5}\right)^n-1} = \left[\begin{array}{c} \text{Geometrické postupnosti} \\ \lim_{n\to\infty} \left(\frac{4}{5}\right)^n=0,\ \lim_{n\to\infty} \left(\frac{4}{3}\right)^n=\infty \end{array} \right]$$

$$\lim_{n\to\infty} \frac{\left(\frac{1}{4}\right)^n}{\left(\frac{1}{5}\right)^n-\left(\frac{1}{3}\right)^n} = \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{\left(\frac{4}{5}\right)^n-\left(\frac{4}{3}\right)^n} = \left[\begin{array}{c} \text{Geometrické postupnosti} \\ \lim_{n\to\infty} \left(\frac{4}{5}\right)^n=0,\ \lim_{n\to\infty} \left(\frac{4}{3}\right)^n=\infty \end{array} \right]$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{5}\right)^n-1} = \left[\begin{array}{c} \text{Geometrické postupnosti} \\ \lim_{n\to\infty} \left(\frac{3}{4}\right)^n=0,\ \lim_{n\to\infty} \left(\frac{3}{5}\right)^n=0 \end{array} \right]$$

$$\lim_{n\to\infty} \frac{\left(\frac{1}{5}\right)^n}{\left(\frac{1}{4}\right)^n-\left(\frac{1}{3}\right)^n} = \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{\left(\frac{4}{5}\right)^n}{1-\left(\frac{4}{3}\right)^n} = \left[\begin{array}{c} \text{Geometrické postupnosti} \\ \lim_{n\to\infty} \left(\frac{4}{5}\right)^n=0,\ \lim_{n\to\infty} \left(\frac{4}{3}\right)^n=\infty \end{array} \right]$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{4}\right)^n-1} = \left[\begin{array}{c} \text{Geometrické postupnosti} \\ \lim_{n\to\infty} \left(\frac{3}{4}\right)^n=0,\ \lim_{n\to\infty} \left(\frac{3}{5}\right)^n=0 \end{array} \right]$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{(\frac{1}{3})^n}{(\frac{1}{5})^n-(\frac{1}{4})^n} = \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti} \\ \lim_{n\to\infty} (\frac{4}{5})^n=0,\ \lim_{n\to\infty} (\frac{4}{3})^n=\infty\end{array}\right] = \frac{\infty}{0-1}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{4})^n}{(\frac{1}{5})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n} = \left[\begin{array}{c}\text{Geometrické postupnosti} \\ \lim_{n\to\infty} (\frac{4}{5})^n=0,\ \lim_{n\to\infty} (\frac{4}{3})^n=\infty\end{array}\right] = \frac{1}{0-\infty}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti} \\ \lim_{n\to\infty} (\frac{3}{4})^n=0,\ \lim_{n\to\infty} (\frac{3}{5})^n=0\end{array}\right] = \frac{0}{0-1}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{5})^n}{(\frac{1}{4})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n} = \left[\begin{array}{c}\text{Geometrické postupnosti} \\ \lim_{n\to\infty} (\frac{4}{5})^n=0,\ \lim_{n\to\infty} (\frac{4}{3})^n=\infty\end{array}\right] = \frac{0}{1-\infty}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}} \cdot \frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti} \\ \lim_{n\to\infty} (\frac{3}{4})^n=0,\ \lim_{n\to\infty} (\frac{3}{5})^n=0\end{array}\right] = \frac{0}{0-1}$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty} \frac{(\frac{1}{3})^n}{(\frac{1}{5})^n-(\frac{1}{4})^n} = \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}\cdot\frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{3})^n}{(\frac{4}{5})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim_{n\to\infty}(\frac{4}{5})^n=0,\ \lim_{n\to\infty}(\frac{4}{3})^n=\infty\end{array}\right] = \frac{\infty}{0-1} = \frac{\infty}{-1}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{4})^n}{(\frac{1}{5})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}\cdot\frac{4^n}{4^n} = \lim_{n\to\infty} \frac{1}{(\frac{4}{5})^n-(\frac{4}{3})^n} = \left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim_{n\to\infty}(\frac{4}{5})^n=0,\ \lim_{n\to\infty}(\frac{4}{3})^n=\infty\end{array}\right] = \frac{1}{0-\infty} = \frac{1}{-\infty}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}\cdot\frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{4})^n}{(\frac{3}{5})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim_{n\to\infty}(\frac{3}{4})^n=0,\ \lim_{n\to\infty}(\frac{3}{5})^n=0\end{array}\right] = \frac{0}{0-1} = \frac{0}{-1}$$

$$\lim_{n\to\infty} \frac{(\frac{1}{5})^n}{(\frac{1}{4})^n-(\frac{1}{3})^n} = \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}\cdot\frac{4^n}{4^n} = \lim_{n\to\infty} \frac{(\frac{4}{5})^n}{1-(\frac{4}{3})^n} = \left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim_{n\to\infty}(\frac{4}{5})^n=0,\ \lim_{n\to\infty}(\frac{4}{3})^n=\infty\end{array}\right] = \frac{0}{1-\infty} = \frac{0}{-\infty}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}\cdot\frac{3^n}{3^n} = \lim_{n\to\infty} \frac{(\frac{3}{5})^n}{(\frac{3}{4})^n-1} = \left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim_{n\to\infty}(\frac{3}{4})^n=0,\ \lim_{n\to\infty}(\frac{3}{5})^n=0\end{array}\right] = \frac{0}{0-1} = \frac{0}{-1}$$

Riešené limity – 66, 67, 68

$$\lim_{n\to\infty}\frac{\left(\frac{1}{3}\right)^n}{\left(\frac{1}{5}\right)^n-\left(\frac{1}{4}\right)^n}=\lim_{n\to\infty}\frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}=-\infty$$

$$=\lim_{n\to\infty}\frac{\frac{1}{3^n}}{\frac{1}{5^n}-\frac{1}{4^n}}\cdot\frac{4^n}{4^n}=\lim_{n\to\infty}\frac{\left(\frac{4}{3}\right)^n}{\left(\frac{4}{5}\right)^n-1}=\left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim\limits_{n\to\infty}\left(\frac{4}{5}\right)^n=0,\ \lim\limits_{n\to\infty}\left(\frac{4}{3}\right)^n=\infty\end{array}\right]=\frac{\infty}{0-1}=\frac{\infty}{-1}=-\infty.$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{4}\right)^n}{\left(\frac{1}{5}\right)^n-\left(\frac{1}{3}\right)^n}=\lim_{n\to\infty}\frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}=0$$

$$=\lim_{n\to\infty}\frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}\cdot\frac{4^n}{4^n}=\lim_{n\to\infty}\frac{1}{\left(\frac{4}{5}\right)^n-\left(\frac{4}{3}\right)^n}=\left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim\limits_{n\to\infty}\left(\frac{4}{5}\right)^n=0,\ \lim\limits_{n\to\infty}\left(\frac{4}{3}\right)^n=\infty\end{array}\right]=\frac{1}{0-\infty}=\frac{1}{-\infty}=0.$$

$$=\lim_{n\to\infty}\frac{\frac{1}{4^n}}{\frac{1}{5^n}-\frac{1}{3^n}}\cdot\frac{3^n}{3^n}=\lim_{n\to\infty}\frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{5}\right)^n-1}=\left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim\limits_{n\to\infty}\left(\frac{3}{4}\right)^n=0,\ \lim\limits_{n\to\infty}\left(\frac{3}{5}\right)^n=0\end{array}\right]=\frac{0}{0-1}=\frac{0}{-1}=0.$$

$$\lim_{n\to\infty}\frac{\left(\frac{1}{5}\right)^n}{\left(\frac{1}{4}\right)^n-\left(\frac{1}{3}\right)^n}=\lim_{n\to\infty}\frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}=0$$

$$=\lim_{n\to\infty}\frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}\cdot\frac{4^n}{4^n}=\lim_{n\to\infty}\frac{\left(\frac{4}{5}\right)^n}{1-\left(\frac{4}{3}\right)^n}=\left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim\limits_{n\to\infty}\left(\frac{4}{5}\right)^n=0,\ \lim\limits_{n\to\infty}\left(\frac{4}{3}\right)^n=\infty\end{array}\right]=\frac{0}{1-\infty}=\frac{0}{-\infty}=0.$$

$$=\lim_{n\to\infty}\frac{\frac{1}{5^n}}{\frac{1}{4^n}-\frac{1}{3^n}}\cdot\frac{3^n}{3^n}=\lim_{n\to\infty}\frac{\left(\frac{3}{5}\right)^n}{\left(\frac{3}{4}\right)^n-1}=\left[\begin{array}{c}\text{Geometrické postupnosti}\\ \lim\limits_{n\to\infty}\left(\frac{3}{4}\right)^n=0,\ \lim\limits_{n\to\infty}\left(\frac{3}{5}\right)^n=0\end{array}\right]=\frac{0}{0-1}=\frac{0}{-1}=0.$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^3} & \lim\limits_{n\to\infty}\sqrt[n]{a_n} = \lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}} = \lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3} = \frac{5}{1^3} = 5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3} = \lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3} = \frac{5}{(1+\frac{1}{\infty})^3} = \frac{5}{(1+0)^3} = 5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{5^n}{n^3} = \infty \right]$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^4} & \lim\limits_{n\to\infty}\sqrt[n]{a_n} = \lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^4}} = \lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^4} = \frac{5}{1^4} = 5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty}\frac{5n^4}{(n+1)^4} = \lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^4} = \frac{5}{(1+\frac{1}{\infty})^4} = \frac{5}{(1+0)^4} = 5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{5^n}{n^4} = \infty \right]$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\text{Označme } a_n=\frac{5^n}{n^3} \text{ pre všetky } n\in N \;\middle|\; \left. \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^3}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^3} = \frac{5}{1^3} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5n^3}{(n+1)^3} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^3} = \frac{5}{(1+\frac{1}{\infty})^3} = \frac{5}{(1+0)^3} = 5>1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{5^n}{n^3} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$= \left[\text{Označme } a_n=\frac{5^n}{n^4} \text{ pre všetky } n\in N \;\middle|\; \left. \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^4}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^4} = \frac{5}{1^4} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5n^4}{(n+1)^4} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^4} = \frac{5}{(1+\frac{1}{\infty})^4} = \frac{5}{(1+0)^4} = 5>1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{5^n}{n^4} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{n^4\left(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4}\right)}{n^3\left(\frac{2}{n}-1\right)}$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^3} \\ \text{pre všetky } n\in N \end{array} \left| \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^3}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^3} = \frac{5}{1^3} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5n^3}{(n+1)^3} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^3} = \frac{5}{(1+\frac{1}{\infty})^3} = \frac{5}{(1+0)^3} = 5>1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{5^n}{n^3} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1}$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^4} \\ \text{pre všetky } n\in N \end{array} \left| \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^4}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^4} = \frac{5}{1^4} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5n^4}{(n+1)^4} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^4} = \frac{5}{(1+\frac{1}{\infty})^4} = \frac{5}{(1+0)^4} = 5>1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{5^n}{n^4} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^3} \\ \text{pre všetky } n\in N \end{array} \left| \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^3}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^3} = \frac{5}{1^3} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5n^3}{(n+1)^3} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^3} = \frac{5}{(1+\frac{1}{\infty})^3} = \frac{5}{(1+0)^3} = 5>1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{5^n}{n^3} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \left[\lim_{n\to\infty} \frac{a}{n} = \frac{a}{\infty} = 0 \text{ pre všetky } a\in R \right]$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^4} \\ \text{pre všetky } n\in N \end{array} \left| \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^4}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^4} = \frac{5}{1^4} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5n^4}{(n+1)^4} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^4} = \frac{5}{(1+\frac{1}{\infty})^4} = \frac{5}{(1+0)^4} = 5>1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{5^n}{n^4} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \left[\lim_{n\to\infty} \frac{a}{n^k} = \frac{a}{\infty} = 0 \text{ pre všetky } a\in R,\ k\in N \right]$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^3} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty}\frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \left[\lim_{n\to\infty}\frac{a}{n}=\frac{a}{\infty}=0 \text{ pre všetky } a\in R\right]$$

$$= \frac{2+3\cdot\infty-0+2\cdot\infty}{0-1}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^4} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^4}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^4}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^4}=\frac{5}{1^4}=5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}}=\lim\limits_{n\to\infty}\frac{5n^4}{(n+1)^4}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^4}=\frac{5}{(1+\frac{1}{\infty})^4}=\frac{5}{(1+0)^4}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^4}=\infty \right]$$

$$= \lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \left[\lim_{n\to\infty}\frac{a}{n^k}=\frac{a}{\infty}=0 \text{ pre všetky } a\in R,\ k\in N\right]$$

$$= \frac{\infty(0+3-0+2\cdot\infty)}{0-1}$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^3} \\ \text{pre všetky } n\in N \end{array} \middle| \begin{array}{l} \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty}\frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \left[\lim_{n\to\infty}\frac{a}{n}=\frac{a}{\infty}=0 \text{ pre všetky } a\in R\right]$$

$$= \frac{2+3\cdot\infty-0+2\cdot\infty}{0-1} = \frac{2+\infty-0+\infty}{-1}$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^4} \\ \text{pre všetky } n\in N \end{array} \middle| \begin{array}{l} \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{5^n}{n^4}}=\lim_{n\to\infty}\frac{5}{\sqrt[n]{n^4}}=\lim_{n\to\infty}\frac{5}{(\sqrt[n]{n})^4}=\frac{5}{1^4}=5>1 \\ \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}}=\lim_{n\to\infty}\frac{5n^4}{(n+1)^4}=\lim_{n\to\infty}\frac{5}{(1+\frac{1}{n})^4}=\frac{5}{(1+\frac{1}{\infty})^4}=\frac{5}{(1+0)^4}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^4}=\infty \right]$$

$$= \lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \left[\lim_{n\to\infty}\frac{a}{n^k}=\frac{a}{\infty}=0 \text{ pre všetky } a\in R,\ k\in N\right]$$

$$= \frac{\infty(0+3-0+2\cdot\infty)}{0-1} = \frac{\infty(0+3-0+\infty)}{-1}$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^3} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^3}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^3}=\frac{5}{1^3}=5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}}=\lim\limits_{n\to\infty}\frac{5n^3}{(n+1)^3}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^3}=\frac{5}{(1+\frac{1}{\infty})^3}=\frac{5}{(1+0)^3}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^3}=\infty \right]$$

$$=\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3}\cdot\frac{\frac{1}{n^3}}{\frac{1}{n^3}}=\lim_{n\to\infty}\frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1}=\left[\lim_{n\to\infty}\frac{a}{n}=\frac{a}{\infty}=0 \text{ pre všetky } a\in R\right]$$

$$=\frac{2+3\cdot\infty-0+2\cdot\infty}{0-1}=\frac{2+\infty-0+\infty}{-1}=\frac{\infty}{-1}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{5^n}{n^4} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{5^n}{n^4}}=\lim\limits_{n\to\infty}\frac{5}{\sqrt[n]{n^4}}=\lim\limits_{n\to\infty}\frac{5}{(\sqrt[n]{n})^4}=\frac{5}{1^4}=5>1 \\ \text{pre všetky } n\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}}=\lim\limits_{n\to\infty}\frac{5n^4}{(n+1)^4}=\lim\limits_{n\to\infty}\frac{5}{(1+\frac{1}{n})^4}=\frac{5}{(1+\frac{1}{\infty})^4}=\frac{5}{(1+0)^4}=5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{5^n}{n^4}=\infty \right]$$

$$=\lim_{n\to\infty}\frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)}=\lim_{n\to\infty}\frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1}=\left[\lim_{n\to\infty}\frac{a}{n^k}=\frac{a}{\infty}=0 \text{ pre všetky } a\in R,\ k\in N\right]$$

$$=\frac{\infty(0+3-0+2\cdot\infty)}{0-1}=\frac{\infty(0+3-0+\infty)}{-1}=\frac{\infty}{-1}$$

Riešené limity – 69

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} = -\infty$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^3} \\ \text{pre všetky } n\in N \end{array} \middle| \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^3}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^3} = \frac{5}{1^3} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^3}}{\frac{5^n}{n^3}} = \lim\limits_{n\to\infty} \frac{5n^3}{(n+1)^3} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^3} = \frac{5}{(1+\frac{1}{\infty})^3} = \frac{5}{(1+0)^3} = 5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5^n}{n^3} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3} \cdot \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{2+3n-\frac{1}{n}+2\cdot\frac{5^n}{n^3}}{\frac{2}{n}-1} = \left[\lim_{n\to\infty} \frac{a}{n} = \frac{a}{\infty} = 0 \text{ pre všetky } a\in R \right]$$

$$= \frac{2+3\cdot\infty-0+2\cdot\infty}{0-1} = \frac{2+\infty-0+\infty}{-1} = \frac{\infty}{-1} = -\infty.$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{5^n}{n^4} \\ \text{pre všetky } n\in N \end{array} \middle| \begin{array}{l} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5}{\sqrt[n]{n^4}} = \lim\limits_{n\to\infty} \frac{5}{(\sqrt[n]{n})^4} = \frac{5}{1^4} = 5>1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)^4}}{\frac{5^n}{n^4}} = \lim\limits_{n\to\infty} \frac{5n^4}{(n+1)^4} = \lim\limits_{n\to\infty} \frac{5}{(1+\frac{1}{n})^4} = \frac{5}{(1+\frac{1}{\infty})^4} = \frac{5}{(1+0)^4} = 5>1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5^n}{n^4} = \infty \right]$$

$$= \lim_{n\to\infty} \frac{n^4(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{n^3(\frac{2}{n}-1)} = \lim_{n\to\infty} \frac{n(\frac{2}{n}+3-\frac{1}{n^2}+2\cdot\frac{5^n}{n^4})}{\frac{2}{n}-1} = \left[\lim_{n\to\infty} \frac{a}{n^k} = \frac{a}{\infty} = 0 \text{ pre všetky } a\in R,\ k\in N \right]$$

$$= \frac{\infty(0+3-0+2\cdot\infty)}{0-1} = \frac{\infty(0+3-0+\infty)}{-1} = \frac{\infty}{-1} = -\infty.$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \cdot \frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \cdot \frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]$$

$$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}\cdot\frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\left[\begin{matrix}n=(n^2)^{\frac{1}{2}}\\ \text{pre } n\in N\end{matrix}\right]$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \begin{bmatrix} n=\sqrt{n^2} \\ \text{pre } n\in N \end{bmatrix} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \cdot \frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \begin{bmatrix} n=(n^2)^{\frac{1}{2}} \\ \text{pre } n\in N \end{bmatrix} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}\cdot\frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\left[\begin{matrix}n=(n^2)^{\frac{1}{2}}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \left[\begin{matrix} n=\sqrt{n^2} \\ \text{pre } n\in N \end{matrix}\right] = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}} = \left[\begin{matrix} n^2=\sqrt{n^4} \\ \text{pre } n\in N \end{matrix}\right]$$

$$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \cdot \frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \left[\begin{matrix} n=(n^2)^{\frac{1}{2}} \\ \text{pre } n\in N \end{matrix}\right] = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} = \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}} = \left[\begin{matrix} n=\sqrt{n^2} \\ \text{pre } n\in N \end{matrix}\right] = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}} = \left[\begin{matrix} n^2=\sqrt{n^4} \\ \text{pre } n\in N \end{matrix}\right] = \lim_{n\to\infty} \frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}$$

$$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}} \cdot \frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}} = \left[\begin{matrix} n=(n^2)^{\frac{1}{2}} \\ \text{pre } n\in N \end{matrix}\right] = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{\left(n+\left(n+n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}=\left[\begin{matrix}n^2=\sqrt{n^4}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{\left(n+\left(n+n^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}\cdot\frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+\left(n+n^{\frac{1}{2}}\right)^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\left[\begin{matrix}n=(n^2)^{\frac{1}{2}}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0 \text{ pre všetky } k,\ m\in N\right]$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}=\left[\begin{matrix}n^2=\sqrt{n^4}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}\cdot\frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\left[\begin{matrix}n=(n^2)^{\frac{1}{2}}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0 \text{ pre všetky } k,\ m\in N\right]=\frac{1}{(1+(0+0)^{\frac{1}{2}})^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\left[\begin{matrix}n=\sqrt{n^2}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}=\left[\begin{matrix}n^2=\sqrt{n^4}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]=\frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}\cdot\frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\left[\begin{matrix}n=(n^2)^{\frac{1}{2}}\\ \text{pre } n\in N\end{matrix}\right]=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0 \text{ pre všetky } k,\ m\in N\right]=\frac{1}{(1+(0+0)^{\frac{1}{2}})^{\frac{1}{2}}}=\frac{1}{(1+0)^{\frac{1}{2}}}$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\left[\begin{array}{l} n=\sqrt{n^2} \\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}=\left[\begin{array}{l} n^2=\sqrt{n^4} \\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]=\frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}=\frac{1}{\sqrt{1+0}}$$

$$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}\cdot\frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\left[\begin{array}{l} n=(n^2)^{\frac{1}{2}} \\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0 \text{ pre všetky } k,\ m\in N\right]=\frac{1}{(1+(0+0)^{\frac{1}{2}})^{\frac{1}{2}}}=\frac{1}{(1+0)^{\frac{1}{2}}}=\frac{1}{1}$$

Riešené limity – 70

$$\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}=1$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n+\sqrt{n}}}}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n+\sqrt{n+\sqrt{n}}}{n}}}=\left[\begin{array}{l}n=\sqrt{n^2}\\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{\sqrt{n+\sqrt{n}}}{\sqrt{n^2}}}}$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{n+\sqrt{n}}{n^2}}}}=\left[\begin{array}{l}n^2=\sqrt{n^4}\\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\frac{\sqrt{n}}{\sqrt{n^4}}}}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\sqrt{\frac{1}{n}+\sqrt{\frac{1}{n^3}}}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]=\frac{1}{\sqrt{1+\sqrt{0+\sqrt{0}}}}=\frac{1}{\sqrt{1+0}}=1.$$

$$=\lim_{n\to\infty}\frac{n^{\frac{1}{2}}}{(n+(n+n^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}}\cdot\frac{1/n^{\frac{1}{2}}}{1/n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+(n+n^{\frac{1}{2}})^{\frac{1}{2}}}{n}\right)^{\frac{1}{2}}}=\left[\begin{array}{l}n=(n^2)^{\frac{1}{2}}\\ \text{pre } n\in N\end{array}\right]=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{n+n^{\frac{1}{2}}}{n^2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{\frac{1}{2}-2}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+n^{-\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{1}{\left(1+\left(\frac{1}{n}+\frac{1}{n^{\frac{3}{2}}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^{\frac{k}{m}}}=\lim_{n\to\infty}\frac{1}{\sqrt[m]{n^k}}=\frac{1}{\sqrt[m]{\infty^k}}=\frac{1}{\sqrt[m]{\infty}}=\frac{1}{\infty}=0 \text{ pre všetky } k,\ m\in N\right]=\frac{1}{(1+(0+0)^{\frac{1}{2}})^{\frac{1}{2}}}=\frac{1}{(1+0)^{\frac{1}{2}}}=\frac{1}{1}=1.$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n}=\sqrt[6]{n^2} & \sqrt[5]{n}=\sqrt[10]{n^2} \\ \sqrt{n}=\sqrt[4]{n^2} & \sqrt{n}=\sqrt[6]{n^3} & \sqrt{n}=\sqrt[10]{n^5} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n}=\sqrt[6]{n^2} & \sqrt[5]{n}=\sqrt[10]{n^2} \\ \sqrt{n}=\sqrt[4]{n^2} & \sqrt{n}=\sqrt[6]{n^3} & \sqrt{n}=\sqrt[10]{n^5} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3} \cdot \frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n}=\sqrt[6]{n^2} & \sqrt[5]{n}=\sqrt[10]{n^2} \\ \sqrt{n}=\sqrt[4]{n^2} & \sqrt{n}=\sqrt[6]{n^3} & \sqrt{n}=\sqrt[10]{n^5} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt[6]{\frac{n^2}{n^3}}+3\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n}=\sqrt[6]{n^2} & \sqrt[5]{n}=\sqrt[10]{n^2} \\ \sqrt{n}=\sqrt[4]{n^2} & \sqrt{n}=\sqrt[6]{n^3} & \sqrt{n}=\sqrt[10]{n^5} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt[6]{\frac{n^2}{n^3}}+3\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt[6]{\frac{1}{n}}+3\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\sqrt{\frac{1}{n}}}$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}} = \lim_{n\to\infty} \frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\text{Pre všetky } n\in N \;\middle|\; \sqrt[3]{n}=\sqrt[6]{n^2} \;\middle|\; \sqrt[5]{n}=\sqrt[10]{n^2};\; \sqrt{n}=\sqrt[4]{n^2} \;\middle|\; \sqrt{n}=\sqrt[6]{n^3} \;\middle|\; \sqrt{n}=\sqrt[10]{n^5}\right]$$

$$= \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt[6]{\frac{n^2}{n^3}}+3\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt[6]{\frac{1}{n}}+3\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\sqrt{\frac{1}{n}}}$$

$$= \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}} = \lim_{n\to\infty} \frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}}$$

Riešené limity – 71

$$\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$=\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}}=\left[\begin{array}{c|c|c}\text{Pre všetky } n\in N & \sqrt[3]{n}=\sqrt[6]{n^2} & \sqrt[5]{n}=\sqrt[10]{n^2}\\ \sqrt{n}=\sqrt[4]{n^2} & \sqrt{n}=\sqrt[6]{n^3} & \sqrt{n}=\sqrt[10]{n^5}\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}}=\lim_{n\to\infty}\frac{2\sqrt[6]{\frac{n^2}{n^3}}+3\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}}=\lim_{n\to\infty}\frac{2\sqrt[6]{\frac{1}{n}}+3\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\sqrt{\frac{1}{n}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]=\frac{2\sqrt[6]{0}+3\sqrt[4]{0}-1}{2-\sqrt[10]{0}+3\sqrt{0}}$$

$$=\lim_{n\to\infty}\frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}}=\lim_{n\to\infty}\frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}}=\lim_{n\to\infty}\frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}}=\lim_{n\to\infty}\frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}}=\lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}}$$

$$=\lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}}=\lim_{n\to\infty}\frac{\frac{2}{\infty}+\frac{3}{\infty}-1}{2-\frac{1}{\infty}+\frac{3}{\infty}}$$

Riešené limity – 71

$$\lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}$$

$$= \lim_{n\to\infty} \frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}} = \left[\begin{array}{c|c|c} \text{Pre všetky } n\in N & \sqrt[3]{n}=\sqrt[6]{n^2} & \sqrt[5]{n}=\sqrt[10]{n^2} \\ \sqrt{n}=\sqrt[4]{n^2} & \sqrt{n}=\sqrt[6]{n^3} & \sqrt{n}=\sqrt[10]{n^5} \end{array}\right]$$

$$= \lim_{n\to\infty} \frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt[6]{\frac{n^2}{n^3}}+3\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}} = \lim_{n\to\infty} \frac{2\sqrt[6]{\frac{1}{n}}+3\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\sqrt{\frac{1}{n}}}$$

$$= \left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right] = \frac{2\sqrt[6]{0}+3\sqrt[4]{0}-1}{2-\sqrt[10]{0}+3\sqrt{0}} = \frac{0+0-1}{2-0+0}$$

$$= \lim_{n\to\infty} \frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

$$= \lim_{n\to\infty} \frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}} = \lim_{n\to\infty} \frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}} = \lim_{n\to\infty} \frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}}$$

$$= \lim_{n\to\infty} \frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}} = \lim_{n\to\infty} \frac{\frac{2}{\infty}+\frac{3}{\infty}-1}{2-\frac{1}{\infty}+\frac{3}{\infty}} = \frac{0+0-1}{2-0+0}$$

Riešené limity – 71

$$\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}=-\frac{1}{2}$$

$$=\lim_{n\to\infty}\frac{2\sqrt[3]{n}+3\sqrt[4]{n}-\sqrt{n}}{2\sqrt{n}-\sqrt[5]{n}+3}\cdot\frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[3]{n}}{\sqrt{n}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt{n}}-1}{2-\frac{\sqrt[5]{n}}{\sqrt{n}}+\frac{3}{\sqrt{n}}}=\left[\begin{array}{c|c|c}\text{Pre všetky } n\in N & \sqrt[3]{n}=\sqrt[6]{n^2} & \sqrt[5]{n}=\sqrt[10]{n^2}\\ \sqrt{n}=\sqrt[4]{n^2} & \sqrt{n}=\sqrt[6]{n^3} & \sqrt{n}=\sqrt[10]{n^5}\end{array}\right]$$

$$=\lim_{n\to\infty}\frac{2\cdot\frac{\sqrt[6]{n^2}}{\sqrt[6]{n^3}}+3\cdot\frac{\sqrt[4]{n}}{\sqrt[4]{n^2}}-1}{2-\frac{\sqrt[10]{n^2}}{\sqrt[10]{n^5}}+\frac{3}{\sqrt{n}}}=\lim_{n\to\infty}\frac{2\sqrt[6]{\frac{n^2}{n^3}}+3\sqrt[4]{\frac{n}{n^2}}-1}{2-\sqrt[10]{\frac{n^2}{n^5}}+\frac{3}{\sqrt{n}}}=\lim_{n\to\infty}\frac{2\sqrt[6]{\frac{1}{n}}+3\sqrt[4]{\frac{1}{n}}-1}{2-\sqrt[10]{\frac{1}{n^3}}+3\sqrt{\frac{1}{n}}}$$

$$=\left[\lim_{n\to\infty}\frac{1}{n^k}=\frac{1}{\infty^k}=\frac{1}{\infty}=0 \text{ pre všetky } k\in N\right]=\frac{2\sqrt[6]{0}+3\sqrt[4]{0}-1}{2-\sqrt[10]{0}+3\sqrt{0}}=\frac{0+0-1}{2-0+0}=-\frac{1}{2}.$$

$$=\lim_{n\to\infty}\frac{2n^{\frac{1}{3}}+3n^{\frac{1}{4}}-n^{\frac{1}{2}}}{2n^{\frac{1}{2}}-n^{\frac{1}{5}}+3}\cdot\frac{n^{-\frac{1}{2}}}{n^{-\frac{1}{2}}}=\lim_{n\to\infty}\frac{2n^{\frac{1}{3}-\frac{1}{2}}+3n^{\frac{1}{4}-\frac{1}{2}}-1}{2-n^{\frac{1}{5}-\frac{1}{2}}+3n^{-\frac{1}{2}}}=\lim_{n\to\infty}\frac{2n^{\frac{2-3}{6}}+3n^{\frac{1-2}{4}}-1}{2-n^{\frac{2-5}{10}}+3n^{-\frac{1}{2}}}$$

$$=\lim_{n\to\infty}\frac{2n^{-\frac{1}{6}}+3n^{-\frac{1}{4}}-1}{2-n^{-\frac{3}{10}}+3n^{-\frac{1}{2}}}=\lim_{n\to\infty}\frac{\frac{2}{n^{\frac{1}{6}}}+\frac{3}{n^{\frac{1}{4}}}-1}{2-\frac{1}{n^{\frac{3}{10}}}+\frac{3}{n^{\frac{1}{2}}}}=\lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{n}}+\frac{3}{\sqrt[4]{n}}-1}{2-\frac{1}{\sqrt[10]{n^3}}+\frac{3}{\sqrt{n}}}$$

$$=\lim_{n\to\infty}\frac{\frac{2}{\sqrt[6]{\infty}}+\frac{3}{\sqrt[4]{\infty}}-1}{2-\frac{1}{\sqrt[10]{\infty}}+\frac{3}{\sqrt{\infty}}}=\lim_{n\to\infty}\frac{\frac{2}{\infty}+\frac{3}{\infty}-1}{2-\frac{1}{\infty}+\frac{3}{\infty}}=\frac{0+0-1}{2-0+0}=-\frac{1}{2}.$$

Riešené limity – 72, 73

$\lim_{n\to\infty} \left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$

$\lim_{n\to\infty} n\left[\sqrt[n]{2} - \sqrt[n+1]{2}\right]$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n\left[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n\left[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right]=\lim_{n\to\infty} n\left[\sqrt[n]{2}-1\right]-\lim_{n\to\infty} n\left[\sqrt[n+1]{2}-1\right]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right]^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n\left[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right]=\lim_{n\to\infty} n\left[\sqrt[n]{2}-1\right]-\lim_{n\to\infty} n\left[\sqrt[n+1]{2}-1\right]=\left[\begin{array}{c}\lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right]^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}=e^{\ln\sqrt{6}}$$

$$\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}=\left[\begin{array}{l|l|l|l}\text{Subst. } m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & n\in N\Rightarrow\sqrt[n]{3}>1,\ \sqrt[n]{2}>1 & \sqrt[n]{3}\to1,\ \sqrt[n]{2}\to1 & n\to\infty\\ \frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\frac{1}{m} & \sqrt[n]{2}+\sqrt[n]{3}-2>1+1-2=0 & \sqrt[n]{2}+\sqrt[n]{3}-2\to0^+ & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^m=e,$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n\left[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right]=\lim_{n\to\infty} n\left[\sqrt[n]{2}-1\right]-\lim_{n\to\infty} n\left[\sqrt[n+1]{2}-1\right]=\left[\begin{array}{c}\lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]$$

$$=\ln 2-\lim_{n\to\infty}(n+1-1)\cdot\left[\sqrt[n+1]{2}-1\right]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right]^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}=e^{\ln\sqrt{6}}$$

$$\lim_{n\to\infty}n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2}=\left[\begin{matrix}\lim_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{matrix}\right]=\frac{\ln 2+\ln 3}{2}=\frac{\ln 6}{2}=\ln 6^{\frac{1}{2}}=\ln\sqrt{6}.$$

$$\lim_{n\to\infty}n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty}n\left[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right]=\lim_{n\to\infty}n\left[\sqrt[n]{2}-1\right]-\lim_{n\to\infty}n\left[\sqrt[n+1]{2}-1\right]=\left[\begin{matrix}\lim_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{matrix}\right]$$

$$=\ln 2-\lim_{n\to\infty}(n+1-1)\cdot\left[\sqrt[n+1]{2}-1\right]=\ln 2-\lim_{n\to\infty}(n+1)\cdot\left[\sqrt[n+1]{2}-1\right]+\lim_{n\to\infty}\left[\sqrt[n+1]{2}-1\right]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$= \lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n = \lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$$

$$= \lim_{n\to\infty}\left[\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right]^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}} = e^{\ln\sqrt{6}}$$

$$\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}} = \left[\begin{array}{l|l|l|l}\text{Subst. } m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & n\in N \Rightarrow \sqrt[n]{3}>1,\ \sqrt[n]{2}>1 & \sqrt[n]{3}\to 1,\ \sqrt[n]{2}\to 1 & n\to\infty \\ \frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\frac{1}{m} & \sqrt[n]{2}+\sqrt[n]{3}-2>1+1-2=0 & \sqrt[n]{2}+\sqrt[n]{3}-2\to 0^+ & m\to\infty\end{array}\right] = \lim_{m\to\infty}\left(1+\frac{1}{m}\right)^m = e,$$

$$\lim_{n\to\infty} n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2} = \lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2} = \left[\begin{array}{c}\lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a \\ \text{pre všetky } a>0\end{array}\right] = \frac{\ln 2+\ln 3}{2} = \frac{\ln 6}{2} = \ln 6^{\frac{1}{2}} = \ln\sqrt{6}.$$

$$\lim_{n\to\infty} n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$= \lim_{n\to\infty} n\left[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right] = \lim_{n\to\infty} n\left[\sqrt[n]{2}-1\right] - \lim_{n\to\infty} n\left[\sqrt[n+1]{2}-1\right] = \left[\begin{array}{c}\lim_{n\to\infty} n(\sqrt[n]{a}-1)=\ln a \\ \text{pre všetky } a>0\end{array}\right]$$

$$= \ln 2 - \lim_{n\to\infty}(n+1-1)\cdot\left[\sqrt[n+1]{2}-1\right] = \ln 2 - \lim_{n\to\infty}(n+1)\cdot\left[\sqrt[n+1]{2}-1\right] + \lim_{n\to\infty}\left[\sqrt[n+1]{2}-1\right]$$

$$= \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=n+1 & m\to\infty\end{array}\right]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\Big(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\Big)^n=\sqrt{6}$$

$$=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\Big)^n=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^n$$

$$=\lim_{n\to\infty}\Bigg[\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\Bigg]^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}=e^{\ln\sqrt{6}}=\sqrt{6}.$$

$$\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}=\left[\begin{array}{l|l|l|l}\text{Subst. } m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & n\in N\Rightarrow\sqrt[n]{3}>1,\ \sqrt[n]{2}>1 & \sqrt[n]{3}\to1,\ \sqrt[n]{2}\to1 & n\to\infty\\ \frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\frac{1}{m} & \sqrt[n]{2}+\sqrt[n]{3}-2>1+1-2=0 & \sqrt[n]{2}+\sqrt[n]{3}-2\to0^{+} & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1+\frac{1}{m}\Big)^m=e,$$

$$\lim_{n\to\infty}n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2}=\left[\begin{array}{c}\lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]=\frac{\ln2+\ln3}{2}=\frac{\ln6}{2}=\ln6^{\frac12}=\ln\sqrt{6}.$$

$$\lim_{n\to\infty}n\Big[\sqrt[n]{2}-\sqrt[n+1]{2}\Big]$$

$$=\lim_{n\to\infty}n\Big[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\Big]=\lim_{n\to\infty}n\Big[\sqrt[n]{2}-1\Big]-\lim_{n\to\infty}n\Big[\sqrt[n+1]{2}-1\Big]=\left[\begin{array}{c}\lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]$$

$$=\ln2-\lim_{n\to\infty}(n+1-1)\cdot\Big[\sqrt[n+1]{2}-1\Big]=\ln2-\lim_{n\to\infty}(n+1)\cdot\Big[\sqrt[n+1]{2}-1\Big]+\lim_{n\to\infty}\Big[\sqrt[n+1]{2}-1\Big]$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n+1 & m\to\infty\end{array}\right]=\ln2-\lim_{m\to\infty}m\Big[\sqrt[m]{2}-1\Big]+\lim_{m\to\infty}\Big[\sqrt[m]{2}-1\Big]$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n=\sqrt{6}$$

$$=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\right)^n=\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^n$$

$$=\lim_{n\to\infty}\left[\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\right]^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}=e^{\ln\sqrt{6}}=\sqrt{6}.$$

$$\lim_{n\to\infty}\left(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\right)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}=\left[\begin{array}{l|l|l|l}\text{Subst. } m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & n\in N\Rightarrow\sqrt[n]{3}>1,\ \sqrt[n]{2}>1 & \sqrt[n]{3}\to1,\ \sqrt[n]{2}\to1 & n\to\infty\\ \frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\frac{1}{m} & \sqrt[n]{2}+\sqrt[n]{3}-2>1+1-2=0 & \sqrt[n]{2}+\sqrt[n]{3}-2\to0^+ & m\to\infty\end{array}\right]=\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^m=e,$$

$$\lim_{n\to\infty}n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2}=\left[\begin{array}{c}\lim_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]=\frac{\ln2+\ln3}{2}=\frac{\ln6}{2}=\ln6^{\frac{1}{2}}=\ln\sqrt{6}.$$

$$\lim_{n\to\infty}n\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty}n\left[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\right]=\lim_{n\to\infty}n\left[\sqrt[n]{2}-1\right]-\lim_{n\to\infty}n\left[\sqrt[n+1]{2}-1\right]=\left[\begin{array}{c}\lim_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]$$

$$=\ln2-\lim_{n\to\infty}(n+1-1)\cdot\left[\sqrt[n+1]{2}-1\right]=\ln2-\lim_{n\to\infty}(n+1)\cdot\left[\sqrt[n+1]{2}-1\right]+\lim_{n\to\infty}\left[\sqrt[n+1]{2}-1\right]$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n+1 & m\to\infty\end{array}\right]=\ln2-\lim_{m\to\infty}m\left[\sqrt[m]{2}-1\right]+\lim_{m\to\infty}\left[\sqrt[m]{2}-1\right]=\ln2-\ln2+(1-1)$$

Riešené limity – 72, 73

$$\lim_{n\to\infty}\Big(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\Big)^n=\sqrt{6}$$

$$=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1\Big)^n=\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^n$$

$$=\lim_{n\to\infty}\Big[\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}\Big]^{n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}}=e^{\ln\sqrt{6}}=\sqrt{6}.$$

$$\lim_{n\to\infty}\Big(1+\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}\Big)^{\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2}}=\left[\begin{array}{l|l|l|l}\text{Subst. } m=\frac{2}{\sqrt[n]{2}+\sqrt[n]{3}-2} & n\in N\Rightarrow\sqrt[n]{3}>1,\ \sqrt[n]{2}>1 & \sqrt[n]{3}\to1,\ \sqrt[n]{2}\to1 & n\to\infty\\ \frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\frac{1}{m} & \sqrt[n]{2}+\sqrt[n]{3}-2>1+1-2=0 & \sqrt[n]{2}+\sqrt[n]{3}-2\to0^+ & m\to\infty\end{array}\right]=\lim_{m\to\infty}\Big(1+\frac{1}{m}\Big)^m=e,$$

$$\lim_{n\to\infty}n\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}=\lim_{n\to\infty}\frac{n(\sqrt[n]{2}-1)+n(\sqrt[n]{3}-1)}{2}=\left[\begin{array}{c}\lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]=\frac{\ln 2+\ln 3}{2}=\frac{\ln 6}{2}=\ln 6^{\frac{1}{2}}=\ln\sqrt{6}.$$

$$\lim_{n\to\infty}n\Big[\sqrt[n]{2}-\sqrt[n+1]{2}\Big]=0$$

$$=\lim_{n\to\infty}n\Big[\sqrt[n]{2}-1-\sqrt[n+1]{2}+1\Big]=\lim_{n\to\infty}n\Big[\sqrt[n]{2}-1\Big]-\lim_{n\to\infty}n\Big[\sqrt[n+1]{2}-1\Big]=\left[\begin{array}{c}\lim\limits_{n\to\infty}n(\sqrt[n]{a}-1)=\ln a\\ \text{pre všetky } a>0\end{array}\right]$$

$$=\ln 2-\lim_{n\to\infty}(n+1-1)\cdot\Big[\sqrt[n+1]{2}-1\Big]=\ln 2-\lim_{n\to\infty}(n+1)\cdot\Big[\sqrt[n+1]{2}-1\Big]+\lim_{n\to\infty}\Big[\sqrt[n+1]{2}-1\Big]$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n+1 & m\to\infty\end{array}\right]=\ln 2-\lim_{m\to\infty}m\Big[\sqrt[m]{2}-1\Big]+\lim_{m\to\infty}\Big[\sqrt[m]{2}-1\Big]=\ln 2-\ln 2+(1-1)=0.$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2 \left[\sqrt[n]{2} - \sqrt[n+1]{2} \right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

Riešené limity – 74

$\lim\limits_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

Riešené limity – 74

$\lim\limits_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

$$=\lim_{n\to\infty} n(n+1)\left[\sqrt[n(n+1)]{2}-1\right]\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

$$=\lim_{n\to\infty} n(n+1)\left[\sqrt[n(n+1)]{2}-1\right]\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n(n+1) & m\to\infty\end{array}\right]$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

$$=\lim_{n\to\infty} n(n+1)\left[\sqrt[n(n+1)]{2}-1\right]\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n(n+1) & m\to\infty\end{array}\right]=\lim_{m\to\infty} m\left[\sqrt[m]{2}-1\right]\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

$$=\lim_{n\to\infty} n(n+1)\left[\sqrt[n(n+1)]{2}-1\right]\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n(n+1) & m\to\infty\end{array}\right]=\lim_{m\to\infty} m\left[\sqrt[m]{2}-1\right]\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$

$$=\ln 2\cdot\frac{1}{1+\frac{1}{\infty}}\cdot 1\cdot 1$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

$$=\lim_{n\to\infty} n(n+1)\left[\sqrt[n(n+1)]{2}-1\right]\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n(n+1) & m\to\infty\end{array}\right]=\lim_{m\to\infty} m\left[\sqrt[m]{2}-1\right]\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$

$$=\ln 2\cdot\frac{1}{1+\frac{1}{\infty}}\cdot 1\cdot 1=\ln 2\cdot\frac{1}{1+0}\cdot 1\cdot 1$$

Riešené limity – 74

$$\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\ln 2$$

$$=\lim_{n\to\infty} n^2\left[\sqrt[n]{2}-\sqrt[n+1]{2}\right]=\lim_{n\to\infty} n^2\left[2^{\frac{1}{n}}-2^{\frac{1}{n+1}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-\frac{2^{\frac{1}{n+1}}}{2^{\frac{1}{n}}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{1}{n+1}-\frac{1}{n}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{n-(n+1)}{n(n+1)}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[1-2^{\frac{-1}{n(n+1)}}\right]$$

$$=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\left[\frac{2^{\frac{1}{n(n+1)}}}{2^{\frac{1}{n(n+1)}}}-\frac{1}{2^{\frac{1}{n(n+1)}}}\right]=\lim_{n\to\infty} n^2\cdot 2^{\frac{1}{n}}\cdot\frac{1}{2^{\frac{1}{n(n+1)}}}\left[2^{\frac{1}{n(n+1)}}-1\right]$$

$$=\lim_{n\to\infty} n^2\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]=\lim_{n\to\infty}\frac{n\cdot n(n+1)}{n+1}\cdot\sqrt[n]{2}\cdot\frac{1}{\sqrt[n(n+1)]{2}}\left[\sqrt[n(n+1)]{2}-1\right]$$

$$=\lim_{n\to\infty} n(n+1)\left[\sqrt[n(n+1)]{2}-1\right]\cdot\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{n\to\infty}\frac{1}{\sqrt[n(n+1)]{2}}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & n\to\infty\\ m=n(n+1) & m\to\infty\end{array}\right]=\lim_{m\to\infty} m\left[\sqrt[m]{2}-1\right]\cdot\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\cdot\lim_{n\to\infty}\sqrt[n]{2}\cdot\lim_{m\to\infty}\frac{1}{\sqrt[m]{2}}$$

$$=\ln 2\cdot\frac{1}{1+\frac{1}{\infty}}\cdot 1\cdot 1=\ln 2\cdot\frac{1}{1+0}\cdot 1\cdot 1=\ln 2.$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right]$$
$$= \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} \right] = \lim_{n\to\infty} \frac{1}{n} \ln \left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} \right]$$

$\forall n \in N,\ n > 1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln \left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

$\forall n \in N,\ n > 1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n,$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln \left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

$\forall n \in N,\ n > 1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$,

t. j. $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln \left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

$\forall n \in N,\ n>1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$,

t. j. $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

$$\Rightarrow \quad \ln 2^n < \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \ln \left[n \cdot 2^n\right]$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

$\forall n \in N,\ n>1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$,

t. j. $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

$$\Rightarrow\quad n \cdot \ln 2 = \ln 2^n < \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \ln\left[n \cdot 2^n\right] = \ln n + \ln 2^n$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

$\forall n \in N,\ n > 1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$,

t. j. $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

$$\Rightarrow\ n \cdot \ln 2 = \ln 2^n < \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \ln\left[n \cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

$\forall n \in N,\ n > 1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$,

t. j. $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

$$\Rightarrow\ n \cdot \ln 2 = \ln 2^n < \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \ln\left[n \cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$$

$$\Rightarrow\ \ln 2 < \frac{1}{n} \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \frac{1}{n} \ln n + \ln 2$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}}\right]$$

$\forall n\in N,\ n>1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}} < 2^n+2^n+2^n+\cdots+2^n = n\cdot 2^n$,

t. j. $2^n < 2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n} < n\cdot 2^n$.

$$\Rightarrow\quad n\cdot\ln 2 = \ln 2^n < \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \ln\left[n\cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n\cdot\ln 2.$$

$$\Rightarrow\quad \ln 2 < \frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \frac{1}{n}\ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right]$$

$\forall n \in N,\ n > 1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$,

t. j. $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

$$\Rightarrow\ n \cdot \ln 2 = \ln 2^n < \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \ln\left[n \cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$$

$$\Rightarrow\ \ln 2 < \frac{1}{n} \ln\left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2.$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}}\right]$$

$\forall n\in N,\ n>1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}} < 2^n+2^n+2^n+\cdots+2^n = n\cdot 2^n$,

t. j. $2^n < 2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n} < n\cdot 2^n$.

$$\Rightarrow\quad n\cdot\ln 2 = \ln 2^n < \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \ln\left[n\cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n\cdot\ln 2.$$

$$\Rightarrow\quad \ln 2 < \frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \frac{1}{n}\ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln\sqrt[n]{n} + \ln 2.$$

$$\Rightarrow\quad \lim_{n\to\infty} \ln 2 \le \lim_{n\to\infty} \frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] \le \lim_{n\to\infty}\left[\ln\sqrt[n]{n} + \ln 2\right]$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}}\right]$$

$\forall n\in N,\ n>1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}} < 2^n+2^n+2^n+\cdots+2^n = n\cdot 2^n,$

t. j. $2^n < 2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n} < n\cdot 2^n.$

$$\Rightarrow\ n\cdot\ln 2 = \ln 2^n < \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \ln\left[n\cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n\cdot\ln 2.$$

$$\Rightarrow\ \ln 2 < \frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \frac{1}{n}\ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln\sqrt[n]{n} + \ln 2.$$

$$\Rightarrow\ \ln 2 = \lim_{n\to\infty}\ln 2 \le \lim_{n\to\infty}\frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] \le \lim_{n\to\infty}\left[\ln\sqrt[n]{n}+\ln 2\right] = \ln 1 + \ln 2$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln\left[2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}}\right]$$

$\forall n\in N,\ n>1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}}+2^{\frac{n}{2}}+2^{\frac{n}{3}}+\cdots+2^{\frac{n}{n}} < 2^n+2^n+2^n+\cdots+2^n = n\cdot 2^n,$

t. j. $2^n < 2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n} < n\cdot 2^n.$

$$\Rightarrow\quad n\cdot\ln 2 = \ln 2^n < \ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \ln\left[n\cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n\cdot\ln 2.$$

$$\Rightarrow\quad \ln 2 < \frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] < \frac{1}{n}\ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln\sqrt[n]{n} + \ln 2.$$

$$\Rightarrow\quad \ln 2 = \lim_{n\to\infty}\ln 2 \le \lim_{n\to\infty}\frac{1}{n}\ln\left[2^n+\sqrt{2^n}+\sqrt[3]{2^n}+\cdots+\sqrt[n]{2^n}\right] \le \lim_{n\to\infty}\left[\ln\sqrt[n]{n}+\ln 2\right] = \ln 1 + \ln 2 = 0 + \ln 2 = \ln 2.$$

Riešené limity – 75

$$\lim_{n\to\infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \lim_{n\to\infty} \frac{1}{n} \ln \left[2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}}\right] = \ln 2$$

$\forall n \in N,\ n>1$ platí: $2^n = 2^{\frac{n}{1}} < 2^{\frac{n}{1}} + 2^{\frac{n}{2}} + 2^{\frac{n}{3}} + \cdots + 2^{\frac{n}{n}} < 2^n + 2^n + 2^n + \cdots + 2^n = n \cdot 2^n$,

t. j. $2^n < 2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n} < n \cdot 2^n$.

$$\Rightarrow\quad n \cdot \ln 2 = \ln 2^n < \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \ln \left[n \cdot 2^n\right] = \ln n + \ln 2^n = \ln n + n \cdot \ln 2.$$

$$\Rightarrow\quad \ln 2 < \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] < \frac{1}{n} \ln n + \ln 2 = \ln n^{\frac{1}{n}} + \ln 2 = \ln \sqrt[n]{n} + \ln 2.$$

$$\Rightarrow\quad \ln 2 = \lim_{n\to\infty} \ln 2 \le \lim_{n\to\infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] \le \lim_{n\to\infty} \left[\ln \sqrt[n]{n} + \ln 2\right] = \ln 1 + \ln 2 = 0 + \ln 2 = \ln 2.$$

$$\Rightarrow\quad \lim_{n\to\infty} \frac{1}{n} \ln \left[2^n + \sqrt{2^n} + \sqrt[3]{2^n} + \cdots + \sqrt[n]{2^n}\right] = \ln 2.$$

Riešené limity – 76, 77

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

Riešené limity – 76, 77

$\lim\limits_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$

$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n^k}{4^n} & \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1 \\ \text{pre } n \in N,\ k \in N & \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+\frac{1}{\infty})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n^k}{4^n} = 0 \right]$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots \right\}$.

Rekurentne: $a_1 = \sqrt{2} = 2^{\frac{1}{2}}$,

Riešené limity – 76, 77

$\lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$

$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n^k}{4^n} & \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1 \\ \text{pre } n \in N,\ k \in N & \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+\frac{1}{\infty})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1 \end{array} \right\} \Rightarrow \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{n^k}{4^n} = 0 \right]$

$= \lim\limits_{n\to\infty} \dfrac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \dfrac{\frac{1}{4^n}}{\frac{1}{4^n}}$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

Rekurentne: $a_1 = \sqrt{2} = 2^{\frac{1}{2}}$, $a_{n+1} = \sqrt{a_n} = a_n^{\frac{1}{2}}$ pre $n \in N$.

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

$$= \left[\begin{array}{l} \text{Označme } a_n=\frac{n^k}{4^n} \\ \text{pre } n\in N,\ k\in N \end{array} \middle| \begin{array}{l} \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{4^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{4}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{4}=\frac{1^k}{4}=\frac{1}{4}<1 \\ \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{4n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{4}=\frac{(1+\frac{1}{\infty})^k}{4}=\frac{(1+0)^k}{4}=\frac{1}{4}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{4^n}=0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}\cdot\frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty}\frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6}$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

Explicitne: $a_1 = 2^{\frac{1}{2}} = \sqrt{2},$

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{4^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{4^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{4}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{4}=\frac{1^k}{4}=\frac{1}{4}<1 \\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{4n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{4}=\frac{(1+\frac{1}{\infty})^k}{4}=\frac{(1+0)^k}{4}=\frac{1}{4}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{4^n}=0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}\cdot\frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty}\frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim_{n\to\infty}\frac{5^n}{4^n}=\lim_{n\to\infty}(\frac{5}{4})^n=\infty \end{array} \right]$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\right\}$.

Explicitne: $a_1=2^{\frac{1}{2}}=\sqrt{2}$, $a_2=a_1^{\frac{1}{2}}=2^{\frac{1}{2^2}}=\sqrt[2^2]{2}=\sqrt[4]{2}$,

Riešené limity – 76, 77

$$\lim_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{4^n} & \left.\begin{array}{l}\lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{4^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{4}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{4}=\frac{1^k}{4}=\frac{1}{4}<1 \\ \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{4n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{4}=\frac{(1+\frac{1}{\infty})^k}{4}=\frac{(1+0)^k}{4}=\frac{1}{4}<1\end{array}\right\} \Rightarrow \lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{n^k}{4^n}=0 \\ \text{pre } n\in N,\ k\in N & \end{array}\right]$

$=\lim\limits_{n\to\infty}\frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}\cdot\frac{\frac{1}{4^n}}{\frac{1}{4^n}}=\lim\limits_{n\to\infty}\frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6}=\left[\begin{array}{l}\text{Geometrická postupnosť} \\ \lim\limits_{n\to\infty}\frac{5^n}{4^n}=\lim\limits_{n\to\infty}(\frac{5}{4})^n=\infty\end{array}\right]$

$=\frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6}$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}=\left\{\sqrt{2},\sqrt{\sqrt{2}},\sqrt{\sqrt{\sqrt{2}}},\ldots\right\}$.

Explicitne: $a_1=2^{\frac{1}{2}}=\sqrt{2},\quad a_2=a_1^{\frac{1}{2}}=2^{\frac{1}{2^2}}=\sqrt[2^2]{2}=\sqrt[4]{2},\quad a_3=a_2^{\frac{1}{2}}=2^{\frac{1}{2^3}}=\sqrt[2^3]{2}=\sqrt[8]{2},$

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n^k}{4^n} & \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+\frac{1}{\infty})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^k}{4^n} = 0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim\limits_{n\to\infty} \frac{5^n}{4^n} = \lim\limits_{n\to\infty} (\frac{5}{4})^n = \infty \end{array} \right]$$

$$= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6}$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

Explicitne: $a_1 = 2^{\frac{1}{2}} = \sqrt{2}$, $a_2 = a_1^{\frac{1}{2}} = 2^{\frac{1}{2^2}} = \sqrt[2^2]{2} = \sqrt[4]{2}$, $a_3 = a_2^{\frac{1}{2}} = 2^{\frac{1}{2^3}} = \sqrt[2^3]{2} = \sqrt[8]{2}$, ...,
$a_n = a_{n-1}^{\frac{1}{2}} = 2^{\frac{1}{2^n}} = \sqrt[2^n]{2}$ pre $n \in N$.

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} = \infty$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{4^n} & \lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{n^k}{4^n}}=\lim\limits_{n\to\infty}\frac{\sqrt[n]{n^k}}{4}=\lim\limits_{n\to\infty}\frac{(\sqrt[n]{n})^k}{4}=\frac{1^k}{4}=\frac{1}{4}<1 \\ \text{pre } n\in N,\ k\in N & \lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}}=\lim\limits_{n\to\infty}\frac{(n+1)^k}{4n^k}=\lim\limits_{n\to\infty}\frac{(1+\frac{1}{n})^k}{4}=\frac{(1+\frac{1}{\infty})^k}{4}=\frac{(1+0)^k}{4}=\frac{1}{4}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{4^n}=0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}\cdot\frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty}\frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{l} \text{Geometrická postupnosť} \\ \lim\limits_{n\to\infty}\frac{5^n}{4^n}=\lim\limits_{n\to\infty}\left(\frac{5}{4}\right)^n=\infty \end{array} \right]$$

$$= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6} = \infty.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[2^n]{2}$$

Rekurentne: $a_1=\sqrt{2}=2^{\frac{1}{2}}$, $a_{n+1}=\sqrt{a_n}=a_n^{\frac{1}{2}}$ pre $n\in N$.

Explicitne: $a_1=2^{\frac{1}{2}}=\sqrt{2}$, $a_2=a_1^{\frac{1}{2}}=2^{\frac{1}{2^2}}=\sqrt[2^2]{2}=\sqrt[4]{2}$, $a_3=a_2^{\frac{1}{2}}=2^{\frac{1}{2^3}}=\sqrt[2^3]{2}=\sqrt[8]{2}$, $\ldots$,

$a_n=a_{n-1}^{\frac{1}{2}}=2^{\frac{1}{2^n}}=\sqrt[2^n]{2}$ pre $n\in N$.

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} = \infty$$

$$= \left[\begin{matrix} \text{Označme } a_n = \frac{n^k}{4^n} \\ \text{pre } n\in N,\ k\in N \end{matrix} \middle| \begin{matrix} \lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim\limits_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1 \\ \lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim\limits_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim\limits_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+\frac{1}{\infty})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1 \end{matrix} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^k}{4^n} = 0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{matrix} \text{Geometrická postupnosť} \\ \lim\limits_{n\to\infty} \frac{5^n}{4^n} = \lim\limits_{n\to\infty} (\frac{5}{4})^n = \infty \end{matrix} \right]$$

$$= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6} = \infty.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[2^n]{2} = \left[\begin{matrix} \text{Subst.} \\ m=2^n \end{matrix} \middle| \begin{matrix} n\to\infty \\ m\to\infty \end{matrix} \right]$$

Rekurentne: $a_1 = \sqrt{2} = 2^{\frac{1}{2}}$, $a_{n+1} = \sqrt{a_n} = a_n^{\frac{1}{2}}$ pre $n\in N$.

Explicitne: $a_1 = 2^{\frac{1}{2}} = \sqrt{2}$, $a_2 = a_1^{\frac{1}{2}} = 2^{\frac{1}{2^2}} = \sqrt[2^2]{2} = \sqrt[4]{2}$, $a_3 = a_2^{\frac{1}{2}} = 2^{\frac{1}{2^3}} = \sqrt[2^3]{2} = \sqrt[8]{2}$, ...,
$a_n = a_{n-1}^{\frac{1}{2}} = 2^{\frac{1}{2^n}} = \sqrt[2^n]{2}$ pre $n\in N$.

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} = \infty$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n=\frac{n^k}{4^n} & \lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{n^k}{4^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n^k}}{4}=\lim_{n\to\infty}\frac{(\sqrt[n]{n})^k}{4}=\frac{1^k}{4}=\frac{1}{4}<1 \\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}}=\lim_{n\to\infty}\frac{(n+1)^k}{4n^k}=\lim_{n\to\infty}\frac{(1+\frac{1}{n})^k}{4}=\frac{(1+\frac{1}{\infty})^k}{4}=\frac{(1+0)^k}{4}=\frac{1}{4}<1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{n^k}{4^n}=0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n}\cdot\frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty}\frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{l}\text{Geometrická postupnosť}\\ \lim_{n\to\infty}\frac{5^n}{4^n}=\lim_{n\to\infty}(\frac{5}{4})^n=\infty\end{array}\right]$$

$$= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6} = \infty.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \ldots\right\}$.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\sqrt[2^n]{2} = \left[\begin{array}{l|l}\text{Subst.} & n\to\infty \\ m=2^n & m\to\infty\end{array}\right] = \lim_{m\to\infty}\sqrt[m]{2}$$

Rekurentne: $a_1=\sqrt{2}=2^{\frac{1}{2}}$, $a_{n+1}=\sqrt{a_n}=a_n^{\frac{1}{2}}$ pre $n\in N$.

Explicitne: $a_1=2^{\frac{1}{2}}=\sqrt{2}$, $a_2=a_1^{\frac{1}{2}}=2^{\frac{1}{2^2}}=\sqrt[2^2]{2}=\sqrt[4]{2}$, $a_3=a_2^{\frac{1}{2}}=2^{\frac{1}{2^3}}=\sqrt[2^3]{2}=\sqrt[8]{2}$, ...,

$a_n=a_{n-1}^{\frac{1}{2}}=2^{\frac{1}{2^n}}=\sqrt[2^n]{2}$ pre $n\in N$.

Riešené limity – 76, 77

$$\lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} = \infty$$

$$= \left[\begin{array}{l|l} \text{Označme } a_n = \frac{n^k}{4^n} & \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n^k}}{4} = \lim_{n\to\infty} \frac{(\sqrt[n]{n})^k}{4} = \frac{1^k}{4} = \frac{1}{4} < 1 \\ \text{pre } n\in N,\ k\in N & \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^k}{4^{n+1}}}{\frac{n^k}{4^n}} = \lim_{n\to\infty} \frac{(n+1)^k}{4n^k} = \lim_{n\to\infty} \frac{(1+\frac{1}{n})^k}{4} = \frac{(1+\frac{1}{\infty})^k}{4} = \frac{(1+0)^k}{4} = \frac{1}{4} < 1 \end{array} \right\} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^k}{4^n} = 0 \right]$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^4-n^2+2\cdot 5^n}{2n^2-n^3+6\cdot 4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{2\cdot\frac{n^3}{4^n}+3\cdot\frac{n^4}{4^n}-\frac{n^2}{4^n}+2\cdot\frac{5^n}{4^n}}{2\cdot\frac{n^2}{4^n}-\frac{n^3}{4^n}+6} = \left[\begin{array}{c} \text{Geometrická postupnosť} \\ \lim_{n\to\infty} \frac{5^n}{4^n} = \lim_{n\to\infty} (\frac{5}{4})^n = \infty \end{array} \right]$$

$$= \frac{2\cdot 0+3\cdot 0-0+2\cdot\infty}{2\cdot 0-0+6} = \frac{\infty}{6} = \infty.$$

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \dots\right\}$.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[2^n]{2} = \left[\begin{array}{c|c} \text{Subst.} & n\to\infty \\ m=2^n & m\to\infty \end{array} \right] = \lim_{m\to\infty} \sqrt[m]{2} = 1.$$

Rekurentne: $a_1 = \sqrt{2} = 2^{\frac{1}{2}}, \quad a_{n+1} = \sqrt{a_n} = a_n^{\frac{1}{2}}$ pre $n \in N$.

Explicitne: $a_1 = 2^{\frac{1}{2}} = \sqrt{2}, \quad a_2 = a_1^{\frac{1}{2}} = 2^{\frac{1}{2^2}} = \sqrt[2^2]{2} = \sqrt[4]{2}, \quad a_3 = a_2^{\frac{1}{2}} = 2^{\frac{1}{2^3}} = \sqrt[2^3]{2} = \sqrt[8]{2}, \quad \dots,$

$a_n = a_{n-1}^{\frac{1}{2}} = 2^{\frac{1}{2^n}} = \sqrt[2^n]{2}$ pre $n \in N$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Rekurentná definícia má tvar: $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2+a_n}$ pre $n \in N$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{ \sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots \right\}$.

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Označme $\lim\limits_{n\to\infty} a_n = a$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

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Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.

Riešené limity – 78

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Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.

$\Rightarrow$ $a^2 = \lim_{n\to\infty} a_n^2 = \lim_{n\to\infty} a_{n+1}^2 = \lim_{n\to\infty} (2+a_n) = 2 + \lim_{n\to\infty} a_n = 2+a$.

Riešené limity – 78

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Riešené limity – 78

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Označme $\lim\limits_{n\to\infty} a_n = a$. Platí $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1}$.

$\Rightarrow \quad a^2 = \lim\limits_{n\to\infty} a_n^2 = \lim\limits_{n\to\infty} a_{n+1}^2 = \lim\limits_{n\to\infty} (2+a_n) = 2 + \lim\limits_{n\to\infty} a_n = 2+a.$

$\Rightarrow$ a je riešením rovnice $a^2 = 2+a$, t. j. $a^2 - a - 2 = (a-2)\cdot(a+1) = 0$.

Riešené limity – 78

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$\Rightarrow$ $a = \lim\limits_{n\to\infty} a_n = 2$ (Koreň $a = -1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$).

Riešené limity – 78

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Ešte musíme ukázať, že limita $\lim\limits_{n\to\infty} a_n$ existuje, t. j. že postupnosť $\{a_n\}_{n=1}^{\infty}$ konverguje.

Riešené limity – 78

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Ešte musíme ukázať, že limita $\lim\limits_{n\to\infty} a_n$ existuje, t. j. že postupnosť $\{a_n\}_{n=1}^{\infty}$ konverguje.

$\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora

$\{a_n\}_{n=1}^{\infty}$ je rastúca

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Rekurentná definícia má tvar: $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2+a_n}$ pre $n \in N$.

Označme $\lim\limits_{n\to\infty} a_n = a$. Platí $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1}$.

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$\Rightarrow$ a je riešením rovnice $a^2 = 2+a$, t. j. $a^2 - a - 2 = (a-2)\cdot(a+1) = 0$.

$\Rightarrow$ $a = \lim\limits_{n\to\infty} a_n = 2$ (Koreň $a=-1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$).

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$\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora

$a_n < 2$, t. j. $a_n - 2 < 0$ pre všetky $n \in N$ (dôkaz matematickou indukciou).

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Rekurentná definícia má tvar: $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2+a_n}$ pre $n \in N$.

Označme $\lim\limits_{n\to\infty} a_n = a$. Platí $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1}$.

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$\Rightarrow$ $a = \lim\limits_{n\to\infty} a_n = 2$ (Koreň $a = -1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$).

Ešte musíme ukázať, že limita $\lim\limits_{n\to\infty} a_n$ existuje, t. j. že postupnosť $\{a_n\}_{n=1}^{\infty}$ konverguje.

$\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora: $a_n < 2$, t. j. $a_n - 2 < 0$ pre všetky $n \in N$ (dôkaz matematickou indukciou).

1. krok: $a_1 = \sqrt{2} < 2$, $a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = \sqrt{4} = 2$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Rekurentná definícia má tvar: $a_1 = \sqrt{2}$, $a_{n+1} = \sqrt{2+a_n}$ pre $n \in N$.

Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.

$\Rightarrow$ $a^2 = \lim_{n\to\infty} a_n^2 = \lim_{n\to\infty} a_{n+1}^2 = \lim_{n\to\infty} (2+a_n) = 2 + \lim_{n\to\infty} a_n = 2+a$.

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1. krok: $a_1 = \sqrt{2} < 2$, $a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = \sqrt{4} = 2$.

2. krok: $a_k < 2$ pre $k \in N$

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Rekurentná definícia má tvar: $a_1 = \sqrt{2}, \quad a_{n+1} = \sqrt{2+a_n}$ pre $n \in N$.

Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.

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$\Rightarrow a$ je riešením rovnice $a^2 = 2+a$, t. j. $a^2 - a - 2 = (a-2)\cdot(a+1) = 0.$

$\Rightarrow a = \lim_{n\to\infty} a_n = 2$ (Koreň $a = -1 < 0$ nevyhovuje, pretože $a_n > 0$ pre všetky $n \in N$).

Ešte musíme ukázať, že limita $\lim_{n\to\infty} a_n$ existuje, t. j. že postupnosť $\{a_n\}_{n=1}^{\infty}$ konverguje.

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1. krok: $a_1 = \sqrt{2} < 2, \quad a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = \sqrt{4} = 2.$

2. krok: $a_k < 2$ pre $k \in N$ $\Rightarrow a_{k+1} = \sqrt{2+a_k} < \sqrt{2+2} = \sqrt{4} = 2$ pre $k+1$.

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

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$\{a_n\}_{n=1}^{\infty}$ je rastúca: $a_n > 0$, $a_n - 2 < 0$ pre všetky $n \in N$,

Riešené limity – 78

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty} = \left\{\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots\right\}$.

Rekurentná definícia má tvar: $a_1 = \sqrt{2}, \quad a_{n+1} = \sqrt{2+a_n}$ pre $n \in N$.

Označme $\lim_{n\to\infty} a_n = a$. Platí $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.

$\Rightarrow \ a^2 = \lim_{n\to\infty} a_n^2 = \lim_{n\to\infty} a_{n+1}^2 = \lim_{n\to\infty} (2+a_n) = 2 + \lim_{n\to\infty} a_n = 2+a.$

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1. krok: $a_1 = \sqrt{2} < 2, \quad a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = \sqrt{4} = 2.$

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Ešte musíme ukázať, že limita $\lim\limits_{n\to\infty} a_n$ existuje, t. j. že postupnosť $\{a_n\}_{n=1}^{\infty}$ konverguje.

$\{a_n\}_{n=1}^{\infty}$ je ohraničená zhora: $a_n<3$, t. j. $a_n-3<0$ pre všetky $n\in N$ (dôkaz matematickou indukciou).

1. krok: $a_1 = 2 < 3,\quad a_2 = \sqrt{2\cdot 2+3} < \sqrt{7} < \sqrt{9} = 3.$

2. krok: $a_k < 3$ pre $k\in N$ $\Rightarrow$ $a_{k+1} = \sqrt{2a_k+3} < \sqrt{2\cdot 3+3} = \sqrt{9} = 3$ pre $k+1$.

$\{a_n\}_{n=1}^{\infty}$ je rastúca: $a_n>0,\quad a_n-3<0$ pre všetky $n\in N$, potom platí:

$a_n^2 - a_{n+1}^2 = a_n^2-(2a_n+3) = a_n^2-2a_n-3 = (a_n-3)(a_n+1) < 0$, t. j. $a_n^2 < a_{n+1}^2$,

Riešené limity – 79

Vypočítajte limitu postupnosti $\{a_n\}_{n=1}^{\infty}$ zadanej rekurentne $a_1=2,\ a_{n+1}=\sqrt{2a_n+3},\ n\in N$.

Označme $\lim\limits_{n\to\infty} a_n = a$. Platí $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1}$.

$\Rightarrow\ a^2 = \lim\limits_{n\to\infty} a_n^2 = \lim\limits_{n\to\infty} a_{n+1}^2 = \lim\limits_{n\to\infty} (2a_n+3) = 2\lim\limits_{n\to\infty} a_n + 3 = 2a+3.$

$\Rightarrow$ a je riešením rovnice $a^2 = 2a+3$, t. j. $a^2-2a-3 = (a-3)\cdot(a+1) = 0$.

$\Rightarrow\ a = \lim\limits_{n\to\infty} a_n = 3$ (Koreň $a=-1<0$ nevyhovuje, pretože $a_n>0$ pre všetky $n\in N$).

Ešte musíme ukázať, že limita $\lim\limits_{n\to\infty} a_n$ existuje, t. j. že postupnosť $\{a_n\}_{n=1}^{\infty}$ konverguje.

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1. krok $a_1=2<3,\quad a_2=\sqrt{2\cdot 2+3}<\sqrt{7}<\sqrt{9}=3.$

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$\{a_n\}_{n=1}^{\infty}$ je rastúca $a_n>0,\quad a_n-3<0$ pre všetky $n\in N$, potom platí:

$a_n^2-a_{n+1}^2 = a_n^2-(2a_n+3) = a_n^2-2a_n-3 = (a_n-3)(a_n+1)<0$, t. j. $a_n^2<a_{n+1}^2$, t. j. $a_n<a_{n+1}$.

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

$32{,}17\overline{71} = 32{,}177171\ldots$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

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$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots \quad \Rightarrow \quad 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots$ $\Rightarrow$ $100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$\Rightarrow$ $10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

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$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right]$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots \quad \Rightarrow \quad 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$$\Rightarrow \quad 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$$

$$\Rightarrow \qquad 10\,000x - 100x$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

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$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot100} + \frac{71}{100\cdot100\cdot100} + \frac{71}{100\cdot100\cdot100\cdot100} + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot100} + \cdots\right] = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q=\frac{1}{100}\\ 1+\frac{1}{100}+\frac{1}{100\cdot100}+\cdots=\frac{1}{1-q}=\frac{1}{1-\frac{1}{100}}\end{array}\right]$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots \quad \Rightarrow 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$$\Rightarrow 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$$

$$\Rightarrow \quad 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right] = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q=\frac{1}{100}\\ 1+\frac{1}{100}+\frac{1}{100\cdot 100}+\cdots=\frac{1}{1-q}=\frac{1}{1-\frac{1}{100}}\end{array}\right]$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}}$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots \quad \Rightarrow \quad 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$$\Rightarrow \quad 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$$

$$\Rightarrow \quad 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$$
$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

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$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right] = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q=\frac{1}{100}\\ 1+\frac{1}{100}+\frac{1}{100\cdot 100}+\cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{100-1}$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots$ $\Rightarrow 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$\Rightarrow 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$

$$\Rightarrow \quad 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot100} + \frac{71}{100\cdot100\cdot100} + \frac{71}{100\cdot100\cdot100\cdot100} + \cdots$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot100} + \cdots\right] = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q=\frac{1}{100}\\ 1+\frac{1}{100}+\frac{1}{100\cdot100}+\cdots=\frac{1}{1-q}=\frac{1}{1-\frac{1}{100}}\end{array}\right]$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{100-1}$$

$$= \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{99}$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots \quad \Rightarrow\ 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$$\Rightarrow\ 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$$

$$\Rightarrow\quad 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100} + \frac{71}{100\cdot 100\cdot 100} + \frac{71}{100\cdot 100\cdot 100\cdot 100} + \cdots$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right] = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{100-1}$$
$$= \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{99} = \frac{3\,217\cdot 99 + 71}{100\cdot 99}$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots$ $\Rightarrow 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$\Rightarrow 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$

$\Rightarrow 9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$

$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots$

Periodické číslo $32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$
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$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right] = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q=\frac{1}{100}\\ 1+\frac{1}{100}+\frac{1}{100\cdot 100}+\cdots=\frac{1}{1-q}=\frac{1}{1-\frac{1}{100}}\end{array}\right]$$
$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{100-1}$$
$$= \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{99} = \frac{3\,217\cdot 99+71}{100\cdot 99} = \frac{318\,483+71}{9\,900}$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots \quad \Rightarrow 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$$\Rightarrow 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$$

$$\Rightarrow 9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$$
$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

$$\Rightarrow 9\,900x = 318\,554$$

Riešené limity – 80

Vyjadrite ako zlomok periodické číslo $32{,}17\overline{71} = 32{,}17717171\ldots = \frac{318\,554}{9\,900}$.

Periodické číslo $32{,}17\overline{71}$ môžeme vyjadriť ako súčet členov postupnosti:

$$32{,}17\overline{71} = 32{,}177171\ldots = 32{,}17 + 0{,}0071 + 0{,}000071 + 0{,}00000071 + \cdots$$

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$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\left[1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots\right] = \left[\begin{array}{l}\text{Geometrický rad s kvocientom } q = \frac{1}{100} \\ 1 + \frac{1}{100} + \frac{1}{100\cdot 100} + \cdots = \frac{1}{1-q} = \frac{1}{1-\frac{1}{100}}\end{array}\right]$$

$$= \frac{3\,217}{100} + \frac{71}{100\cdot 100}\cdot\frac{1}{1-\frac{1}{100}} = \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{100-1}$$

$$= \frac{3\,217}{100} + \frac{71}{100}\cdot\frac{1}{99} = \frac{3\,217\cdot 99 + 71}{100\cdot 99} = \frac{318\,483+71}{9\,900} = \frac{318\,554}{9\,900}.$$

Označme $x = 32{,}17\overline{71} = 32{,}17717171\ldots \quad \Rightarrow 100x = 3\,217{,}\overline{71} = 3217{,}7171\ldots$

$$\Rightarrow 10\,000x = 321\,771{,}\overline{71} = 321771{,}7171\ldots$$

$$\Rightarrow 9\,900x = 10\,000x - 100x = 321\,771{,}\overline{71} - 3\,217{,}\overline{71}$$

$$= 321\,771{,}717171\ldots - 3\,217{,}717171\ldots = 321\,771 - 3\,217 = 318\,554.$$

$$\Rightarrow 9\,900x = 318\,554 \quad \Rightarrow x = \frac{318\,554}{9\,900}.$$