Matematická analýza 1

2018/2019

6. Limita funkcie

Obsah

Zoznam riešených limít – príklady 01–44

01. $\lim\limits_{x\to 0} \frac{a^x-1}{x}$ **02.** $\lim\limits_{x\to 2} \frac{x-2}{x^2-3x+2}$ **03.** $\lim\limits_{x\to\infty} \left(\frac{5x^2}{x^2-1}+2^{\frac{1}{x}}\right)$ **04.** $\lim\limits_{x\to 0} \frac{1-3^x}{x}$ **05.** $\lim\limits_{x\to-\infty} \frac{1-3^x}{x}$

06. $\lim\limits_{x\to 1} \frac{1-3^x}{x}$ **07.** $\lim\limits_{x\to\infty} \frac{1-3^x}{x}$ **08.** $\lim\limits_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$ **09.** $\lim\limits_{x\to\infty} \left(\frac{1-x}{x^2}+\frac{1-2x}{2-3x}\right)$

10. $\lim\limits_{x\to 1} \left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$ **11.** $\lim\limits_{x\to\infty} \left(\frac{3x-2}{3x+1}\right)^x$ **12.** $\lim\limits_{x\to-\infty} \left(\frac{x+1}{x}\right)^x$ **13.** $\lim\limits_{x\to 0} (1+3\operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$

14. $\lim\limits_{x\to 0} (1+3\operatorname{tg}^2 x)^{\operatorname{tg}^2 x}$ **15.** $\lim\limits_{x\to\infty} (1-\frac{1}{x})^{x^2}$ **16.** $\lim\limits_{x\to\infty} x\left[\ln(x+2)-\ln x\right]$

17. $\lim\limits_{x\to 0} \frac{x}{\ln(1+tx)}$ **18.** $\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x}$ **19.** $\lim\limits_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$ **20.** $\lim\limits_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$

21. $\lim\limits_{x\to\infty} \left(x-\sqrt{x^2-1}\right)$ **22.** $\lim\limits_{x\to a} \frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$ **23.** $\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ **24.** $\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$ **25.** $\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$

26. $\lim\limits_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$ **27.** $\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$ **28.** $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ **29.** $\lim\limits_{x\to 0} \frac{\sin 5x}{x}$ **30.** $\lim\limits_{x\to 0} \frac{\operatorname{tg} x}{x}$ **31.** $\lim\limits_{x\to 0} \frac{\operatorname{arctg} x}{x}$

32. $\lim\limits_{x\to 0} \frac{\operatorname{tg} x}{\sin 2x}$ **33.** $\lim\limits_{x\to\pi} \frac{\operatorname{tg} x}{\sin 2x}$ **34.** $\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx}$ **35.** $\lim\limits_{x\to\pi} \frac{\sin mx}{\sin nx}$ **36.** $\lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2}$

37. $\lim\limits_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x-\sin x}$ **38.** $\lim\limits_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$ **39.** $\lim\limits_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$ **40.** $\lim\limits_{x\to\infty} e^x(2+\cos x)$

41. $\lim\limits_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$ **42.** $\lim\limits_{x\to 0} \frac{x^n-\sin^n x}{x-\sin x}$ **43.** $\lim\limits_{x\to a} \frac{\sin x-\sin a}{x-a}$ **44.** $\lim\limits_{x\to a} \frac{\cos x-\cos a}{x-a}$

Limita funkcie – Definícia v zmysle Heineho

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Ekvivalentná definícia limity pomocou okolí

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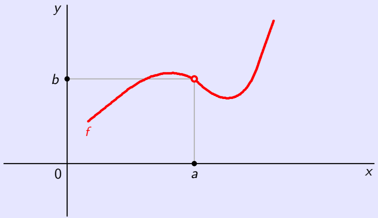
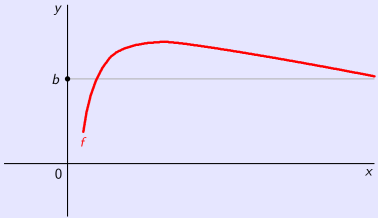
Ekvivalentná definícia limity pomocou okolí $a, b \in R^*$

$\lim_{x \to a} f(x) = b$ práve vtedy, ak:





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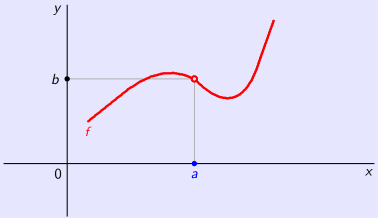

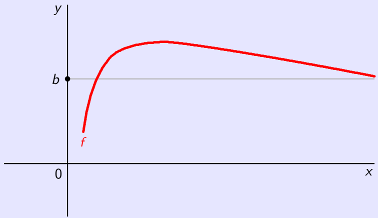

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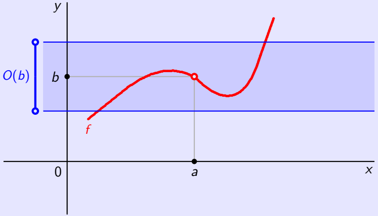

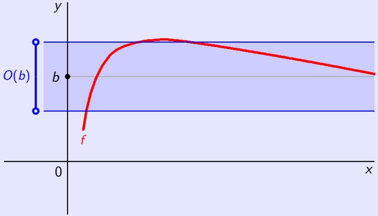

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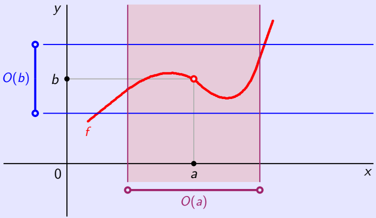

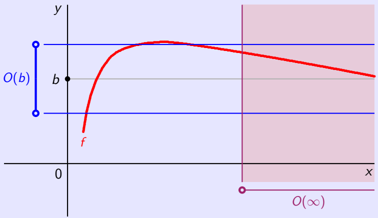

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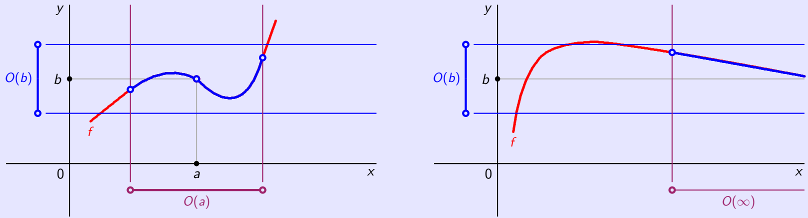

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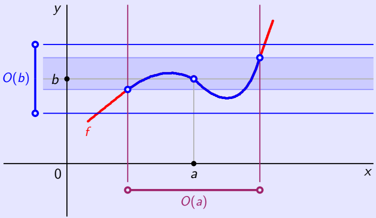

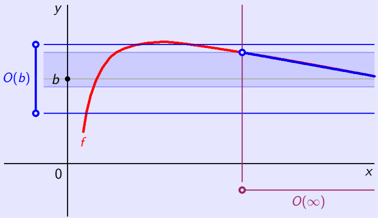

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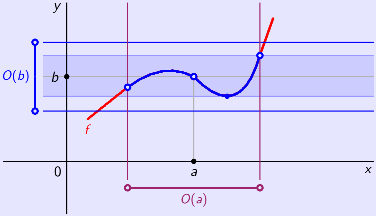

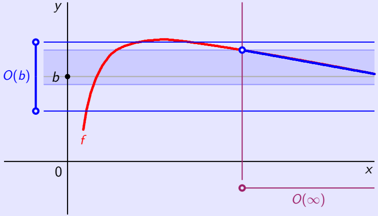

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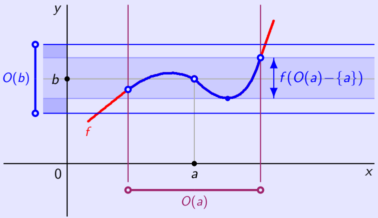
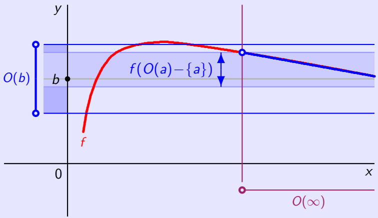
  t. j. $f(O(a) - \{a\}) \subset O(b)$.





Vlastnosti limít

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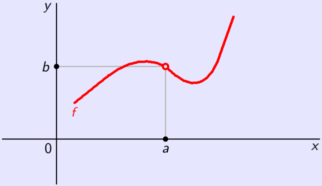

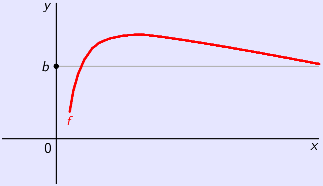

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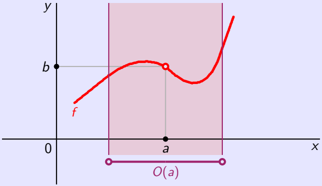

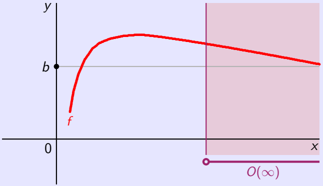

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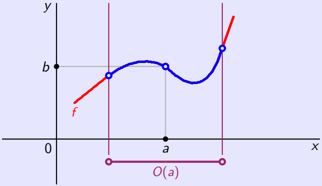

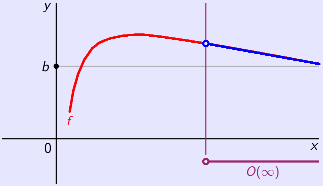

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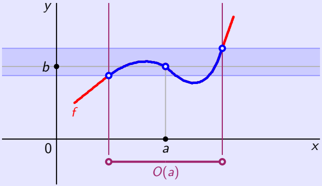

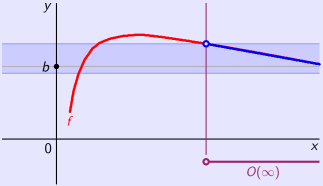

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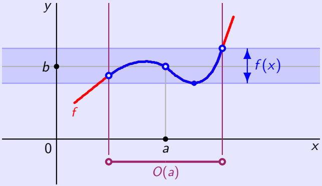

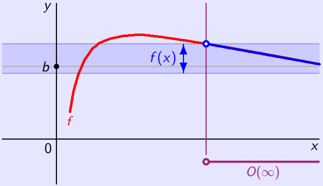

Vlastnosti limít

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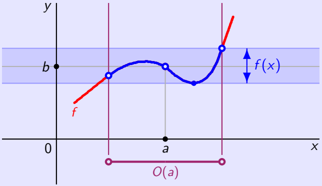

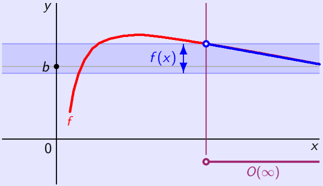

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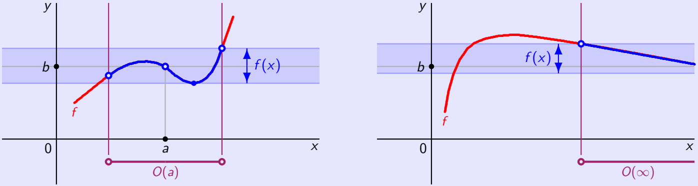

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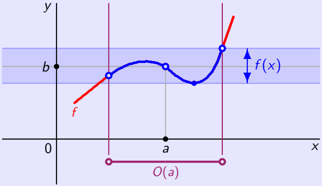

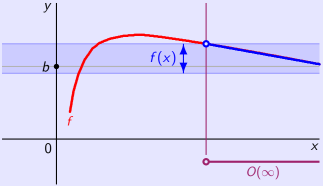

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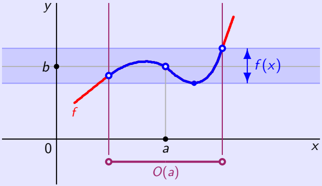

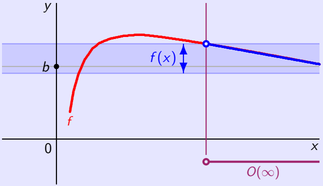

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$\lim_{x \to a} x^2$ $a \in R^*$ (konštanta).

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$\lim\limits_{x\to 0} \chi(x)$

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$\{\frac{1}{n}\}_{n=1}^{\infty} \longmapsto 0$, $\frac{1}{n} \neq 0$, $\frac{1}{n} \in Q$, t. j. $\chi(\frac{1}{n}) = 1$ pre $n \in N$

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$\lim\limits_{x\to 0} \chi(x)$ neexistuje.

Bod 0 je hromadným bodom $D(\chi) = R$.

$$\left.\begin{array}{l} \{\frac{1}{n}\}_{n=1}^{\infty} \longmapsto 0,\ \frac{1}{n}\neq 0,\ \frac{1}{n}\in Q, \text{ t. j. } \chi(\frac{1}{n}) = 1 \text{ pre } n\in N \Rightarrow \{\chi(\frac{1}{n})\}_{n=1}^{\infty} \longmapsto 1. \\ \{\frac{\pi}{n}\}_{n=1}^{\infty} \longmapsto 0,\ \frac{\pi}{n}\neq 0,\ \frac{\pi}{n}\notin Q, \text{ t. j. } \chi(\frac{\pi}{n}) = 0 \text{ pre } n\in N \Rightarrow \{\chi(\frac{\pi}{n})\}_{n=1}^{\infty} \longmapsto 0. \end{array}\right\}$$

To znamená, že limita neexistuje.

Vlastnosti limít

$f(x)=g(x)$ pre všetky $x\in O(a)$, $x\neq a$,
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Vlastnosti limít

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Predchádzajúce tvrdenie sa pri výpočte limít využíva prakticky neustále.

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Predchádzajúce tvrdenie sa pri výpočte limít využíva prakticky neustále.

$\lim\limits_{x\to 2} \frac{x^2-4}{x-2}$

Vlastnosti limít

$f(x) = g(x)$ pre všetky $x \in O(a)$, $x \neq a$,
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$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \left[\begin{array}{l} \text{pre všetky } x \in O(2),\ x \neq 2 \text{ (pre ľubovoľné okolie)} \\ \text{platí } f(x) = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2 = g(x) \end{array} \right] = \lim_{x\to 2} (x+2)$$

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Vlastnosti limít

$f(x)=g(x)$ pre všetky $x\in O(a)$, $x\neq a$,
$a\in R^*$ je hromadný bod $D(f)$, $D(g)$, $O(a)$ je okolie.

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Predchádzajúce tvrdenie sa pri výpočte limít využíva prakticky neustále.

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \left[\begin{array}{l}\text{pre všetky } x\in O(2),\ x\neq 2 \text{ (pre ľubovoľné okolie)}\\ \text{platí } f(x)=\frac{x^2-4}{x-2}=\frac{(x-2)(x+2)}{x-2}=x+2=g(x)\end{array}\right] = \lim_{x\to 2}(x+2)=4.$$

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a mnohé „zaručené kuchárske recepty tzv. vzdelancov“ ju ignorujú.

Dôležité limity

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Pre limity týchto funkcií potom platí:

$$-\lim_{x\to\infty} \frac{1}{x} \leq \lim_{x\to\infty} \frac{\sin x}{x} \leq \lim_{x\to\infty} \frac{1}{x}$$

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$$0=-\lim_{x\to\infty}\frac{1}{x}\leq\lim_{x\to\infty}\frac{\sin x}{x}\leq\lim_{x\to\infty}\frac{1}{x}=0$$

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Výpočet limít – zaručené kuchárske recepty

$$\lim_{x\to 0} \frac{\arcsin x}{x}$$

$$\lim_{x\to 1} \frac{\arcsin x}{x}$$

Výpočet limít – zaručené kuchárske recepty

$$\lim_{x\to 0}\frac{\arcsin x}{x}=\lim_{x\to 0}\frac{\sin\arcsin x}{\sin x}=\left[\begin{array}{l}\text{tzv. sínusovanie}\\ \sin\arcsin x = x\end{array}\right]$$

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I keď náhodou vyšiel správny výsledok, uvedený postup je úplne mimo realitu!

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$$\lim_{x\to 1}\frac{\arcsin x}{x}$$

resp. $\lim_{x\to 1}\frac{\arcsin x}{x}$

Výpočet limít – zaručené kuchárske recepty

$$\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{x\to 0} \frac{\sin \arcsin x}{\sin x} = \left[\begin{array}{c} \text{tzv. sínusovanie} \\ \sin \arcsin x = x \end{array} \right] = \lim_{x\to 0} \frac{x}{\sin x} = 1.$$

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$$\lim_{x\to 1} \frac{\arcsin x}{x} = \left[\begin{array}{r|l} \text{Subst. } x = \sin u & x \to 1 \\ u = \arcsin x & u \to \frac{\pi}{2} \end{array} \right]$$

resp. $\lim_{x\to 1} \frac{\arcsin x}{x}$

Výpočet limít – zaručené kuchárske recepty

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resp. $\lim_{x\to 1}\frac{\arcsin x}{x} = \frac{\arcsin 1}{1}$

Výpočet limít – zaručené kuchárske recepty

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Výpočet limít – zaručené kuchárske recepty

Postupy patriace medzi kuchárske recepty zaručené

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Jeden zo správnych postupov

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Výpočet limít – zaručené kuchárske recepty

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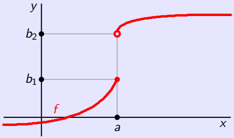
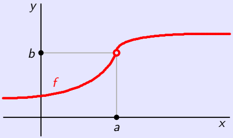
Označme zúženia $f(x)$, $x \in D(f)$ na intervaly $(-\infty; a)$, $(a; \infty)$, kde $a \in R$:





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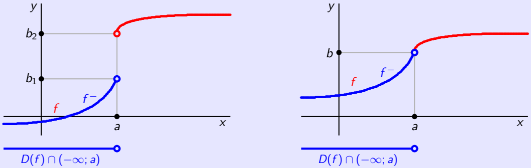

$\lim\limits_{x\to a} f^-(x)$ sa nazýva limita zľava funkcie f v bode a

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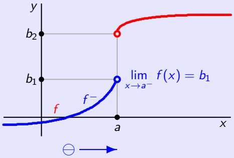

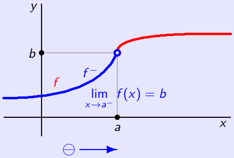

$\lim_{x\to a} f^-(x)$ sa nazýva limita zľava funkcie f v bode a, ozn. $\lim_{x\to a^-} f(x)$,

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Označme zúženia $f(x)$, $x \in D(f)$ na intervaly $(-\infty; a)$, $(a; \infty)$, kde $a \in R$:

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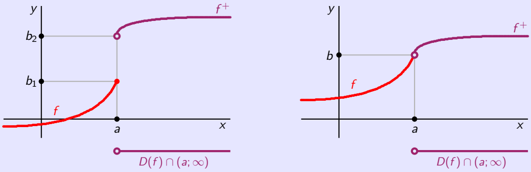

$\lim\limits_{x\to a} f^{+}(x)$ sa nazýva limita sprava funkcie f v bode a

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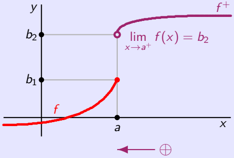

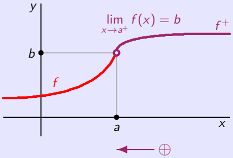

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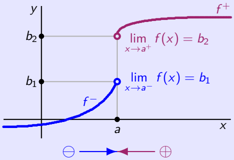

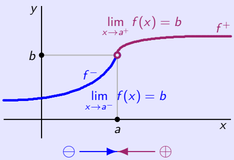

$\lim\limits_{x\to a} f^{-}(x)$ sa nazýva limita zľava funkcie f v bode a, ozn. $\lim\limits_{x\to a^-} f(x)$,

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$a \in R$, a je hromadný bod $D(f)$, $b \in R^*$

Obojstranná $\lim\limits_{x \to a} f(x) = b$ existuje

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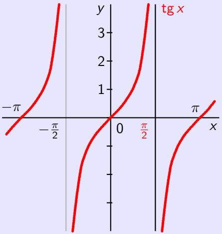

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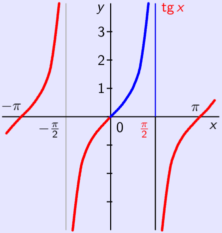

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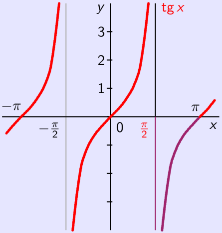

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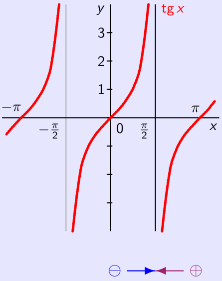

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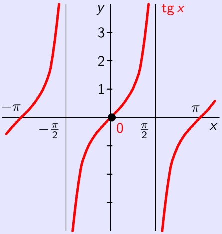

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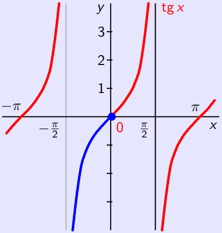

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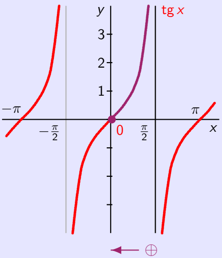

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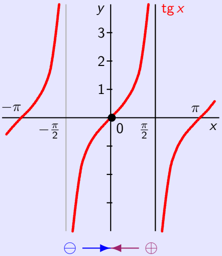

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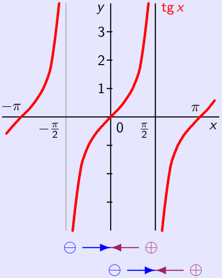

Riešené limity – 01, 02, 03

$\lim_{x\to 0} \frac{a^x-1}{x}$ $\quad a>0,\ a\neq 1$

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Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x}$ $\qquad a > 0,\ a \neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x \ln a = \ln a^x = \ln(z+1) & \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z \ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)}$$

Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x}$ $\qquad a>0,\ a\neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x \ln a = \ln a^x = \ln(z+1) & \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z \ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}}$$

Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x}$ $\qquad a>0,\ a\neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x \ln a = \ln a^x = \ln(z+1) & \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z \ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to e \\ \text{pre } z \to 0 \end{array} \right] = \frac{\ln a}{\ln e}$$

Riešené limity – 01, 02, 03

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a \qquad a>0,\ a\neq 1$$

$$=\left[\begin{array}{r|l}\text{Subst. } z=a^x-1 & x\to 0\\ z+1=a^x & z\to 0\\ x\ln a=\ln a^x=\ln(z+1) & \end{array}\right]=\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln a}}=\lim_{z\to 0}\frac{z\ln a}{\ln(z+1)}$$

$$=\lim_{z\to 0}\frac{\ln a}{\frac{1}{z}\ln(z+1)}=\lim_{z\to 0}\frac{\ln a}{\ln(z+1)^{\frac{1}{z}}}=\left[\begin{array}{c}(1+z)^{\frac{1}{z}}\to e\\ \text{pre } z\to 0\end{array}\right]=\frac{\ln a}{\ln e}=\frac{\ln a}{1}=\ln a.$$

Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x} = \ln a$ $\quad a>0,\ a\neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x\ln a = \ln a^x = \ln(z+1) & \end{array}\right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z\ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to e \\ \text{pre } z \to 0 \end{array}\right] = \frac{\ln a}{\ln e} = \frac{\ln a}{1} = \ln a.$$

$\lim\limits_{x\to 2} \frac{x-2}{x^2-3x+2}$

Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x} = \ln a$ $\quad a > 0,\ a \neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x \ln a = \ln a^x = \ln(z+1) & \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z \ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to e \\ \text{pre } z \to 0 \end{array} \right] = \frac{\ln a}{\ln e} = \frac{\ln a}{1} = \ln a.$$

$\lim\limits_{x\to 2} \frac{x-2}{x^2-3x+2}$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x-1)}$$

Riešené limity – 01, 02, 03

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a \qquad a>0,\ a\neq 1$$

$$=\left[\begin{array}{r|l}\text{Subst. } z=a^x-1 & x\to 0\\ z+1=a^x & z\to 0\\ x\ln a=\ln a^x=\ln(z+1) & \end{array}\right]=\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln a}}=\lim_{z\to 0}\frac{z\ln a}{\ln(z+1)}$$

$$=\lim_{z\to 0}\frac{\ln a}{\frac{1}{z}\ln(z+1)}=\lim_{z\to 0}\frac{\ln a}{\ln(z+1)^{\frac{1}{z}}}=\left[\begin{array}{c}(1+z)^{\frac{1}{z}}\to e\\ \text{pre } z\to 0\end{array}\right]=\frac{\ln a}{\ln e}=\frac{\ln a}{1}=\ln a.$$

$$\lim_{x\to 2}\frac{x-2}{x^2-3x+2}$$

$$=\lim_{x\to 2}\frac{x-2}{(x-2)(x-1)}=\lim_{x\to 2}\frac{1}{x-1}$$

Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x} = \ln a$ $\qquad a>0,\ a\neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x\to 0 \\ z+1 = a^x & z\to 0 \\ x\ln a = \ln a^x = \ln(z+1) & \end{array}\right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z\ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to e \\ \text{pre } z\to 0 \end{array}\right] = \frac{\ln a}{\ln e} = \frac{\ln a}{1} = \ln a.$$

$\lim\limits_{x\to 2} \frac{x-2}{x^2-3x+2} = 1$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x-1)} = \lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1} = 1.$$

Riešené limity – 01, 02, 03

$$\lim_{x\to 0} \frac{a^x-1}{x} = \ln a \qquad a>0,\ a\neq 1$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x\ln a = \ln a^x = \ln(z+1) & \end{array}\right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z\ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to \mathrm{e} \\ \text{pre } z \to 0 \end{array}\right] = \frac{\ln a}{\ln \mathrm{e}} = \frac{\ln a}{1} = \ln a.$$

$$\lim_{x\to 2} \frac{x-2}{x^2-3x+2} = 1$$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x-1)} = \lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1} = 1.$$

$$\lim_{x\to\infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}}\right)$$

Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x} = \ln a$ $\qquad a>0,\ a\neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x\to 0 \\ z+1 = a^x & z\to 0 \\ x\ln a = \ln a^x = \ln(z+1) & \end{array}\right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z\ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to \mathrm{e} \\ \text{pre } z\to 0 \end{array}\right] = \frac{\ln a}{\ln \mathrm{e}} = \frac{\ln a}{1} = \ln a.$$

$\lim\limits_{x\to 2} \frac{x-2}{x^2-3x+2} = 1$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x-1)} = \lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1} = 1.$$

$\lim\limits_{x\to\infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}}\right)$

$$= \lim_{x\to\infty} \frac{5}{1-\frac{1}{x^2}} + \lim_{x\to\infty} 2^{\frac{1}{x}}$$

Riešené limity – 01, 02, 03

$$\lim_{x\to 0} \frac{a^x-1}{x} = \ln a \qquad a>0,\ a\neq 1$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x\ln a = \ln a^x = \ln(z+1) & \end{array}\right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z\ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to e \\ \text{pre } z \to 0 \end{array}\right] = \frac{\ln a}{\ln e} = \frac{\ln a}{1} = \ln a.$$

$$\lim_{x\to 2} \frac{x-2}{x^2-3x+2} = 1$$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x-1)} = \lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1} = 1.$$

$$\lim_{x\to\infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}}\right)$$

$$= \lim_{x\to\infty} \frac{5}{1-\frac{1}{x^2}} + \lim_{x\to\infty} 2^{\frac{1}{x}} = \frac{5}{1-0} + 2^0$$

Riešené limity – 01, 02, 03

$\lim\limits_{x\to 0} \frac{a^x-1}{x} = \ln a$ $\quad a>0,\ a\neq 1$

$$= \left[\begin{array}{r|l} \text{Subst. } z = a^x - 1 & x \to 0 \\ z+1 = a^x & z \to 0 \\ x\ln a = \ln a^x = \ln(z+1) & \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln a}} = \lim_{z\to 0} \frac{z\ln a}{\ln(z+1)}$$

$$= \lim_{z\to 0} \frac{\ln a}{\frac{1}{z}\ln(z+1)} = \lim_{z\to 0} \frac{\ln a}{\ln(z+1)^{\frac{1}{z}}} = \left[\begin{array}{c} (1+z)^{\frac{1}{z}} \to e \\ \text{pre } z \to 0 \end{array} \right] = \frac{\ln a}{\ln e} = \frac{\ln a}{1} = \ln a.$$

$\lim\limits_{x\to 2} \frac{x-2}{x^2-3x+2} = 1$

$$= \lim_{x\to 2} \frac{x-2}{(x-2)(x-1)} = \lim_{x\to 2} \frac{1}{x-1} = \frac{1}{2-1} = 1.$$

$\lim\limits_{x\to\infty} \left(\frac{5x^2}{x^2-1} + 2^{\frac{1}{x}}\right) = 6$

$$= \lim_{x\to\infty} \frac{5}{1-\frac{1}{x^2}} + \lim_{x\to\infty} 2^{\frac{1}{x}} = \frac{5}{1-0} + 2^0 = 5+1 = 6.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0} \frac{1-3^x}{x} = -\lim_{x\to 0} \frac{3^x-1}{x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \\ & \end{array} \right]$$

Riešené limity – 04, 05, 06, 07

$\lim\limits_{x\to 0} \frac{1-3^x}{x} = -\lim\limits_{x\to 0} \frac{3^x-1}{x}$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z + 1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x \ln 3 & \end{array} \right] = -\lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x\ln 3 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z + 1 = 3^x & z \to 0 \\ \multicolumn{2}{c}{\ln(z+1) = \ln 3^x = x\ln 3} \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0} \frac{1-3^x}{x} = -\lim_{x\to 0} \frac{3^x-1}{x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \\ \multicolumn{2}{c}{\ln(z+1) = \ln 3^x = x \ln 3} \end{array} \right] = -\lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0} \frac{z \ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0} \frac{\ln 3}{\frac{1}{z} \ln(z+1)} = -\lim_{z\to 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x}=-\lim_{x\to 0}\frac{3^x-1}{x}$$

$$=\left[\begin{array}{r|l}\text{Subst. } z=3^x-1 & x\to 0\\ z+1=3^x & z\to 0\\ \multicolumn{2}{c}{\ln(z+1)=\ln 3^x=x\ln 3}\end{array}\right]=-\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}}=-\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$=-\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)}=-\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}}=-\frac{\ln 3}{\ln e}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0} \frac{1-3^x}{x} = -\lim_{x\to 0} \frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z + 1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x \ln 3 & \end{array} \right] = -\lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0} \frac{z \ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0} \frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x\ln 3 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

$$\lim_{x\to -\infty}\frac{1-3^x}{x}$$

$$\lim_{x\to 1}\frac{1-3^x}{x}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x\ln 3 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

$$\lim_{x\to -\infty}\frac{1-3^x}{x}$$

$$= \frac{1-3^{-\infty}}{-\infty}$$

$$\lim_{x\to 1}\frac{1-3^x}{x}$$

$$= \frac{1-3^1}{1}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0} \frac{1-3^x}{x} = -\lim_{x\to 0} \frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z + 1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x \ln 3 & \end{array} \right] = -\lim_{z\to 0} \frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0} \frac{z \ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0} \frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0} \frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

$$\lim_{x\to -\infty} \frac{1-3^x}{x}$$

$$= \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty}$$

$$\lim_{x\to 1} \frac{1-3^x}{x}$$

$$= \frac{1-3^1}{1} = \frac{1-3}{1}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z + 1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x \ln 3 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

$$\lim_{x\to -\infty}\frac{1-3^x}{x} = 0$$

$$= \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x\to 1}\frac{1-3^x}{x} = -2$$

$$= \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x\ln 3 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

$$\lim_{x\to -\infty}\frac{1-3^x}{x} = 0$$

$$= \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^\infty}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x\to 1}\frac{1-3^x}{x} = -2$$

$$= \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

$$\lim_{x\to \infty}\frac{1-3^x}{x}$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x \to 0 \\ z+1 = 3^x & z \to 0 \\ \ln(z+1) = \ln 3^x = x\ln 3 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

$$\lim_{x\to -\infty}\frac{1-3^x}{x} = 0$$

$$= \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^\infty}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x\to 1}\frac{1-3^x}{x} = -2$$

$$= \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

$$\lim_{x\to \infty}\frac{1-3^x}{x}$$

$$= \lim_{x\to\infty}\left(\frac{1}{x} - \frac{3^x}{x}\right)$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x}=-\lim_{x\to 0}\frac{3^x-1}{x}=-\ln 3$$

$$=\left[\begin{array}{r|l}\text{Subst. } z=3^x-1 & x\to 0\\ z+1=3^x & z\to 0\\ \multicolumn{2}{c}{\ln(z+1)=\ln 3^x=x\ln 3}\end{array}\right]=-\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}}=-\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$=-\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)}=-\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}}=-\frac{\ln 3}{\ln e}=-\frac{\ln 3}{1}=-\ln 3.$$

$$\lim_{x\to -\infty}\frac{1-3^x}{x}=0$$

$$=\frac{1-3^{-\infty}}{-\infty}=\frac{1-\frac{1}{3^{\infty}}}{-\infty}=\frac{1-0}{-\infty}=0.$$

$$\lim_{x\to 1}\frac{1-3^x}{x}=-2$$

$$=\frac{1-3^1}{1}=\frac{1-3}{1}=-\frac{2}{1}=-2.$$

$$\lim_{x\to\infty}\frac{1-3^x}{x}$$

$$=\lim_{x\to\infty}\left(\frac{1}{x}-\frac{3^x}{x}\right)=\frac{1}{\infty}-\infty$$

Riešené limity – 04, 05, 06, 07

$$\lim_{x\to 0}\frac{1-3^x}{x} = -\lim_{x\to 0}\frac{3^x-1}{x} = -\ln 3$$

$$= \left[\begin{array}{r|l} \text{Subst. } z = 3^x - 1 & x\to 0 \\ z+1 = 3^x & z\to 0 \\ \ln(z+1) = \ln 3^x = x\ln 3 & \end{array}\right] = -\lim_{z\to 0}\frac{z}{\frac{\ln(z+1)}{\ln 3}} = -\lim_{z\to 0}\frac{z\ln 3}{\ln(z+1)}$$

$$= -\lim_{z\to 0}\frac{\ln 3}{\frac{1}{z}\ln(z+1)} = -\lim_{z\to 0}\frac{\ln 3}{\ln(z+1)^{\frac{1}{z}}} = -\frac{\ln 3}{\ln e} = -\frac{\ln 3}{1} = -\ln 3.$$

$$\lim_{x\to -\infty}\frac{1-3^x}{x} = 0$$

$$= \frac{1-3^{-\infty}}{-\infty} = \frac{1-\frac{1}{3^{\infty}}}{-\infty} = \frac{1-0}{-\infty} = 0.$$

$$\lim_{x\to 1}\frac{1-3^x}{x} = -2$$

$$= \frac{1-3^1}{1} = \frac{1-3}{1} = -\frac{2}{1} = -2.$$

$$\lim_{x\to\infty}\frac{1-3^x}{x} = -\infty$$

$$= \lim_{x\to\infty}\left(\frac{1}{x} - \frac{3^x}{x}\right) = \frac{1}{\infty} - \infty = 0 - \infty = -\infty.$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$$

Riešené limity – 08, 09, 10

$\lim\limits_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right]$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x} \right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to \infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right)$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right)$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right]$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right)$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+ \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+ \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+\lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+\lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

$$= \lim_{x\to 1} \frac{1+x+x^2-3}{1-x^3}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+\lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

$$= \lim_{x\to 1} \frac{1+x+x^2-3}{1-x^3} = \lim_{x\to 1} \frac{x^2+x-2}{1-x^3}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+ \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

$$= \lim_{x\to 1} \frac{1+x+x^2-3}{1-x^3} = \lim_{x\to 1} \frac{x^2+x-2}{1-x^3} = \lim_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+\lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

$$= \lim_{x\to 1} \frac{1+x+x^2-3}{1-x^3} = \lim_{x\to 1} \frac{x^2+x-2}{1-x^3} = \lim_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$$

$$= -\lim_{x\to 1} \frac{x+2}{1+x+x^2}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+ \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right)$$

$$= \lim_{x\to 1} \frac{1+x+x^2-3}{1-x^3} = \lim_{x\to 1} \frac{x^2+x-2}{1-x^3} = \lim_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$$

$$= -\lim_{x\to 1} \frac{x+2}{1+x+x^2} = -\frac{1+2}{1+1+1}$$

Riešené limity – 08, 09, 10

$$\lim_{x\to 0} \frac{3x+\frac{2}{x}}{x+\frac{4}{x}} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{3x+\frac{2}{x}}{x+\frac{4}{x}} \cdot \frac{x}{x}\right] = \lim_{x\to 0} \frac{3x^2+2}{x^2+4} = \frac{0+2}{0+4} = \frac{1}{2}.$$

$$\lim_{x\to\infty} \left(\frac{1-x}{x^2} + \frac{1-2x}{2-3x}\right) = \frac{2}{3}$$

$$= \lim_{x\to\infty} \frac{1}{x^2} - \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \left[\frac{1-2x}{2-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = 0-0+ \lim_{x\to\infty} \frac{\frac{1}{x}-2}{\frac{2}{x}-3} = \frac{0-2}{0-3} = \frac{2}{3}.$$

$$\lim_{x\to 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right) = -1$$

$$= \lim_{x\to 1} \frac{1+x+x^2-3}{1-x^3} = \lim_{x\to 1} \frac{x^2+x-2}{1-x^3} = \lim_{x\to 1} \frac{(x+2)(x-1)}{(1-x)(1+x+x^2)}$$

$$= -\lim_{x\to 1} \frac{x+2}{1+x+x^2} = -\frac{1+2}{1+1+1} = -\frac{3}{3} = -1.$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

Riešené limity – 11

$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$

$$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}}$$

$$= \left[\begin{array}{r|c|c} \text{Subst. } z = 3x+1 & & \\ z \to \infty & & \end{array}\right]$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

$$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}}$$

$$= \left[\begin{array}{r|c|c} \text{Subst. } z = 3x+1 & 3x = z-1 & x = \frac{z-1}{3} \\ z\to\infty & 3x-2 = z-3 & x\to\infty \end{array}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}}$$

Riešené limity – 11

$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$

$$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{c|c} \text{Subst.} & z = 3x+1 \\ x\to\infty & z\to\infty \end{array}\right]$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}}$$

$$= \left[\begin{array}{r|c|c} \text{Subst. } z = 3x+1 & 3x = z-1 & x = \frac{z-1}{3} \\ z\to\infty & 3x-2 = z-3 & x\to\infty \end{array}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}}$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}}$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

$$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l|l} \text{Subst.} & z=3x+1 \\ x\to\infty & z\to\infty \end{array}\right]$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}}$$

$$= \left[\begin{array}{r|r|r} \text{Subst. } z=3x+1 & 3x=z-1 & x=\frac{z-1}{3} \\ z\to\infty & 3x-2=z-3 & x\to\infty \end{array}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}}$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right]$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

$$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l|l} \text{Subst.} & z = 3x+1 \\ x\to\infty & z\to\infty \end{array}\right]$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}} = \left[\lim_{z\to\infty}\frac{z-1}{3z} = \frac{1}{3}\right]$$

$$= \left(e^{-3}\right)^{\frac{1}{3}}$$

$$= \left[\begin{array}{r|c|c} \text{Subst. } z = 3x+1 & 3x = z-1 & x = \frac{z-1}{3} \\ z\to\infty & 3x-2 = z-3 & x\to\infty \end{array}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}}$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right]$$

$$= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x}$$

$$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l|l}\text{Subst.} & z=3x+1\\ x\to\infty & z\to\infty\end{array}\right]$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}} = \left[\lim_{z\to\infty}\frac{z-1}{3z}=\frac{1}{3}\right]$$

$$= \left(e^{-3}\right)^{\frac{1}{3}}$$

$$= \left[\begin{array}{r|c|c}\text{Subst. } z=3x+1 & 3x=z-1 & x=\frac{z-1}{3}\\ z\to\infty & 3x-2=z-3 & x\to\infty\end{array}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}}$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right]$$

$$= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}} = \left(e^{-3}\right)^{\frac{1}{3}}\cdot(1+0)^{-\frac{1}{3}}$$

Riešené limity – 11

$$\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{x} = e^{-1} = \frac{1}{e}$$

$$= \lim_{x\to\infty}\left(\frac{3x+1-3}{3x+1}\right)^{\frac{3x}{3}} = \lim_{x\to\infty}\left(1+\frac{-3}{3x+1}\right)^{\frac{3x+1-1}{3}} = \left[\begin{array}{l|l} \text{Subst.} & z = 3x+1 \\ x\to\infty & z\to\infty \end{array}\right]$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z-1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{z-1}{3\cdot z}} = \left[\lim_{z\to\infty}\frac{z-1}{3z} = \frac{1}{3}\right]$$

$$= \left(e^{-3}\right)^{\frac{1}{3}} = e^{-1} = \frac{1}{e}.$$

$$= \left[\begin{array}{r|r|r} \text{Subst. } z = 3x+1 & 3x = z-1 & x = \frac{z-1}{3} \\ z\to\infty & 3x-2 = z-3 & x\to\infty \end{array}\right] = \lim_{z\to\infty}\left(\frac{z-3}{z}\right)^{\frac{z-1}{3}}$$

$$= \lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{\frac{z}{3}-\frac{1}{3}} = \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{\frac{z}{3}}\cdot\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}}\right]$$

$$= \lim_{z\to\infty}\left[\left(1+\frac{-3}{z}\right)^{z}\right]^{\frac{1}{3}}\cdot\lim_{z\to\infty}\left(1+\frac{-3}{z}\right)^{-\frac{1}{3}} = \left(e^{-3}\right)^{\frac{1}{3}}\cdot(1+0)^{-\frac{1}{3}}$$

$$= e^{-1} = \frac{1}{e}.$$

Riešené limity – 12, 13, 14

$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^x$$

$$= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x$$

Riešené limity – 12, 13, 14

$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$

$$= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\begin{array}{l|l} \text{Subst.} & x=-z \\ x\to-\infty & z\to\infty \end{array}\right] = \lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$$

$$= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\begin{array}{l|l} \text{Subst.} & x=-z \\ x\to-\infty & z\to\infty \end{array}\right] = \lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$= \lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}$$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}$$

$$= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\begin{array}{l|l} \text{Subst.} & x=-z \\ x\to-\infty & z\to\infty \end{array}\right] = \lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$= \lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z} = \lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}$$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}=\mathrm{e}$$

$$=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}=\left[\begin{array}{l|l}\text{Subst.} & x=-z\\ x\to-\infty & z\to\infty\end{array}\right]=\lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$=\lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}=\lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}=\left(\mathrm{e}^{-1}\right)^{-1}=\mathrm{e}.$$

Riešené limity – 12, 13, 14

$\lim\limits_{x\to-\infty} \left(\frac{x+1}{x}\right)^x = e$

$$= \lim_{x\to-\infty} \left(1+\frac{1}{x}\right)^x = \left[\begin{array}{l|l} \text{Subst.} & x=-z \\ x\to-\infty & z\to\infty \end{array}\right] = \lim_{z\to\infty} \left(1+\frac{1}{-z}\right)^{-z}$$

$$= \lim_{z\to\infty} \left(1+\frac{-1}{z}\right)^{-z} = \lim_{z\to\infty} \left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1} = \left(e^{-1}\right)^{-1} = e.$$

$\lim\limits_{x\to 0} (1+3\operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}=e$$

$$=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}=\left[\begin{array}{l|l}\text{Subst.} & x=-z\\ x\to-\infty & z\to\infty\end{array}\right]=\lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$=\lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}=\lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}=\left(e^{-1}\right)^{-1}=e.$$

$$\lim_{x\to0}(1+3\operatorname{tg}^2 x)^{\operatorname{cotg}^2 x}$$

$$=\lim_{x\to0}(1+3\operatorname{tg}^2 x)^{\frac{1}{\operatorname{tg}^2 x}}$$

Riešené limity – 12, 13, 14

$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}=\mathrm{e}$

$$=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}=\left[\begin{array}{l|l}\text{Subst.} & x=-z\\ x\to-\infty & z\to\infty\end{array}\right]=\lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$=\lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}=\lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}=\left(\mathrm{e}^{-1}\right)^{-1}=\mathrm{e}.$$

$\lim_{x\to 0}(1+3\,\mathrm{tg}^2 x)^{\mathrm{cotg}^2 x}$

$$=\lim_{x\to 0}(1+3\,\mathrm{tg}^2 x)^{\frac{1}{\mathrm{tg}^2 x}}=\left[\begin{array}{l|l}\text{Subst.} & z=\mathrm{tg}^2 x\\ x\to 0 & z\to 0\end{array}\right]=\lim_{z\to 0}(1+3z)^{\frac{1}{z}}$$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}=\mathrm{e}$$

$$=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}=\left[\begin{array}{l|l}\text{Subst.} & x=-z\\ x\to-\infty & z\to\infty\end{array}\right]=\lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$=\lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}=\lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}=\left(\mathrm{e}^{-1}\right)^{-1}=\mathrm{e}.$$

$$\lim_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\mathrm{cotg}^2 x}=\mathrm{e}^3$$

$$=\lim_{x\to0}\left(1+3\,\mathrm{tg}^2 x\right)^{\frac{1}{\mathrm{tg}^2 x}}=\left[\begin{array}{l|l}\text{Subst.} & z=\mathrm{tg}^2 x\\ x\to0 & z\to0\end{array}\right]=\lim_{z\to0}\left(1+3z\right)^{\frac{1}{z}}=\mathrm{e}^3.$$

Riešené limity – 12, 13, 14

$\lim\limits_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x} = e$

$$= \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x} = \left[\begin{array}{l|l}\text{Subst.} & x=-z \\ x\to-\infty & z\to\infty\end{array}\right] = \lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$= \lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z} = \lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1} = \left(e^{-1}\right)^{-1} = e.$$

$\lim\limits_{x\to0}(1+3\,\mathrm{tg}^2 x)^{\mathrm{cotg}^2 x} = e^3$

$$= \lim_{x\to0}(1+3\,\mathrm{tg}^2 x)^{\frac{1}{\mathrm{tg}^2 x}} = \left[\begin{array}{l|l}\text{Subst.} & z=\mathrm{tg}^2 x \\ x\to0 & z\to0\end{array}\right] = \lim_{z\to0}(1+3z)^{\frac{1}{z}} = e^3.$$

$\lim\limits_{x\to0}(1+3\,\mathrm{tg}^2 x)^{\mathrm{tg}^2 x}$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}=\mathrm{e}$$

$$=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}=\left[\begin{array}{l|l}\text{Subst.} & x=-z\\ x\to-\infty & z\to\infty\end{array}\right]=\lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$=\lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}=\lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}=\left(\mathrm{e}^{-1}\right)^{-1}=\mathrm{e}.$$

$$\lim_{x\to0}(1+3\operatorname{tg}^2x)^{\operatorname{cotg}^2x}=\mathrm{e}^3$$

$$=\lim_{x\to0}(1+3\operatorname{tg}^2x)^{\frac{1}{\operatorname{tg}^2x}}=\left[\begin{array}{l|l}\text{Subst.} & z=\operatorname{tg}^2x\\ x\to0 & z\to0\end{array}\right]=\lim_{z\to0}(1+3z)^{\frac{1}{z}}=\mathrm{e}^3.$$

$$\lim_{x\to0}(1+3\operatorname{tg}^2x)^{\operatorname{tg}^2x}$$

$$=(1+3\cdot0)^0$$

Riešené limity – 12, 13, 14

$$\lim_{x\to-\infty}\left(\frac{x+1}{x}\right)^{x}=\mathrm{e}$$

$$=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{x}=\left[\begin{array}{l|l}\text{Subst.} & x=-z\\ x\to-\infty & z\to\infty\end{array}\right]=\lim_{z\to\infty}\left(1+\frac{1}{-z}\right)^{-z}$$

$$=\lim_{z\to\infty}\left(1+\frac{-1}{z}\right)^{-z}=\lim_{z\to\infty}\left[\left(1+\frac{-1}{z}\right)^{z}\right]^{-1}=\left(\mathrm{e}^{-1}\right)^{-1}=\mathrm{e}.$$

$$\lim_{x\to0}(1+3\operatorname{tg}^2x)^{\operatorname{cotg}^2x}=\mathrm{e}^3$$

$$=\lim_{x\to0}(1+3\operatorname{tg}^2x)^{\frac{1}{\operatorname{tg}^2x}}=\left[\begin{array}{l|l}\text{Subst.} & z=\operatorname{tg}^2x\\ x\to0 & z\to0\end{array}\right]=\lim_{z\to0}(1+3z)^{\frac{1}{z}}=\mathrm{e}^3.$$

$$\lim_{x\to0}(1+3\operatorname{tg}^2x)^{\operatorname{tg}^2x}=1$$

$$=(1+3\cdot0)^0=1^0=1.$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1 - \frac{1}{x})^{x^2}$

Riešené limity – 15, 16, 17

$\lim_{x\to\infty} (1 - \frac{1}{x})^{x^2}$

$$= \lim_{x\to\infty} (1-\frac{1}{x})^{x\cdot x}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty}(1-\frac{1}{x})^{x^2}$

$$= \lim_{x\to\infty}(1-\frac{1}{x})^{x\cdot x} = \lim_{x\to\infty}\left[(1+\frac{-1}{x})^{x}\right]^{x}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1 - \frac{1}{x})^{x^2}$

$$= \lim_{x\to\infty} (1-\tfrac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\tfrac{-1}{x})^{x}\right]^{x} = \left[\, \mathrm{e}^{-1} \,\right]^{\infty}$$

Riešené limity – 15, 16, 17

$\lim_{x\to\infty} (1 - \frac{1}{x})^{x^2} = 0$

$$= \lim_{x\to\infty} (1-\frac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\frac{-1}{x})^{x}\right]^{x} = \left[\,e^{-1}\,\right]^{\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty} \left(1-\tfrac{1}{x}\right)^{x^2} = 0$$

$$= \lim_{x\to\infty} \left(1-\tfrac{1}{x}\right)^{x\cdot x} = \lim_{x\to\infty} \left[\left(1+\tfrac{-1}{x}\right)^{x}\right]^{x} = \left[\,e^{-1}\,\right]^{\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$$

$$\lim_{x\to\infty} x\left[\,\ln(x+2) - \ln x\right]$$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty} \left(1-\frac{1}{x}\right)^{x^2} = 0$$

$$= \lim_{x\to\infty} \left(1-\frac{1}{x}\right)^{x\cdot x} = \lim_{x\to\infty} \left[\left(1+\frac{-1}{x}\right)^{x}\right]^{x} = \left[\,e^{-1}\,\right]^{\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$$

$$\lim_{x\to\infty} x\left[\ln(x+2) - \ln x\right]$$

$$= \lim_{x\to\infty} x \ln\frac{x+2}{x}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1 - \frac{1}{x})^{x^2} = 0$

$$= \lim_{x\to\infty} (1-\tfrac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\tfrac{-1}{x})^x\right]^x = \left[\mathrm{e}^{-1}\right]^\infty = \frac{1}{\mathrm{e}^\infty} = \frac{1}{\infty} = 0.$$

$\lim\limits_{x\to\infty} x\left[\ln(x+2) - \ln x\right]$

$$= \lim_{x\to\infty} x \ln \frac{x+2}{x} = \lim_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^x$$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^{x^2} = 0$$

$$= \lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^{x\cdot x} = \lim_{x\to\infty} \left[\left(1 + \frac{-1}{x}\right)^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}} = \frac{1}{\infty} = 0.$$

$$\lim_{x\to\infty} x\left[\ln(x+2) - \ln x\right]$$

$$= \lim_{x\to\infty} x \ln \frac{x+2}{x} = \lim_{x\to\infty} \ln \left(\frac{x+2}{x}\right)^{x} = \lim_{x\to\infty} \ln \left(1 + \frac{2}{x}\right)^{x}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1-\frac{1}{x})^{x^2} = 0$

$$= \lim_{x\to\infty} (1-\tfrac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\tfrac{-1}{x})^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}} = \frac{1}{\infty} = 0.$$

$\lim\limits_{x\to\infty} x\left[\ln(x+2)-\ln x\right] = 2$

$$= \lim_{x\to\infty} x\ln\frac{x+2}{x} = \lim_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^{x} = \lim_{x\to\infty} \ln\left(1+\frac{2}{x}\right)^{x} = \ln \mathrm{e}^{2} = 2.$$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x^2}=0$$

$$=\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x\cdot x}=\lim_{x\to\infty}\left[\left(1+\frac{-1}{x}\right)^{x}\right]^{x}=\left[\mathrm{e}^{-1}\right]^{\infty}=\frac{1}{\mathrm{e}^{\infty}}=\frac{1}{\infty}=0.$$

$$\lim_{x\to\infty}x\left[\ln(x+2)-\ln x\right]=2$$

$$=\lim_{x\to\infty}x\ln\frac{x+2}{x}=\lim_{x\to\infty}\ln\left(\frac{x+2}{x}\right)^{x}=\lim_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^{x}=\ln\mathrm{e}^2=2.$$

$$\lim_{x\to 0}\frac{x}{\ln(1+tx)}\qquad t\in R-\{0\}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1-\frac{1}{x})^{x^2} = 0$

$$= \lim_{x\to\infty} (1-\tfrac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\tfrac{-1}{x})^x\right]^x = \left[e^{-1}\right]^\infty = \frac{1}{e^\infty} = \frac{1}{\infty} = 0.$$

$\lim\limits_{x\to\infty} x\left[\ln(x+2) - \ln x\right] = 2$

$$= \lim_{x\to\infty} x \ln \frac{x+2}{x} = \lim_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^x = \lim_{x\to\infty} \ln\left(1+\frac{2}{x}\right)^x = \ln e^2 = 2.$$

$\lim\limits_{x\to 0} \frac{x}{\ln(1+tx)}$ $\quad t \in R - \{0\}$

$$= \lim_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty}\left(1-\frac{1}{x}\right)^{x^2}=0$

$$=\lim_{x\to\infty}\left(1-\tfrac{1}{x}\right)^{x\cdot x}=\lim_{x\to\infty}\left[\left(1+\tfrac{-1}{x}\right)^{x}\right]^{x}=\left[e^{-1}\right]^{\infty}=\frac{1}{e^{\infty}}=\frac{1}{\infty}=0.$$

$\lim\limits_{x\to\infty}x\left[\ln(x+2)-\ln x\right]=2$

$$=\lim_{x\to\infty}x\ln\frac{x+2}{x}=\lim_{x\to\infty}\ln\left(\frac{x+2}{x}\right)^{x}=\lim_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^{x}=\ln e^{2}=2.$$

$\lim\limits_{x\to 0}\frac{x}{\ln(1+tx)}$ $\qquad t\in R-\{0\}$

$$=\lim_{x\to 0}\frac{1}{\frac{1}{x}\ln(1+tx)}=\lim_{x\to 0}\frac{1}{\frac{t}{tx}\ln(1+tx)}$$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty} (1-\tfrac{1}{x})^{x^2} = 0$$

$$= \lim_{x\to\infty} (1-\tfrac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\tfrac{-1}{x})^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}} = \frac{1}{\infty} = 0.$$

$$\lim_{x\to\infty} x\left[\ln(x+2) - \ln x\right] = 2$$

$$= \lim_{x\to\infty} x\ln\frac{x+2}{x} = \lim_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^{x} = \lim_{x\to\infty} \ln\left(1+\frac{2}{x}\right)^{x} = \ln \mathrm{e}^2 = 2.$$

$$\lim_{x\to 0} \frac{x}{\ln(1+tx)} \qquad t \in R-\{0\}$$

$$= \lim_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty} \left(1-\tfrac{1}{x}\right)^{x^2} = 0$$

$$= \lim_{x\to\infty} \left(1-\tfrac{1}{x}\right)^{x\cdot x} = \lim_{x\to\infty} \left[\left(1+\tfrac{-1}{x}\right)^{x}\right]^{x} = \left[\,e^{-1}\,\right]^{\infty} = \tfrac{1}{e^{\infty}} = \tfrac{1}{\infty} = 0.$$

$$\lim_{x\to\infty} x\left[\ln(x+2) - \ln x\right] = 2$$

$$= \lim_{x\to\infty} x \ln \tfrac{x+2}{x} = \lim_{x\to\infty} \ln\left(\tfrac{x+2}{x}\right)^{x} = \lim_{x\to\infty} \ln\left(1+\tfrac{2}{x}\right)^{x} = \ln e^2 = 2.$$

$$\lim_{x\to 0} \frac{x}{\ln(1+tx)} \qquad t\in R-\{0\}$$

$$= \lim_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = tx \\ x\to 0 & z\to 0 \end{array}\right] = \lim_{z\to 0} \frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}$$

Riešené limity – 15, 16, 17

$\lim\limits_{x\to\infty} (1-\frac{1}{x})^{x^2} = 0$

$$= \lim_{x\to\infty} (1-\frac{1}{x})^{x\cdot x} = \lim_{x\to\infty} \left[(1+\frac{-1}{x})^{x}\right]^{x} = \left[\mathrm{e}^{-1}\right]^{\infty} = \frac{1}{\mathrm{e}^{\infty}} = \frac{1}{\infty} = 0.$$

$\lim\limits_{x\to\infty} x\left[\ln(x+2) - \ln x\right] = 2$

$$= \lim_{x\to\infty} x \ln\frac{x+2}{x} = \lim_{x\to\infty} \ln\left(\frac{x+2}{x}\right)^{x} = \lim_{x\to\infty} \ln\left(1+\frac{2}{x}\right)^{x} = \ln \mathrm{e}^2 = 2.$$

$\lim\limits_{x\to 0} \frac{x}{\ln(1+tx)}$ $\qquad t \in R - \{0\}$

$$= \lim_{x\to 0} \frac{1}{\frac{1}{x}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{\frac{t}{tx}\ln(1+tx)} = \lim_{x\to 0} \frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$$

$$= \left[\begin{array}{c|c} \text{Subst.} & z = tx \\ x\to 0 & z\to 0 \end{array}\right] = \lim_{z\to 0} \frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}} = \frac{1}{t\cdot\ln \mathrm{e}}$$

Riešené limity – 15, 16, 17

$$\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x^2}=0$$

$$=\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{x\cdot x}=\lim_{x\to\infty}\left[\left(1+\frac{-1}{x}\right)^{x}\right]^{x}=\left[\mathrm{e}^{-1}\right]^{\infty}=\frac{1}{\mathrm{e}^{\infty}}=\frac{1}{\infty}=0.$$

$$\lim_{x\to\infty}x\left[\ln(x+2)-\ln x\right]=2$$

$$=\lim_{x\to\infty}x\ln\frac{x+2}{x}=\lim_{x\to\infty}\ln\left(\frac{x+2}{x}\right)^{x}=\lim_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^{x}=\ln\mathrm{e}^2=2.$$

$$\lim_{x\to 0}\frac{x}{\ln(1+tx)}=\frac{1}{t}\qquad t\in R-\{0\}$$

$$=\lim_{x\to 0}\frac{1}{\frac{1}{x}\ln(1+tx)}=\lim_{x\to 0}\frac{1}{\frac{t}{tx}\ln(1+tx)}=\lim_{x\to 0}\frac{1}{t\cdot\ln(1+tx)^{\frac{1}{tx}}}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & z=tx\\ x\to 0 & z\to 0\end{array}\right]=\lim_{z\to 0}\frac{1}{t\cdot\ln(1+z)^{\frac{1}{z}}}=\frac{1}{t\cdot\ln\mathrm{e}}=\frac{1}{t\cdot 1}=\frac{1}{t}.$$

Riešené limity – 18, 19

$$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x}$$

Riešené limity – 18, 19

$$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x}$$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right]$$

Riešené limity – 18, 19

$$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x}$$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})}$$

Riešené limity – 18, 19

$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x}$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

Riešené limity – 18, 19

$\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x}$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{x}{x(1+\sqrt{1-x})}$$

Riešené limity – 18, 19

$\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x}$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{x}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1}{1+\sqrt{1-x}}$$

Riešené limity – 18, 19

$\lim\limits_{x\to 0} \frac{1-\sqrt{1-x}}{x} = \frac{1}{2}$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{x}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-0}} = \frac{1}{2}.$$

Riešené limity – 18, 19

$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x} = \frac{1}{2}$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{x}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-0}} = \frac{1}{2}.$$

$\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$

Riešené limity – 18, 19

$$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{x}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-0}} = \frac{1}{2}.$$

$$\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

$$= \lim_{x\to 0} \left[\frac{x}{\sqrt{1+x}-\sqrt{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$$

Riešené limity – 18, 19

$$\lim_{x\to 0}\frac{1-\sqrt{1-x}}{x}=\frac{1}{2}$$

$$=\lim_{x\to 0}\left[\frac{1-\sqrt{1-x}}{x}\cdot\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$=\lim_{x\to 0}\frac{x}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1}{1+\sqrt{1-x}}=\frac{1}{1+\sqrt{1-0}}=\frac{1}{2}.$$

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

$$=\lim_{x\to 0}\left[\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}$$

Riešené limity – 18, 19

$$\lim_{x\to 0}\frac{1-\sqrt{1-x}}{x}=\frac{1}{2}$$

$$=\lim_{x\to 0}\left[\frac{1-\sqrt{1-x}}{x}\cdot\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$=\lim_{x\to 0}\frac{x}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1}{1+\sqrt{1-x}}=\frac{1}{1+\sqrt{1-0}}=\frac{1}{2}.$$

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

$$=\lim_{x\to 0}\left[\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}$$

$$=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}$$

Riešené limity – 18, 19

$$\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x} = \frac{1}{2}$$

$$= \lim_{x\to 0} \left[\frac{1-\sqrt{1-x}}{x} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1-x}} \right] = \lim_{x\to 0} \frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{x}{x(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-0}} = \frac{1}{2}.$$

$$\lim_{x\to 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

$$= \lim_{x\to 0} \left[\frac{x}{\sqrt{1+x}-\sqrt{1-x}} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \right] = \lim_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}$$

$$= \lim_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)} = \lim_{x\to 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}$$

Riešené limity – 18, 19

$$\lim_{x\to 0}\frac{1-\sqrt{1-x}}{x}=\frac{1}{2}$$

$$=\lim_{x\to 0}\left[\frac{1-\sqrt{1-x}}{x}\cdot\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$=\lim_{x\to 0}\frac{x}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1}{1+\sqrt{1-x}}=\frac{1}{1+\sqrt{1-0}}=\frac{1}{2}.$$

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}$$

$$=\lim_{x\to 0}\left[\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}$$

$$=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}$$

$$=\lim_{x\to 0}\frac{\sqrt{1+x}+\sqrt{1-x}}{2}$$

Riešené limity – 18, 19

$$\lim_{x\to 0}\frac{1-\sqrt{1-x}}{x}=\frac{1}{2}$$

$$=\lim_{x\to 0}\left[\frac{1-\sqrt{1-x}}{x}\cdot\frac{1+\sqrt{1-x}}{1+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{1-(\sqrt{1-x})^2}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1-(1-x)}{x(1+\sqrt{1-x})}$$

$$=\lim_{x\to 0}\frac{x}{x(1+\sqrt{1-x})}=\lim_{x\to 0}\frac{1}{1+\sqrt{1-x}}=\frac{1}{1+\sqrt{1-0}}=\frac{1}{2}.$$

$$\lim_{x\to 0}\frac{x}{\sqrt{1+x}-\sqrt{1-x}}=1$$

$$=\lim_{x\to 0}\left[\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\cdot\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}$$

$$=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{(1+x)-(1-x)}=\lim_{x\to 0}\frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x}$$

$$=\lim_{x\to 0}\frac{\sqrt{1+x}+\sqrt{1-x}}{2}=\frac{\sqrt{1+0}+\sqrt{1-0}}{2}=1.$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

Riešené limity – 20, 21

$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right)$$

Riešené limity – 20, 21

$\lim\limits_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c} x\to\infty,\quad x>0 \\ \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{c} x \to \infty, \quad x > 0 \\ \sqrt{x^2} = |x| = x \end{array} \right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$$

$$= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right)$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{c} x \to \infty,\ \ x > 0 \\ \sqrt{x^2} = |x| = x \end{array} \right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$$

$$= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}} \right)$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2+1}}{x}\right) = \begin{bmatrix} x\to\infty,\ \ x>0 \\ \sqrt{x^2}=|x|=x \end{bmatrix} = \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2-1}{x^2}}+\sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty}\left(\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)$$

$$= \sqrt{1-\frac{1}{\infty}}+\sqrt{1+\frac{1}{\infty}}$$

Riešené limity – 20, 21

$\lim\limits_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x}$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2+1}}{x}\right) = \begin{bmatrix} x\to\infty,\ \ x>0 \\ \sqrt{x^2}=|x|=x \end{bmatrix} = \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$$
$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2-1}{x^2}}+\sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty}\left(\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)$$
$$= \sqrt{1-\frac{1}{\infty}}+\sqrt{1+\frac{1}{\infty}} = \sqrt{1-0}+\sqrt{1+0}$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{matrix} x \to \infty, \quad x > 0 \\ \sqrt{x^2} = |x| = x \end{matrix} \right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$$

$$= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}} \right)$$

$$= \sqrt{1 - \frac{1}{\infty}} + \sqrt{1 + \frac{1}{\infty}} = \sqrt{1-0} + \sqrt{1+0} = 2.$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \begin{bmatrix} x\to\infty, \;\; x>0 \\ \sqrt{x^2} = |x| = x \end{bmatrix} = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$$

$$= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}} \right)$$

$$= \sqrt{1-\frac{1}{\infty}} + \sqrt{1+\frac{1}{\infty}} = \sqrt{1-0} + \sqrt{1+0} = 2.$$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1} \right)$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c} x\to\infty,\quad x>0 \\ \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2-1}{x^2}}+\sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty}\left(\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)$$

$$= \sqrt{1-\frac{1}{\infty}}+\sqrt{1+\frac{1}{\infty}} = \sqrt{1-0}+\sqrt{1+0} = 2.$$

$$\lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right)$$

$$= \lim_{x\to\infty}\left[\left(x-\sqrt{x^2-1}\right)\cdot\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right]$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{array}{c} x\to\infty,\ \ x>0 \\ \sqrt{x^2}=|x|=x \end{array} \right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$$

$$= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}} \right)$$

$$= \sqrt{1-\frac{1}{\infty}} + \sqrt{1+\frac{1}{\infty}} = \sqrt{1-0} + \sqrt{1+0} = 2.$$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1} \right)$$

$$= \lim_{x\to\infty} \left[\left(x - \sqrt{x^2-1} \right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} \right] = \lim_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c} x\to\infty,\quad x>0 \\ \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2-1}{x^2}}+\sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty}\left(\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)$$

$$= \sqrt{1-\frac{1}{\infty}}+\sqrt{1+\frac{1}{\infty}} = \sqrt{1-0}+\sqrt{1+0} = 2.$$

$$\lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right)$$

$$= \lim_{x\to\infty}\left[\left(x-\sqrt{x^2-1}\right)\cdot\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right] = \lim_{x\to\infty}\frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty}\frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}}$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c} x\to\infty,\ \ x>0 \\ \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2-1}{x^2}}+\sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty}\left(\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)$$

$$= \sqrt{1-\frac{1}{\infty}}+\sqrt{1+\frac{1}{\infty}} = \sqrt{1-0}+\sqrt{1+0} = 2.$$

$$\lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right)$$

$$= \lim_{x\to\infty}\left[\left(x-\sqrt{x^2-1}\right)\cdot\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right] = \lim_{x\to\infty}\frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty}\frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim_{x\to\infty}\frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}}$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2+1}}{x}\right) = \left[\begin{array}{c} x\to\infty,\ \ x>0 \\ \sqrt{x^2}=|x|=x \end{array}\right] = \lim_{x\to\infty}\left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}}+\frac{\sqrt{x^2+1}}{\sqrt{x^2}}\right)$$

$$= \lim_{x\to\infty}\left(\sqrt{\frac{x^2-1}{x^2}}+\sqrt{\frac{x^2+1}{x^2}}\right) = \lim_{x\to\infty}\left(\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right)$$

$$= \sqrt{1-\frac{1}{\infty}}+\sqrt{1+\frac{1}{\infty}} = \sqrt{1-0}+\sqrt{1+0} = 2.$$

$$\lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right)$$

$$= \lim_{x\to\infty}\left[\left(x-\sqrt{x^2-1}\right)\cdot\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}\right] = \lim_{x\to\infty}\frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty}\frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim_{x\to\infty}\frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim_{x\to\infty}\frac{1}{x+\sqrt{x^2-1}}$$

Riešené limity – 20, 21

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}+\sqrt{x^2+1}}{x} = 2$$

$$= \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{x} + \frac{\sqrt{x^2+1}}{x} \right) = \left[\begin{matrix} x\to\infty, \quad x>0 \\ \sqrt{x^2}=|x|=x \end{matrix} \right] = \lim_{x\to\infty} \left(\frac{\sqrt{x^2-1}}{\sqrt{x^2}} + \frac{\sqrt{x^2+1}}{\sqrt{x^2}} \right)$$

$$= \lim_{x\to\infty} \left(\sqrt{\frac{x^2-1}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right) = \lim_{x\to\infty} \left(\sqrt{1-\frac{1}{x^2}} + \sqrt{1+\frac{1}{x^2}} \right)$$

$$= \sqrt{1-\frac{1}{\infty}} + \sqrt{1+\frac{1}{\infty}} = \sqrt{1-0} + \sqrt{1+0} = 2.$$

$$\lim_{x\to\infty} \left(x - \sqrt{x^2-1}\right) = 0$$

$$= \lim_{x\to\infty} \left[\left(x-\sqrt{x^2-1}\right) \cdot \frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} \right] = \lim_{x\to\infty} \frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{x+\sqrt{x^2-1}}$$

$$= \lim_{x\to\infty} \frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{x^2-(x^2-1)}{x+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{1}{x+\sqrt{x^2-1}}$$

$$= \frac{1}{\infty+\sqrt{\infty^2-1}} = \frac{1}{\infty} = 0.$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \qquad a>0$$

Riešené limity – 22

$\lim\limits_{x\to a} \frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$ $\quad a>0$

$$= \lim_{x\to a} \left[\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \cdot \frac{\sqrt{ax}+a}{\sqrt{ax}+a} \right]$$

$$= \left[\begin{array}{l|c|c} \text{Subst.} & z=\sqrt{ax} & z^2=ax \\ x\to a & z\to a & x^2=\frac{z^4}{a^2} \end{array} \right] = \lim_{z\to a} \frac{\frac{z^4}{a^2}-az}{z-a}$$

Riešené limity – 22

$$\lim_{x\to a} \frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \qquad a > 0$$

$$= \lim_{x\to a} \left[\frac{x^2 - a\sqrt{ax}}{\sqrt{ax} - a} \cdot \frac{\sqrt{ax} + a}{\sqrt{ax} + a} \right] = \lim_{x\to a} \frac{x^2\sqrt{ax} - a(\sqrt{ax})^2 + ax^2 - aa\sqrt{ax}}{(\sqrt{ax})^2 - a^2}$$

$$= \left[\begin{array}{l|c|l} \text{Subst.} & z = \sqrt{ax} & z^2 = ax \\ x \to a & z \to a & x^2 = \frac{z^4}{a^2} \end{array} \right] = \lim_{z\to a} \frac{\frac{z^4}{a^2} - az}{z - a} = \lim_{z\to a} \frac{z^4 - a^3 z}{a^2(z - a)}$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}$$ $a>0$

$$=\lim_{x\to a}\left[\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right]=\lim_{x\to a}\frac{x^2\sqrt{ax}-a(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x^2-a^2)+ax^2-a^2x}{ax-a^2}$$

$$=\left[\begin{array}{l|c|c}\text{Subst.} & z=\sqrt{ax} & z^2=ax\\ x\to a & z\to a & x^2=\frac{z^4}{a^2}\end{array}\right]=\lim_{z\to a}\frac{\frac{z^4}{a^2}-az}{z-a}=\lim_{z\to a}\frac{z^4-a^3z}{a^2(z-a)}$$

$$=\lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \qquad a>0$$

$$=\lim_{x\to a}\left[\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right]=\lim_{x\to a}\frac{x^2\sqrt{ax}-a(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x^2-a^2)+ax^2-a^2x}{ax-a^2}=\lim_{x\to a}\frac{\sqrt{ax}(x-a)(x+a)+ax(x-a)}{a(x-a)}$$

$$=\left[\begin{array}{c|c|c}\text{Subst.} & z=\sqrt{ax} & z^2=ax\\ x\to a & z\to a & x^2=\frac{z^4}{a^2}\end{array}\right]=\lim_{z\to a}\frac{\frac{z^4}{a^2}-az}{z-a}=\lim_{z\to a}\frac{z^4-a^3z}{a^2(z-a)}$$

$$=\lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)}$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \qquad a>0$$

$$=\lim_{x\to a}\left[\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right]=\lim_{x\to a}\frac{x^2\sqrt{ax}-a(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x^2-a^2)+ax^2-a^2x}{ax-a^2}=\lim_{x\to a}\frac{\sqrt{ax}(x-a)(x+a)+ax(x-a)}{a(x-a)}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x+a)+ax}{a}$$

$$=\left[\begin{array}{l|c|c}\text{Subst.} & z=\sqrt{ax} & z^2=ax\\ x\to a & z\to a & x^2=\frac{z^4}{a^2}\end{array}\right]=\lim_{z\to a}\frac{\frac{z^4}{a^2}-az}{z-a}=\lim_{z\to a}\frac{z^4-a^3z}{a^2(z-a)}$$

$$=\lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z^2+za+a^2)}{a^2}$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a} \qquad a>0$$

$$=\lim_{x\to a}\left[\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right]=\lim_{x\to a}\frac{x^2\sqrt{ax}-a(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x^2-a^2)+ax^2-a^2x}{ax-a^2}=\lim_{x\to a}\frac{\sqrt{ax}(x-a)(x+a)+ax(x-a)}{a(x-a)}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x+a)+ax}{a}=\frac{\sqrt{a\cdot a}(a+a)+a\cdot a}{a}$$

$$=\left[\begin{array}{l|c|c} \text{Subst.} & z=\sqrt{ax} & z^2=ax \\ x\to a & z\to a & x^2=\frac{z^4}{a^2}\end{array}\right]=\lim_{z\to a}\frac{\frac{z^4}{a^2}-az}{z-a}=\lim_{z\to a}\frac{z^4-a^3z}{a^2(z-a)}$$

$$=\lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z^2+za+a^2)}{a^2}$$

$$=\frac{a(a^2+a\cdot a+a^2)}{a^2}$$

Riešené limity – 22

$$\lim_{x\to a}\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}=3a \qquad a>0$$

$$=\lim_{x\to a}\left[\frac{x^2-a\sqrt{ax}}{\sqrt{ax}-a}\cdot\frac{\sqrt{ax}+a}{\sqrt{ax}+a}\right]=\lim_{x\to a}\frac{x^2\sqrt{ax}-a(\sqrt{ax})^2+ax^2-aa\sqrt{ax}}{(\sqrt{ax})^2-a^2}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x^2-a^2)+ax^2-a^2x}{ax-a^2}=\lim_{x\to a}\frac{\sqrt{ax}(x-a)(x+a)+ax(x-a)}{a(x-a)}$$

$$=\lim_{x\to a}\frac{\sqrt{ax}(x+a)+ax}{a}=\frac{\sqrt{a\cdot a}(a+a)+a\cdot a}{a}=\frac{2a^2+a^2}{a}=3a.$$

$$=\left[\begin{array}{l|c|l}\text{Subst.} & z=\sqrt{ax} & z^2=ax\\ x\to a & z\to a & x^2=\frac{z^4}{a^2}\end{array}\right]=\lim_{z\to a}\frac{\frac{z^4}{a^2}-az}{z-a}=\lim_{z\to a}\frac{z^4-a^3z}{a^2(z-a)}$$

$$=\lim_{z\to a}\frac{z(z^3-a^3)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z-a)(z^2+za+a^2)}{a^2(z-a)}=\lim_{z\to a}\frac{z(z^2+za+a^2)}{a^2}$$

$$=\frac{a(a^2+a\cdot a+a^2)}{a^2}=\frac{a\cdot 3a^2}{a^2}=3a.$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ $a \geq 0$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ $a \geq 0$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}}$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}}$ $a \geq 0$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x})$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a}$ $\qquad a \geq 0$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a}\frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \qquad a \geq 0$$

$$= \lim_{x\to a}\frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a}(\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$$\lim_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a}$ $\quad a \geq 0$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a} + \sqrt{x}) = 2\sqrt{a}.$$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \right]$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a}$ $a \geq 0$

$$= \lim\limits_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim\limits_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$

$$= \lim\limits_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = \lim\limits_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \qquad a \geq 0$$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$$\lim_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1}$$

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}}$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \qquad a \geq 0$$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$$\lim_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} = -2$$

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \qquad a \geq 0$$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$$\lim_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} = -2$$

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$$

$$\lim_{x\to 1} \frac{x^m-1}{x^n-1} \qquad m, n \in N$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \qquad a \geq 0$$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$$\lim_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} = -2$$

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$$

$$\lim_{x\to 1} \frac{x^m-1}{x^n-1} \qquad m, n \in N$$

$$= \lim_{x\to 1} \frac{(x-1)(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)}$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a}\frac{a-x}{\sqrt{a}-\sqrt{x}}=2\sqrt{a}$$ $a\geq 0$

$$=\lim_{x\to a}\frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}}=\lim_{x\to a}(\sqrt{a}+\sqrt{x})=2\sqrt{a}.$$

$$\lim_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}=-2$$

$$=\lim_{x\to\infty}\left[\frac{\sqrt{x}-6x}{3x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\right]=\lim_{x\to\infty}\frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}=\frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}}=\frac{0-6}{3+0}=-2.$$

$$\lim_{x\to 1}\frac{x^m-1}{x^n-1}$$ $m,n\in N$

$$=\lim_{x\to 1}\frac{(x-1)(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)}=\lim_{x\to 1}\frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a} \qquad a \geq 0$$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$$\lim_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} = -2$$

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$$

$$\lim_{x\to 1} \frac{x^m-1}{x^n-1} \qquad m, n \in N$$

$$= \lim_{x\to 1} \frac{(x-1)(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)} = \lim_{x\to 1} \frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$$

$$= \frac{1^{m-1}+1^{m-2}+\cdots+1+1}{1^{n-1}+1^{n-2}+\cdots+1+1}$$

Riešené limity – 23, 24, 25

$\lim\limits_{x\to a} \frac{a-x}{\sqrt{a}-\sqrt{x}} = 2\sqrt{a}$ $\quad a \geq 0$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}} = \lim_{x\to a} (\sqrt{a}+\sqrt{x}) = 2\sqrt{a}.$$

$\lim\limits_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} = -2$

$$= \lim_{x\to\infty} \left[\frac{\sqrt{x}-6x}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\right] = \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}} = \frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}} = \frac{0-6}{3+0} = -2.$$

$\lim\limits_{x\to 1} \frac{x^m-1}{x^n-1}$ $\quad m, n \in N$

$$= \lim_{x\to 1} \frac{(x-1)(x^{m-1}+x^{m-2}+\cdots+x+1)}{(x-1)(x^{n-1}+x^{n-2}+\cdots+x+1)} = \lim_{x\to 1} \frac{x^{m-1}+x^{m-2}+\cdots+x+1}{x^{n-1}+x^{n-2}+\cdots+x+1}$$

$$= \frac{1^{m-1}+1^{m-2}+\cdots+1+1}{1^{n-1}+1^{n-2}+\cdots+1+1} = \frac{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}$$

Riešené limity – 23, 24, 25

$$\lim_{x\to a}\frac{a-x}{\sqrt{a}-\sqrt{x}}=2\sqrt{a}\qquad a\geq 0$$

$$=\lim_{x\to a}\frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{a}-\sqrt{x}}=\lim_{x\to a}(\sqrt{a}+\sqrt{x})=2\sqrt{a}.$$

$$\lim_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}=-2$$

$$=\lim_{x\to\infty}\left[\frac{\sqrt{x}-6x}{3x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}\right]=\lim_{x\to\infty}\frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}=\frac{\frac{1}{\sqrt{\infty}}-6}{3+\frac{1}{\infty}}=\frac{0-6}{3+0}=-2.$$

$$\lim_{x\to 1}\frac{x^m-1}{x^n-1}=\frac{m}{n}\qquad m,n\in N$$

$$=\lim_{x\to 1}\frac{(x-1)(x^{m-1}+x^{m-2}+\dots+x+1)}{(x-1)(x^{n-1}+x^{n-2}+\dots+x+1)}=\lim_{x\to 1}\frac{x^{m-1}+x^{m-2}+\dots+x+1}{x^{n-1}+x^{n-2}+\dots+x+1}$$

$$=\frac{1^{m-1}+1^{m-2}+\dots+1+1}{1^{n-1}+1^{n-2}+\dots+1+1}=\frac{1^{m-1}+1^{m-2}+\dots+1^1+1^0}{1^{n-1}+1^{n-2}+\dots+1^1+1^0}=\frac{m}{n}.$$

Riešené limity – 26

$$\lim_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \qquad m, n \in N$$

Riešené limity – 26

$\lim\limits_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$ $\qquad m, n \in N$

$$= \lim_{x\to 1} \left[\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \right]$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & x = z^{mn} & \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ x \to 1 & z \to 1 & \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{array} \right] = \lim_{z\to 1} \frac{z^n-1}{z^m-1}$$

Riešené limity – 26

$$\lim_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \qquad m, n\in N$$

$$= \lim_{x\to 1}\left[\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}\cdot\frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right]$$

$$= \lim_{x\to 1}\frac{(x-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]}{(x-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & x=z^{mn} & \sqrt[m]{x}=\sqrt[m]{z^{mn}}=z^{\frac{mn}{m}}=z^{n} \\ x\to 1 & z\to 1 & \sqrt[n]{x}=\sqrt[n]{z^{mn}}=z^{\frac{mn}{n}}=z^{m}\end{array}\right] = \lim_{z\to 1}\frac{z^n-1}{z^m-1}$$

$$= \lim_{z\to 1}\frac{(z-1)(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)(z^{m-1}+z^{m-2}+\cdots+z+1)}$$

Riešené limity – 26

$$\lim_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \qquad m,n\in N$$

$$= \lim_{x\to 1}\left[\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}\cdot\frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right]$$

$$= \lim_{x\to 1}\frac{(x-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]}{(x-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}$$

$$= \lim_{x\to 1}\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & x=z^{mn} & \sqrt[m]{x}=\sqrt[m]{z^{mn}}=z^{\frac{mn}{m}}=z^n \\ x\to 1 & z\to 1 & \sqrt[n]{x}=\sqrt[n]{z^{mn}}=z^{\frac{mn}{n}}=z^m \end{array}\right] = \lim_{z\to 1}\frac{z^n-1}{z^m-1}$$

$$= \lim_{z\to 1}\frac{(z-1)(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)(z^{m-1}+z^{m-2}+\cdots+z+1)} = \lim_{z\to 1}\frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

Riešené limity – 26

$\lim\limits_{x\to 1} \frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}$ $\quad m, n \in N$

$$= \lim_{x\to 1} \left[\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1} \cdot \frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1} \cdot \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} \right]$$

$$= \lim_{x\to 1} \frac{(x-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]}{(x-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}$$

$$= \lim_{x\to 1} \frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1} = \frac{1^{n-1}+1^{n-2}+\cdots+1^1+1}{1^{m-1}+1^{m-2}+\cdots+1^1+1}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & x = z^{mn} & \sqrt[m]{x} = \sqrt[m]{z^{mn}} = z^{\frac{mn}{m}} = z^n \\ x \to 1 & z \to 1 & \sqrt[n]{x} = \sqrt[n]{z^{mn}} = z^{\frac{mn}{n}} = z^m \end{array} \right] = \lim_{z\to 1} \frac{z^n-1}{z^m-1}$$

$$= \lim_{z\to 1} \frac{(z-1)(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)(z^{m-1}+z^{m-2}+\cdots+z+1)} = \lim_{z\to 1} \frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

$$= \frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}$$

Riešené limity – 26

$$\lim_{x\to 1}\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}=\frac{n}{m}\qquad m,n\in N$$

$$=\lim_{x\to 1}\left[\frac{\sqrt[m]{x}-1}{\sqrt[n]{x}-1}\cdot\frac{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}\cdot\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}\right]$$

$$=\lim_{x\to 1}\frac{(x-1)\cdot\left[(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1\right]}{(x-1)\cdot\left[(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1\right]}$$

$$=\lim_{x\to 1}\frac{(\sqrt[n]{x})^{n-1}+(\sqrt[n]{x})^{n-2}+\cdots+\sqrt[n]{x}+1}{(\sqrt[m]{x})^{m-1}+(\sqrt[m]{x})^{m-2}+\cdots+\sqrt[m]{x}+1}=\frac{1^{n-1}+1^{n-2}+\cdots+1^1+1}{1^{m-1}+1^{m-2}+\cdots+1^1+1}=\frac{n}{m}.$$

$$=\left[\begin{array}{c|c|c}\text{Subst.} & x=z^{mn} & \sqrt[m]{x}=\sqrt[m]{z^{mn}}=z^{\frac{mn}{m}}=z^n\\ x\to 1 & z\to 1 & \sqrt[n]{x}=\sqrt[n]{z^{mn}}=z^{\frac{mn}{n}}=z^m\end{array}\right]=\lim_{z\to 1}\frac{z^n-1}{z^m-1}$$

$$=\lim_{z\to 1}\frac{(z-1)(z^{n-1}+z^{n-2}+\cdots+z+1)}{(z-1)(z^{m-1}+z^{m-2}+\cdots+z+1)}=\lim_{z\to 1}\frac{z^{n-1}+z^{n-2}+\cdots+z+1}{z^{m-1}+z^{m-2}+\cdots+z+1}$$

$$=\frac{1^{n-1}+1^{n-2}+\cdots+1+1}{1^{m-1}+1^{m-2}+\cdots+1+1}=\frac{1^{n-1}+1^{n-2}+\cdots+1^1+1^0}{1^{m-1}+1^{m-2}+\cdots+1^1+1^0}=\frac{n}{m}.$$

Riešené limity – 27

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

Riešené limity – 27

$\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

$$= \lim_{x\to 1} \left[\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right]$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & x = z^6 & \sqrt{x} = \sqrt{z^6} = z^3 \\ x \to 1 & z \to 1 & \sqrt[3]{x} = \sqrt[3]{z^6} = z^2 \end{array} \right] = \lim_{z\to 1} \frac{z^3-1}{z^2-1}$$

$$= \left[\begin{array}{c|c} x = (\sqrt[6]{x})^6 & \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ x \to 1,\ x > 0 & \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right] = \lim_{x\to 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1}$$

Riešené limity – 27

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$$

$$= \lim_{x\to 1} \left[\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right] = \lim_{x\to 1} \frac{(x-1)\cdot[(\sqrt[3]{x})^2+\sqrt[3]{x}+1]}{(x-1)\cdot[\sqrt{x}+1]}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & x = z^6 & \sqrt{x} = \sqrt{z^6} = z^3 \\ x \to 1 & z \to 1 & \sqrt[3]{x} = \sqrt[3]{z^6} = z^2 \end{array} \right] = \lim_{z\to 1} \frac{z^3-1}{z^2-1} = \lim_{z\to 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)}$$

$$= \left[\begin{array}{c|c} x = (\sqrt[6]{x})^6 & \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ x \to 1,\ x > 0 & \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right] = \lim_{x\to 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1}$$

$$= \lim_{x\to 1} \frac{(\sqrt[6]{x}-1)[(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1)(\sqrt[6]{x}+1)}$$

Riešené limity – 27

$\lim\limits_{x\to 1} \frac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

$$= \lim_{x\to 1} \left[\frac{\sqrt{x}-1}{\sqrt[3]{x}-1} \cdot \frac{\sqrt{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1} \cdot \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1} \right] = \lim_{x\to 1} \frac{(x-1)\cdot[(\sqrt[3]{x})^2+\sqrt[3]{x}+1]}{(x-1)\cdot[\sqrt{x}+1]}$$

$$= \lim_{x\to 1} \frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & x = z^6 & \sqrt{x} = \sqrt{z^6} = z^3 \\ x \to 1 & z \to 1 & \sqrt[3]{x} = \sqrt[3]{z^6} = z^2 \end{array} \right] = \lim_{z\to 1} \frac{z^3-1}{z^2-1} = \lim_{z\to 1} \frac{(z-1)(z^2+z+1)}{(z-1)(z+1)}$$

$$= \lim_{z\to 1} \frac{z^2+z+1}{z+1}$$

$$= \left[\begin{array}{c|c} x = (\sqrt[6]{x})^6 & \sqrt{x} = (\sqrt[6]{x})^{\frac{6}{2}} = (\sqrt[6]{x})^3 \\ x \to 1,\ x > 0 & \sqrt[3]{x} = (\sqrt[6]{x})^{\frac{6}{3}} = (\sqrt[6]{x})^2 \end{array} \right] = \lim_{x\to 1} \frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1}$$

$$= \lim_{x\to 1} \frac{(\sqrt[6]{x}-1)[(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1)(\sqrt[6]{x}+1)} = \lim_{x\to 1} \frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1}$$

Riešené limity – 27

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}=\frac{3}{2}$$

$$=\lim_{x\to 1}\left[\frac{\sqrt{x}-1}{\sqrt[3]{x}-1}\cdot\frac{\sqrt{x}+1}{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}\cdot\frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1}\right]=\lim_{x\to 1}\frac{(x-1)\cdot[(\sqrt[3]{x})^2+\sqrt[3]{x}+1]}{(x-1)\cdot[\sqrt{x}+1]}$$

$$=\lim_{x\to 1}\frac{(\sqrt[3]{x})^2+\sqrt[3]{x}+1}{\sqrt{x}+1}=\frac{1+1+1}{1+1}=\frac{3}{2}.$$

$$=\left[\begin{array}{c|c|c}\text{Subst.} & x=z^6 & \sqrt{x}=\sqrt{z^6}=z^3\\ x\to 1 & z\to 1 & \sqrt[3]{x}=\sqrt[3]{z^6}=z^2\end{array}\right]=\lim_{z\to 1}\frac{z^3-1}{z^2-1}=\lim_{z\to 1}\frac{(z-1)(z^2+z+1)}{(z-1)(z+1)}$$

$$=\lim_{z\to 1}\frac{z^2+z+1}{z+1}=\frac{1^2+1+1}{1+1}=\frac{3}{2}.$$

$$=\left[\begin{array}{c|c}x=(\sqrt[6]{x})^6 & \sqrt{x}=(\sqrt[6]{x})^{\frac{6}{2}}=(\sqrt[6]{x})^3\\ x\to 1,\ x>0 & \sqrt[3]{x}=(\sqrt[6]{x})^{\frac{6}{3}}=(\sqrt[6]{x})^2\end{array}\right]=\lim_{x\to 1}\frac{(\sqrt[6]{x})^3-1}{(\sqrt[6]{x})^2-1}$$

$$=\lim_{x\to 1}\frac{(\sqrt[6]{x}-1)[(\sqrt[6]{x})^2+\sqrt[6]{x}+1]}{(\sqrt[6]{x}-1)(\sqrt[6]{x}+1)}=\lim_{x\to 1}\frac{(\sqrt[6]{x})^2+\sqrt[6]{x}+1}{\sqrt[6]{x}+1}=\frac{1+1+1}{1+1}=\frac{3}{2}.$$

Riešené limity – 28, 29

$$\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} \qquad m \in R$$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ $m \in R$

$\boxed{m=0}$ $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0 = 0.$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \dfrac{\sqrt[3]{1+mx}-1}{x}$ $\quad m \in R$

$m = 0$ $\quad \lim\limits_{x\to 0} \dfrac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \dfrac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0 = 0.$

$m \neq 0$ $\quad \lim\limits_{x\to 0} \dfrac{\sqrt[3]{1+mx}-1}{x}$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ $m \in R$

$m = 0$ $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0 = 0.$

$m \neq 0$ $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\begin{array}{r|l} \text{Subst. } z^3 = 1+mx & z \to 1 \\ x = \frac{z^3-1}{m} & x \to 0 \end{array} \right] = \lim\limits_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}}$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ $m \in R$

$m = 0$ $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0 = 0.$

$m \neq 0$ $\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\text{Subst. } z^3 = 1+mx,\ x = \frac{z^3-1}{m} \;\middle|\; z \to 1,\ x \to 0 \right] = \lim\limits_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}}$

$= \lim\limits_{z\to 1} \frac{m(z-1)}{(z-1)(z^2+z+1)}$

Riešené limity – 28, 29

$\lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x}$ $\qquad m\in R$

$\boxed{m=0}$ $\quad \lim\limits_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim\limits_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim\limits_{x\to 0} 0 = 0.$

$\boxed{m\neq 0}$ $\quad \lim\limits_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\begin{array}{r|l} \text{Subst. } z^3 = 1+mx & z\to 1 \\ x = \frac{z^3-1}{m} & x\to 0 \end{array} \right] = \lim\limits_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}}$

$= \lim\limits_{z\to 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim\limits_{z\to 1} \frac{m}{z^2+z+1}$

Riešené limity – 28, 29

$$\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \frac{m}{3} \qquad m \in R$$

$m = 0$ $$\lim_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim_{x\to 0} 0 = 0 = \frac{0}{3}.$$

$m \neq 0$ $$\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\begin{array}{r|l} \text{Subst. } z^3 = 1+mx & z \to 1 \\ x = \frac{z^3-1}{m} & x \to 0 \end{array} \right] = \lim_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}}$$

$$= \lim_{z\to 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim_{z\to 1} \frac{m}{z^2+z+1} = \frac{m}{1^2+1+1} = \frac{m}{3}.$$

Riešené limity – 28, 29

$$\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \frac{m}{3} \qquad m \in R$$

$m = 0$

$$\lim_{x\to 0} \frac{\sqrt[3]{1+0\cdot x}-1}{x} = \lim_{x\to 0} \frac{\sqrt[3]{1}-1}{x} = \lim_{x\to 0} 0 = 0 = \frac{0}{3}.$$

$m \neq 0$

$$\lim_{x\to 0} \frac{\sqrt[3]{1+mx}-1}{x} = \left[\text{Subst. } z^3 = 1+mx, \ x = \frac{z^3-1}{m} \;\middle|\; z \to 1, \ x \to 0 \right] = \lim_{z\to 1} \frac{z-1}{\frac{z^3-1}{m}}$$

$$= \lim_{z\to 1} \frac{m(z-1)}{(z-1)(z^2+z+1)} = \lim_{z\to 1} \frac{m}{z^2+z+1} = \frac{m}{1^2+1+1} = \frac{m}{3}.$$

$$\lim_{x\to 0} \frac{\sin 5x}{x}$$

Riešené limity – 28, 29

$$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\frac{m}{3}\qquad m\in R$$

$\boxed{m=0}$ $$\lim_{x\to 0}\frac{\sqrt[3]{1+0\cdot x}-1}{x}=\lim_{x\to 0}\frac{\sqrt[3]{1}-1}{x}=\lim_{x\to 0}0=0=\frac{0}{3}.$$

$\boxed{m\neq 0}$ $$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\left[\begin{array}{r|l}\text{Subst. } z^3=1+mx & z\to 1\\ x=\frac{z^3-1}{m} & x\to 0\end{array}\right]=\lim_{z\to 1}\frac{z-1}{\frac{z^3-1}{m}}$$

$$=\lim_{z\to 1}\frac{m(z-1)}{(z-1)(z^2+z+1)}=\lim_{z\to 1}\frac{m}{z^2+z+1}=\frac{m}{1^2+1+1}=\frac{m}{3}.$$

$$\lim_{x\to 0}\frac{\sin 5x}{x}$$

$$=\left[\begin{array}{r|l}\text{Subst. } z=5x & z\to 0\\ x=\frac{z}{5} & x\to 0\end{array}\right]=\lim_{z\to 0}\frac{\sin z}{\frac{z}{5}}$$

$$=\lim_{x\to 0}\frac{5\sin 5x}{5x}$$

Riešené limity – 28, 29

$$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\frac{m}{3} \qquad m\in R$$

$\boxed{m=0}$ $$\lim_{x\to 0}\frac{\sqrt[3]{1+0\cdot x}-1}{x}=\lim_{x\to 0}\frac{\sqrt[3]{1}-1}{x}=\lim_{x\to 0}0=0=\frac{0}{3}.$$

$\boxed{m\neq 0}$ $$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\left[\begin{array}{r|l}\text{Subst. } z^3=1+mx & z\to 1\\ x=\frac{z^3-1}{m} & x\to 0\end{array}\right]=\lim_{z\to 1}\frac{z-1}{\frac{z^3-1}{m}}$$

$$=\lim_{z\to 1}\frac{m(z-1)}{(z-1)(z^2+z+1)}=\lim_{z\to 1}\frac{m}{z^2+z+1}=\frac{m}{1^2+1+1}=\frac{m}{3}.$$

$$\lim_{x\to 0}\frac{\sin 5x}{x}$$

$$=\left[\begin{array}{r|l}\text{Subst. } z=5x & z\to 0\\ x=\frac{z}{5} & x\to 0\end{array}\right]=\lim_{z\to 0}\frac{\sin z}{\frac{z}{5}}=5\lim_{z\to 0}\frac{\sin z}{z}$$

$$=\lim_{x\to 0}\frac{5\sin 5x}{5x}=\left[\begin{array}{l|l}\text{Subst.} & z=5x\\ x\to 0 & z\to 0\end{array}\right]=5\lim_{z\to 0}\frac{\sin z}{z}$$

Riešené limity – 28, 29

$$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\frac{m}{3} \qquad m\in R$$

$m=0$: $$\lim_{x\to 0}\frac{\sqrt[3]{1+0\cdot x}-1}{x}=\lim_{x\to 0}\frac{\sqrt[3]{1}-1}{x}=\lim_{x\to 0}0=0=\frac{0}{3}.$$

$m\neq 0$: $$\lim_{x\to 0}\frac{\sqrt[3]{1+mx}-1}{x}=\left[\begin{array}{r|l}\text{Subst. } z^3=1+mx & z\to 1\\ x=\frac{z^3-1}{m} & x\to 0\end{array}\right]=\lim_{z\to 1}\frac{z-1}{\frac{z^3-1}{m}}$$

$$=\lim_{z\to 1}\frac{m(z-1)}{(z-1)(z^2+z+1)}=\lim_{z\to 1}\frac{m}{z^2+z+1}=\frac{m}{1^2+1+1}=\frac{m}{3}.$$

$$\lim_{x\to 0}\frac{\sin 5x}{x}=5$$

$$=\left[\begin{array}{r|l}\text{Subst. } z=5x & z\to 0\\ x=\frac{z}{5} & x\to 0\end{array}\right]=\lim_{z\to 0}\frac{\sin z}{\frac{z}{5}}=5\lim_{z\to 0}\frac{\sin z}{z}=5\cdot 1=5.$$

$$=\lim_{x\to 0}\frac{5\sin 5x}{5x}=\left[\begin{array}{l|l}\text{Subst.} & z=5x\\ x\to 0 & z\to 0\end{array}\right]=5\lim_{z\to 0}\frac{\sin z}{z}=5\cdot 1=5.$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x}$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x}$$

Riešené limity – 30, 31, 32, 33

$\lim\limits_{x \to 0} \frac{\operatorname{tg} x}{x}$

$$= \lim_{x \to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x \to 0} \left[\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right]$$

Riešené limity – 30, 31, 32, 33

$\lim\limits_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0} \left[\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right] = 1 \cdot \frac{1}{1} = 1.$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0} \left[\frac{\sin x}{x} \cdot \frac{1}{\cos x}\right] = 1 \cdot \frac{1}{1} = 1.$$

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0} \left[\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right] = 1 \cdot \frac{1}{1} = 1.$$

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x}$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad z = \operatorname{arctg} x & z \to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x \to 0 \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\sin z}{\cos z}}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0} \frac{\operatorname{tg} x}{x} = 1$$

$$= \lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0} \left[\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right] = 1 \cdot \frac{1}{1} = 1.$$

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x}$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad z = \operatorname{arctg} x & z \to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x \to 0 \end{array} \right] = \lim_{z\to 0} \frac{z}{\frac{\sin z}{\cos z}} = \lim_{z\to 0} \left[\frac{z}{\sin z} \cdot \cos z \right]$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = 1$$

$$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\left[\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right] = 1\cdot\frac{1}{1} = 1.$$

$$\lim_{x\to 0}\frac{\operatorname{arctg} x}{x} = 1$$

$$= \left[\begin{array}{r|l}\text{Subst.}\quad z = \operatorname{arctg} x & z\to 0\\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x\to 0\end{array}\right] = \lim_{z\to 0}\frac{z}{\frac{\sin z}{\cos z}} = \lim_{z\to 0}\left[\frac{z}{\sin z}\cdot\cos z\right] = 1\cdot 1 = 1.$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x}=1$$

$$=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x}=\lim_{x\to 0}\left[\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right]=1\cdot\frac{1}{1}=1.$$

$$\lim_{x\to 0}\frac{\operatorname{arctg} x}{x}=1$$

$$=\left[\begin{array}{r|l}\text{Subst.}\quad z=\operatorname{arctg} x & z\to 0\\ x=\operatorname{tg} z=\frac{\sin z}{\cos z} & x\to 0\end{array}\right]=\lim_{z\to 0}\frac{z}{\frac{\sin z}{\cos z}}=\lim_{z\to 0}\left[\frac{z}{\sin z}\cdot\cos z\right]=1\cdot 1=1.$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 2x}$$

$$\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = 1$$

$$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\left[\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right] = 1\cdot\frac{1}{1} = 1.$$

$$\lim_{x\to 0}\frac{\operatorname{arctg} x}{x} = 1$$

$$= \left[\begin{array}{r} \text{Subst.}\quad z = \operatorname{arctg} x \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} \end{array}\middle|\begin{array}{l} z\to 0 \\ x\to 0 \end{array}\right] = \lim_{z\to 0}\frac{z}{\frac{\sin z}{\cos z}} = \lim_{z\to 0}\left[\frac{z}{\sin z}\cdot\cos z\right] = 1\cdot 1 = 1.$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 2x}$$

$$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x}$$

$$\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x}$$

$$= \lim_{x\to \pi}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x}=1$$

$$=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x}=\lim_{x\to 0}\left[\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right]=1\cdot\frac{1}{1}=1.$$

$$\lim_{x\to 0}\frac{\operatorname{arctg} x}{x}=1$$

$$=\left[\begin{array}{r|l}\text{Subst.}\quad z=\operatorname{arctg} x & z\to 0\\ x=\operatorname{tg} z=\frac{\sin z}{\cos z} & x\to 0\end{array}\right]=\lim_{z\to 0}\frac{z}{\frac{\sin z}{\cos z}}=\lim_{z\to 0}\left[\frac{z}{\sin z}\cdot\cos z\right]=1\cdot 1=1.$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 2x}$$

$$=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x}$$

$$=\lim_{x\to 0}\frac{1}{2\cos^2 x}$$

$$\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x}$$

$$=\lim_{x\to \pi}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x}$$

$$=\lim_{x\to \pi}\frac{1}{2\cos^2 x}$$

Riešené limity – 30, 31, 32, 33

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = 1$$

$$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x} = \lim_{x\to 0}\left[\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right] = 1\cdot\frac{1}{1} = 1.$$

$$\lim_{x\to 0}\frac{\operatorname{arctg} x}{x} = 1$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad z = \operatorname{arctg} x & z\to 0 \\ x = \operatorname{tg} z = \frac{\sin z}{\cos z} & x\to 0 \end{array}\right] = \lim_{z\to 0}\frac{z}{\frac{\sin z}{\cos z}} = \lim_{z\to 0}\left[\frac{z}{\sin z}\cdot\cos z\right] = 1\cdot 1 = 1.$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

$$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x}$$

$$= \lim_{x\to 0}\frac{1}{2\cos^2 x} = \frac{1}{2\cdot 1^2} = \frac{1}{2}.$$

$$\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}$$

$$= \lim_{x\to \pi}\frac{\frac{\sin x}{\cos x}}{2\sin x\cos x}$$

$$= \lim_{x\to \pi}\frac{1}{2\cos^2 x} = \frac{1}{2(-1)^2} = \frac{1}{2}.$$

Riešené limity – 34, 35

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} \qquad m,n \in R - \{0\}$$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx}$ $m,n \in R - \{0\}$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right]$$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx}$ $m, n \in R - \{0\}$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right] = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$$

Riešené limity – 34, 35

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} \qquad m, n \in R - \{0\}$$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right] = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & u = mx & v = nx \\ x \to 0 & u \to 0 & v \to 0 \end{array} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v}$$

Riešené limity – 34, 35

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n} \qquad m,n \in R - \{0\}$$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right] = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & u = mx & v = nx \\ x \to 0 & u \to 0 & v \to 0 \end{array} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}.$$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \dfrac{\sin mx}{\sin nx} = \dfrac{m}{n}$ $\qquad m,n \in R - \{0\}$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right] = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & u = mx & v = nx \\ x \to 0 & u \to 0 & v \to 0 \end{array} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}.$$

$\lim\limits_{x\to \pi} \dfrac{\sin mx}{\sin nx}$ $\qquad m,n \in R - \{0\}$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n}$ $\qquad m,n \in R - \{0\}$

$$= \lim_{x\to 0}\left[\frac{\sin mx}{mx}\cdot\frac{mx}{nx}\cdot\frac{nx}{\sin nx}\right] = \lim_{x\to 0}\frac{mx}{nx}\cdot\lim_{x\to 0}\frac{\sin mx}{mx}\cdot\lim_{x\to 0}\frac{nx}{\sin nx}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} & u = mx & v = nx \\ x\to 0 & u\to 0 & v\to 0\end{array}\right] = \frac{m}{n}\cdot\lim_{u\to 0}\frac{\sin u}{u}\cdot\lim_{v\to 0}\frac{v}{\sin v} = \frac{m}{n}\cdot 1\cdot 1 = \frac{m}{n}.$$

$\lim\limits_{x\to \pi} \frac{\sin mx}{\sin nx}$ $\qquad m,n \in R - \{0\}$

$$= \left[\begin{array}{r|r} \text{Subst. } x = \pi + z & z = \pi - x \\ x\to \pi & z\to 0\end{array}\right] = \lim_{z\to 0}\frac{\sin m(\pi+z)}{\sin n(\pi+z)}$$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n}$ $\qquad m,n \in R - \{0\}$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right] = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.} & u = mx & v = nx \\ x \to 0 & u \to 0 & v \to 0 \end{array} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}.$$

$\lim\limits_{x\to \pi} \frac{\sin mx}{\sin nx}$ $\qquad m,n \in R - \{0\}$

$$= \left[\begin{array}{c|c} \text{Subst. } x = \pi + z & z = \pi - x \\ x \to \pi & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z\to 0} \frac{\sin (m\pi+mz)}{\sin (n\pi+nz)}$$

Riešené limity – 34, 35

$\lim\limits_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n}$ $\qquad m,n \in R - \{0\}$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right] = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} & u = mx & v = nx \\ x \to 0 & u \to 0 & v \to 0 \end{array} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}.$$

$\lim\limits_{x\to \pi} \frac{\sin mx}{\sin nx}$ $\qquad m,n \in R - \{0\}$

$$= \left[\begin{array}{r|l} \text{Subst. } x = \pi + z & z = \pi - x \\ x \to \pi & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z\to 0} \frac{\sin(m\pi+mz)}{\sin(n\pi+nz)}$$

$$= \lim_{z\to 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz}$$

Riešené limity – 34, 35

$$\lim_{x\to 0}\frac{\sin mx}{\sin nx}=\frac{m}{n} \qquad m,n\in R-\{0\}$$

$$=\lim_{x\to 0}\left[\frac{\sin mx}{mx}\cdot\frac{mx}{nx}\cdot\frac{nx}{\sin nx}\right]=\lim_{x\to 0}\frac{mx}{nx}\cdot\lim_{x\to 0}\frac{\sin mx}{mx}\cdot\lim_{x\to 0}\frac{nx}{\sin nx}$$

$$=\left[\begin{array}{c|c|c}\text{Subst.} & u=mx & v=nx\\ x\to 0 & u\to 0 & v\to 0\end{array}\right]=\frac{m}{n}\cdot\lim_{u\to 0}\frac{\sin u}{u}\cdot\lim_{v\to 0}\frac{v}{\sin v}=\frac{m}{n}\cdot 1\cdot 1=\frac{m}{n}.$$

$$\lim_{x\to \pi}\frac{\sin mx}{\sin nx} \qquad m,n\in R-\{0\}$$

$$=\left[\begin{array}{c|c}\text{Subst. } x=\pi+z & z=\pi-x\\ x\to\pi & z\to 0\end{array}\right]=\lim_{z\to 0}\frac{\sin m(\pi+z)}{\sin n(\pi+z)}=\lim_{z\to 0}\frac{\sin(m\pi+mz)}{\sin(n\pi+nz)}$$

$$=\lim_{z\to 0}\frac{\sin m\pi\cdot\cos mz+\cos m\pi\cdot\sin mz}{\sin n\pi\cdot\cos nz+\cos n\pi\cdot\sin nz}=\lim_{z\to 0}\frac{0\cdot\cos mz+(-1)^m\sin mz}{0\cdot\cos nz+(-1)^n\sin nz}$$

Riešené limity – 34, 35

$$\lim_{x\to 0} \frac{\sin mx}{\sin nx} = \frac{m}{n} \qquad m,n \in R - \{0\}$$

$$= \lim_{x\to 0} \left[\frac{\sin mx}{mx} \cdot \frac{mx}{nx} \cdot \frac{nx}{\sin nx} \right] = \lim_{x\to 0} \frac{mx}{nx} \cdot \lim_{x\to 0} \frac{\sin mx}{mx} \cdot \lim_{x\to 0} \frac{nx}{\sin nx}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} & u = mx & v = nx \\ x \to 0 & u \to 0 & v \to 0 \end{array} \right] = \frac{m}{n} \cdot \lim_{u\to 0} \frac{\sin u}{u} \cdot \lim_{v\to 0} \frac{v}{\sin v} = \frac{m}{n} \cdot 1 \cdot 1 = \frac{m}{n}.$$

$$\lim_{x\to \pi} \frac{\sin mx}{\sin nx} \qquad m,n \in R - \{0\}$$

$$= \left[\begin{array}{r|r} \text{Subst. } x = \pi + z & z = \pi - x \\ x \to \pi & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin m(\pi+z)}{\sin n(\pi+z)} = \lim_{z\to 0} \frac{\sin (m\pi+mz)}{\sin (n\pi+nz)}$$

$$= \lim_{z\to 0} \frac{\sin m\pi \cdot \cos mz + \cos m\pi \cdot \sin mz}{\sin n\pi \cdot \cos nz + \cos n\pi \cdot \sin nz} = \lim_{z\to 0} \frac{0 \cdot \cos mz + (-1)^m \sin mz}{0 \cdot \cos nz + (-1)^n \sin nz}$$

$$= (-1)^{m-n} \lim_{z\to 0} \frac{\sin mz}{\sin nz}$$

Riešené limity – 34, 35

$$\lim_{x\to 0}\frac{\sin mx}{\sin nx}=\frac{m}{n} \qquad m,n\in R-\{0\}$$

$$=\lim_{x\to 0}\left[\frac{\sin mx}{mx}\cdot\frac{mx}{nx}\cdot\frac{nx}{\sin nx}\right]=\lim_{x\to 0}\frac{mx}{nx}\cdot\lim_{x\to 0}\frac{\sin mx}{mx}\cdot\lim_{x\to 0}\frac{nx}{\sin nx}$$

$$=\left[\begin{array}{c|c|c}\text{Subst.} & u=mx & v=nx\\ x\to 0 & u\to 0 & v\to 0\end{array}\right]=\frac{m}{n}\cdot\lim_{u\to 0}\frac{\sin u}{u}\cdot\lim_{v\to 0}\frac{v}{\sin v}=\frac{m}{n}\cdot 1\cdot 1=\frac{m}{n}.$$

$$\lim_{x\to \pi}\frac{\sin mx}{\sin nx}=(-1)^{m-n}\frac{m}{n} \qquad m,n\in R-\{0\}$$

$$=\left[\begin{array}{r|c}\text{Subst. } x=\pi+z & z=\pi-x\\ x\to\pi & z\to 0\end{array}\right]=\lim_{z\to 0}\frac{\sin m(\pi+z)}{\sin n(\pi+z)}=\lim_{z\to 0}\frac{\sin(m\pi+mz)}{\sin(n\pi+nz)}$$

$$=\lim_{z\to 0}\frac{\sin m\pi\cdot\cos mz+\cos m\pi\cdot\sin mz}{\sin n\pi\cdot\cos nz+\cos n\pi\cdot\sin nz}=\lim_{z\to 0}\frac{0\cdot\cos mz+(-1)^m\sin mz}{0\cdot\cos nz+(-1)^n\sin nz}$$

$$=(-1)^{m-n}\lim_{z\to 0}\frac{\sin mz}{\sin nz}=\left[\begin{array}{c}\text{Predchádzajúci}\\ \text{príklad}\end{array}\right]=(-1)^{m-n}\frac{m}{n}.$$

Riešené limity – 36

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

Riešené limity – 36

$\lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2}$

$$= \lim_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2}$$

$$= \lim_{x\to 0} \left[\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right]$$

Riešené limity – 36

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

$$= \lim_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2}$$

$$= \lim_{x\to 0} \left[\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right] = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)}$$

Riešené limity – 36

$\lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2}$

$$= \lim_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$$= \lim_{x\to 0} \left[\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right] = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)}$$

$$= \left[\sin^2 x = \frac{1-\cos 2x}{2}\right] = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

Riešené limity – 36

$\lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2}$

$$= \lim_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$$= \lim_{x\to 0} \left[\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right] = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)}$$

$$= \lim_{x\to 0} \left[\frac{4}{1+\cos 2x} \cdot \left(\frac{\sin 2x}{2x}\right)^2\right]$$

$$= \left[\sin^2 x = \frac{1-\cos 2x}{2}\right] = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

Riešené limity – 36

$$\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$$

$$= \lim_{x\to 0} \frac{1-(\cos^2 x-\sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x+\sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$$= \lim_{x\to 0} \left[\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right] = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)}$$

$$= \lim_{x\to 0} \left[\frac{4}{1+\cos 2x} \cdot \left(\frac{\sin 2x}{2x}\right)^2\right] = \lim_{x\to 0} \frac{4}{1+\cos 2x} \cdot \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2$$

$$= \left[\sin^2 x = \frac{1-\cos 2x}{2}\right] = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

Riešené limity – 36

$\lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2}$

$$= \lim_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$$= 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2$$

$$= \lim_{x\to 0} \left[\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right] = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)}$$

$$= \lim_{x\to 0} \left[\frac{4}{1+\cos 2x} \cdot \left(\frac{\sin 2x}{2x}\right)^2\right] = \lim_{x\to 0} \frac{4}{1+\cos 2x} \cdot \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2$$

$$= \left[\begin{array}{c|c} \text{Subst.} & z = 2x \\ x \to 0 & z \to 0 \end{array}\right] = \lim_{x\to 0} \frac{4}{1+\cos 2x} \cdot \lim_{z\to 0} \left(\frac{\sin z}{z}\right)^2$$

$$= \left[\sin^2 x = \frac{1-\cos 2x}{2}\right] = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2$$

Riešené limity – 36

$\lim\limits_{x\to 0} \frac{1-\cos 2x}{x^2} = 2$

$$= \lim_{x\to 0} \frac{1-(\cos^2 x - \sin^2 x)}{x^2} = \lim_{x\to 0} \frac{\sin^2 x + \sin^2 x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$$

$$= 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 2\cdot 1^2 = 2.$$

$$= \lim_{x\to 0} \left[\frac{1-\cos 2x}{x^2} \cdot \frac{1+\cos 2x}{1+\cos 2x}\right] = \lim_{x\to 0} \frac{1-\cos^2 2x}{x^2(1+\cos 2x)} = \lim_{x\to 0} \frac{\sin^2 2x}{x^2(1+\cos 2x)}$$

$$= \lim_{x\to 0} \left[\frac{4}{1+\cos 2x} \cdot \left(\frac{\sin 2x}{2x}\right)^2\right] = \lim_{x\to 0} \frac{4}{1+\cos 2x} \cdot \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = 2x \\ x \to 0 & z \to 0 \end{array}\right] = \lim_{x\to 0} \frac{4}{1+\cos 2x} \cdot \lim_{z\to 0} \left(\frac{\sin z}{z}\right)^2 = \frac{4}{1+1} \cdot 1^2 = 2.$$

$$= \left[\sin^2 x = \frac{1-\cos 2x}{2}\right] = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2 \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 = 2\cdot 1^2 = 2.$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

Riešené limity – 37, 38

$$\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$$

Riešené limity – 37, 38

$$\lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x \to \frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x)$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1} - 1}$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

$$= \lim_{x\to 0} \left[\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} \right]$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

$$= \lim_{x\to 0} \left[\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right] = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1}$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

$$= \lim_{x\to 0} \left[\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right] = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1}$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

$$= \lim_{x\to 0} \left[\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right] = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1}$$

$$= \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x}$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1}$$

$$= \lim_{x\to 0} \left[\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right] = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1}$$

$$= \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x\to 0} \left[4(\sqrt{x+1}+1) \cdot \frac{\sin 4x}{4x}\right]$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1} - 1}$$

$$= \lim_{x\to 0} \left[\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right] = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(\sqrt{x+1})^2 - 1} = \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{(x+1)-1}$$

$$= \lim_{x\to 0} \frac{\sin 4x \cdot (\sqrt{x+1}+1)}{x} = \lim_{x\to 0} \left[4(\sqrt{x+1}+1) \cdot \frac{\sin 4x}{4x}\right] = \left[\begin{array}{c|c} \text{Subst.} & z = 4x \\ x \to 0 & z \to 0 \end{array}\right]$$

$$= 4 \lim_{x\to 0} (\sqrt{x+1}+1) \cdot \lim_{z\to 0} \frac{\sin z}{z}$$

Riešené limity – 37, 38

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos 2x}{\cos x - \sin x} = \sqrt{2}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \lim_{x\to\frac{\pi}{4}} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x - \sin x}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cos x + \sin x) = \lim_{x\to\frac{\pi}{4}} \cos x + \lim_{x\to\frac{\pi}{4}} \sin x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}.$$

$$\lim_{x\to 0} \frac{\sin 4x}{\sqrt{x+1}-1} = 8$$

$$= \lim_{x\to 0} \left[\frac{\sin 4x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right] = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-1} = \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{(x+1)-1}$$

$$= \lim_{x\to 0} \frac{\sin 4x\cdot(\sqrt{x+1}+1)}{x} = \lim_{x\to 0} \left[4(\sqrt{x+1}+1)\cdot\frac{\sin 4x}{4x}\right] = \left[\begin{array}{c|c} \text{Subst.} & z = 4x \\ x \to 0 & z \to 0 \end{array}\right]$$

$$= 4 \lim_{x\to 0} (\sqrt{x+1}+1) \cdot \lim_{z\to 0} \frac{\sin z}{z} = 4\cdot(\sqrt{0+1}+1)\cdot 1 = 4\cdot 2\cdot 1 = 8.$$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right]$$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x}$$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array} \right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z}$$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x\to\infty} e^x(2+\cos x)$$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

$$= \lim_{x\to 2} \left[\frac{1}{x}\cdot\frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right]$$

$$= \lim_{x\to 2}\frac{1}{x}\cdot\lim_{z\to 0}\frac{z}{\sin z} = \frac{1}{2}\cdot 1 = \frac{1}{2}.$$

$$\lim_{x\to\infty} e^x(2+\cos x)$$

Pre $x\in R$ platí $-1 \leq \cos x \leq 1$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x\to\infty} e^x (2+\cos x)$$

Pre $x \in R$ platí $-1 \leq \cos x \leq 1$, t. j. $1 \leq 2+\cos x \leq 3$

Riešené limity – 39, 40

$\lim\limits_{x\to 2} \dfrac{\arcsin(x-2)}{x^2-2x} = \dfrac{1}{2}$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array} \right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$\lim\limits_{x\to\infty} e^x(2+\cos x)$

Pre $x \in R$ platí $\quad -1 \leq \cos x \leq 1, \quad$ t. j. $1 \leq 2+\cos x \leq 3,$

$e^x > 0$ pre všetky $x \in R$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x\to\infty} e^x(2+\cos x)$$

Pre $x \in R$ platí $\quad -1 \le \cos x \le 1$, t. j. $1 \le 2+\cos x \le 3$,

$e^x > 0$ pre všetky $x \in R$, t. j. $e^x \le e^x(2+\cos x) \le 3\,e^x$.

Riešené limity – 39, 40

$\lim\limits_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array}\right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$\lim\limits_{x\to\infty} e^x(2+\cos x)$

Pre $x \in R$ platí $-1 \leq \cos x \leq 1$, t. j. $1 \leq 2+\cos x \leq 3$,

$e^x > 0$ pre všetky $x \in R$, t. j. $e^x \leq e^x(2+\cos x) \leq 3e^x$.

Pre limity týchto funkcií potom platí:

$$\lim_{x\to\infty} e^x \leq \lim_{x\to\infty} e^x(2+\cos x) \leq \lim_{x\to\infty} 3e^x$$

Riešené limity – 39, 40

$\lim\limits_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$

$$= \lim_{x\to 2} \left[\frac{1}{x}\cdot\frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z\to 0 \\ x-2 = \sin z & x\to 2 \end{array}\right]$$

$$= \lim_{x\to 2}\frac{1}{x}\cdot\lim_{z\to 0}\frac{z}{\sin z} = \frac{1}{2}\cdot 1 = \frac{1}{2}.$$

$\lim\limits_{x\to\infty} e^x(2+\cos x)$

Pre $x\in R$ platí $-1 \leq \cos x \leq 1$, t. j. $1 \leq 2+\cos x \leq 3$,

$e^x > 0$ pre všetky $x\in R$, t. j. $e^x \leq e^x(2+\cos x) \leq 3\,e^x$.

Pre limity týchto funkcií potom platí:

$$\infty = \lim_{x\to\infty} e^x \leq \lim_{x\to\infty} e^x(2+\cos x) \leq \lim_{x\to\infty} 3\,e^x = \infty$$

Riešené limity – 39, 40

$$\lim_{x\to 2} \frac{\arcsin(x-2)}{x^2-2x} = \frac{1}{2}$$

$$= \lim_{x\to 2} \left[\frac{1}{x} \cdot \frac{\arcsin(x-2)}{x-2}\right] = \left[\begin{array}{r|l} \text{Subst. } z = \arcsin(x-2) & z \to 0 \\ x-2 = \sin z & x \to 2 \end{array} \right]$$

$$= \lim_{x\to 2} \frac{1}{x} \cdot \lim_{z\to 0} \frac{z}{\sin z} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

$$\lim_{x\to\infty} e^x(2+\cos x) = \infty$$

Pre $x \in R$ platí $-1 \leq \cos x \leq 1$, t. j. $1 \leq 2+\cos x \leq 3$,

$e^x > 0$ pre všetky $x \in R$, t. j. $e^x \leq e^x(2+\cos x) \leq 3\,e^x$.

Pre limity týchto funkcií potom platí:

$$\infty = \lim_{x\to\infty} e^x \leq \lim_{x\to\infty} e^x(2+\cos x) \leq \lim_{x\to\infty} 3\,e^x = \infty,$$

t. j. $\lim_{x\to\infty} e^x(2+\cos x) = \infty$.

Riešené limity – 41, 42

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

Riešené limity – 41, 42

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[-1 \leq \cos x \leq 1 \right]$$

Riešené limity – 41, 42

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[\ -1\leq \cos x\leq 1 \ \Rightarrow\ 0\leq 1\pm\cos x \ \Rightarrow \ \right]$$

Riešené limity – 41, 42

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[\begin{array}{l} -1\le \cos x\le 1 \;\Rightarrow\; 0\le 1\pm\cos x \;\Rightarrow \\ \quad \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \end{array} \right]$$

Riešené limity – 41, 42

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[\begin{array}{l} -1 \leq \cos x \leq 1 \;\Rightarrow\; 0 \leq 1 \pm \cos x \;\Rightarrow \\ \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \\ \sin x = -|\sin x| \text{ pre } x \in (-\pi; 0)\text{, t. j. } x \to 0^- \end{array}\right]$$

$$\lim_{x\to 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = -\lim_{x\to 0^-} \sqrt{1+\cos x}$$

Riešené limity – 41, 42

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[-1 \leq \cos x \leq 1 \;\Rightarrow\; 0 \leq 1 \pm \cos x \;\Rightarrow\; \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \right.$$

$$\left. \sin x = |\sin x| \text{ pre } x \in (0;\pi)\text{, t. j. } x \to 0^+ \right]$$

$$\lim_{x\to 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = -\lim_{x\to 0^-} \sqrt{1+\cos x}$$

$$\lim_{x\to 0^+} \frac{\sin x}{\sqrt{1-\cos x}} = +\lim_{x\to 0^+} \sqrt{1+\cos x}$$

Riešené limity – 41, 42

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{1-\cos x}}$$

$$\left[\begin{array}{l} -1\leq \cos x\leq 1 \;\Rightarrow\; 0\leq 1\pm\cos x \;\Rightarrow \\ \frac{|\sin x|}{\sqrt{1-\cos x}} = \frac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \frac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \frac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \\ \sin x = -|\sin x| \text{ pre } x\in(-\pi;0)\text{, t. j. } x\to 0^- \qquad \sin x = |\sin x| \text{ pre } x\in(0;\pi)\text{, t. j. } x\to 0^+ \end{array}\right]$$

$$\lim_{x\to 0^-} \frac{\sin x}{\sqrt{1-\cos x}} = -\lim_{x\to 0^-} \sqrt{1+\cos x} = -\sqrt{2}$$

$$\lim_{x\to 0^+} \frac{\sin x}{\sqrt{1-\cos x}} = +\lim_{x\to 0^+} \sqrt{1+\cos x} = +\sqrt{2}$$

Riešené limity – 41, 42

$\lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}}$ neexistuje

$$\left[\begin{array}{l} -1 \leq \cos x \leq 1 \;\Rightarrow\; 0 \leq 1 \pm \cos x \;\Rightarrow \\ \dfrac{|\sin x|}{\sqrt{1-\cos x}} = \dfrac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \\ \sin x = -|\sin x| \text{ pre } x \in (-\pi; 0) \text{, t. j. } x \to 0^- \qquad \sin x = |\sin x| \text{ pre } x \in (0; \pi) \text{, t. j. } x \to 0^+ \end{array} \right]$$

$$\left. \begin{array}{l} \lim\limits_{x\to 0^-} \dfrac{\sin x}{\sqrt{1-\cos x}} = -\lim\limits_{x\to 0^-} \sqrt{1+\cos x} = -\sqrt{2} \\ \lim\limits_{x\to 0^+} \dfrac{\sin x}{\sqrt{1-\cos x}} = +\lim\limits_{x\to 0^+} \sqrt{1+\cos x} = +\sqrt{2} \end{array} \right\} \Rightarrow \lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}} \text{ neexistuje.}$$

Riešené limity – 41, 42

$\lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}}$ neexistuje

$$\left[\begin{array}{l} -1\leq \cos x\leq 1 \;\Rightarrow\; 0\leq 1\pm\cos x \;\Rightarrow \\ \dfrac{|\sin x|}{\sqrt{1-\cos x}} = \dfrac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \\ \sin x = -\,|\sin x| \text{ pre } x\in(-\pi;0)\text{, t. j. } x\to 0^- \qquad \sin x = |\sin x| \text{ pre } x\in(0;\pi)\text{, t. j. } x\to 0^+ \end{array}\right]$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} \dfrac{\sin x}{\sqrt{1-\cos x}} = -\lim\limits_{x\to 0^-} \sqrt{1+\cos x} = -\sqrt{2} \\ \lim\limits_{x\to 0^+} \dfrac{\sin x}{\sqrt{1-\cos x}} = +\lim\limits_{x\to 0^+} \sqrt{1+\cos x} = +\sqrt{2} \end{array}\right\} \Rightarrow \lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}} \text{ neexistuje.}$$

$\lim\limits_{x\to 0} \dfrac{x^n-\sin^n x}{x-\sin x}$ $\qquad n\in N,\ n\neq 1$

Riešené limity – 41, 42

$\lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}}$ neexistuje

$$\left[\begin{array}{l} -1\leq \cos x\leq 1 \;\Rightarrow\; 0\leq 1\pm\cos x \;\Rightarrow \\ \dfrac{|\sin x|}{\sqrt{1-\cos x}} = \dfrac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \\ \sin x = -\,|\sin x| \text{ pre } x\in(-\pi;0)\text{, t. j. } x\to 0^- \qquad \sin x = |\sin x| \text{ pre } x\in(0;\pi)\text{, t. j. } x\to 0^+ \end{array}\right]$$

$$\left.\begin{array}{l} \lim\limits_{x\to 0^-} \dfrac{\sin x}{\sqrt{1-\cos x}} = -\lim\limits_{x\to 0^-} \sqrt{1+\cos x} = -\sqrt{2} \\ \lim\limits_{x\to 0^+} \dfrac{\sin x}{\sqrt{1-\cos x}} = +\lim\limits_{x\to 0^+} \sqrt{1+\cos x} = +\sqrt{2} \end{array}\right\} \Rightarrow \lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}} \text{ neexistuje.}$$

$\lim\limits_{x\to 0} \dfrac{x^n-\sin^n x}{x-\sin x}$ $\qquad n\in N,\ n\neq 1$

$$= \lim_{x\to 0} \frac{(x-\sin x)(x^{n-1}+x^{n-2}\sin x+\cdots+x\sin^{n-2} x+\sin^{n-1} x)}{x-\sin x}$$

Riešené limity – 41, 42

$\lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}}$ neexistuje

$$\left[\begin{array}{l} -1\leq \cos x\leq 1 \ \Rightarrow\ 0\leq 1\pm\cos x \ \Rightarrow \\ \dfrac{|\sin x|}{\sqrt{1-\cos x}} = \dfrac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \\ \sin x = -\,|\sin x| \text{ pre } x\in(-\pi;0) \text{, t. j. } x\to 0^- \qquad \sin x = |\sin x| \text{ pre } x\in(0;\pi) \text{, t. j. } x\to 0^+ \end{array} \right]$$

$$\left. \begin{array}{l} \lim\limits_{x\to 0^-} \dfrac{\sin x}{\sqrt{1-\cos x}} = -\lim\limits_{x\to 0^-} \sqrt{1+\cos x} = -\sqrt{2} \\ \lim\limits_{x\to 0^+} \dfrac{\sin x}{\sqrt{1-\cos x}} = +\lim\limits_{x\to 0^+} \sqrt{1+\cos x} = +\sqrt{2} \end{array} \right\} \Rightarrow \lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}} \text{ neexistuje.}$$

$\lim\limits_{x\to 0} \dfrac{x^n-\sin^n x}{x-\sin x}$ $\qquad n\in N,\ n\neq 1$

$$= \lim_{x\to 0} \frac{(x-\sin x)(x^{n-1}+x^{n-2}\sin x+\cdots+x\sin^{n-2}x+\sin^{n-1}x)}{x-\sin x}$$

$$= \lim_{x\to 0} \left(x^{n-1}+x^{n-2}\sin x+\cdots+x\sin^{n-2}x+\sin^{n-1}x\right)$$

Riešené limity – 41, 42

$\lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}}$ neexistuje

$$\left[\begin{array}{l} -1\leq \cos x\leq 1 \;\Rightarrow\; 0\leq 1\pm\cos x \;\Rightarrow \\ \dfrac{|\sin x|}{\sqrt{1-\cos x}} = \dfrac{\sqrt{\sin^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos^2 x}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{(1-\cos x)(1+\cos x)}}{\sqrt{1-\cos x}} = \dfrac{\sqrt{1-\cos x}\sqrt{1+\cos x}}{\sqrt{1-\cos x}} = \sqrt{1+\cos x} \\ \sin x = -|\sin x| \text{ pre } x\in(-\pi;0)\text{, t. j. } x\to 0^- \qquad \sin x = |\sin x| \text{ pre } x\in(0;\pi)\text{, t. j. } x\to 0^+ \end{array} \right]$$

$$\left. \begin{array}{l} \lim\limits_{x\to 0^-} \dfrac{\sin x}{\sqrt{1-\cos x}} = -\lim\limits_{x\to 0^-} \sqrt{1+\cos x} = -\sqrt{2} \\ \lim\limits_{x\to 0^+} \dfrac{\sin x}{\sqrt{1-\cos x}} = +\lim\limits_{x\to 0^+} \sqrt{1+\cos x} = +\sqrt{2} \end{array} \right\} \Rightarrow \lim\limits_{x\to 0} \dfrac{\sin x}{\sqrt{1-\cos x}} \text{ neexistuje.}$$

$\lim\limits_{x\to 0} \dfrac{x^n-\sin^n x}{x-\sin x} = 0$ $\qquad n\in N,\ n\neq 1$

$$= \lim_{x\to 0} \frac{(x-\sin x)(x^{n-1}+x^{n-2}\sin x+\cdots+x\sin^{n-2}x+\sin^{n-1}x)}{x-\sin x}$$

$$= \lim_{x\to 0} (x^{n-1}+x^{n-2}\sin x+\cdots+x\sin^{n-2}x+\sin^{n-1}x)$$

$$= 0^{n-1}+0^{n-2}\cdot 0+\cdots+0^{n-2}\cdot 0+0^{n-1} = 0.$$

Riešené limity – 43, 44

$\lim\limits_{x \to a} \frac{\sin x - \sin a}{x - a}$ $a \in R$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a}$ $a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}}$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a}$ $a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a}$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x \to a & z \to 0 \end{array}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a}$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x \to a & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= 1 \cdot \cos\frac{a+a}{2}$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a} = \cos a$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x \to a & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= 1 \cdot \cos\frac{a+a}{2} = 1 \cdot \cos a = \cos a.$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a} = \cos a$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x \to a & z \to 0 \end{array}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= 1 \cdot \cos\frac{a+a}{2} = 1 \cdot \cos a = \cos a.$$

$\lim\limits_{x\to a} \frac{\cos x - \cos a}{x-a}$ $\qquad a \in R$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a} = \cos a$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x \to a & z \to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= 1 \cdot \cos\frac{a+a}{2} = 1 \cdot \cos a = \cos a.$$

$\lim\limits_{x\to a} \frac{\cos x - \cos a}{x-a}$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}}$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a} = \cos a$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x\to a & z\to 0 \end{array}\right] = \lim_{z\to 0}\frac{\sin z}{z} \cdot \lim_{x\to a}\cos\frac{x+a}{2}$$

$$= 1\cdot\cos\frac{a+a}{2} = 1\cdot\cos a = \cos a.$$

$\lim\limits_{x\to a} \frac{\cos x - \cos a}{x-a}$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \sin\frac{x+a}{2}$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a} = \cos a$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x \to a & z \to 0 \end{array}\right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= 1 \cdot \cos\frac{a+a}{2} = 1 \cdot \cos a = \cos a.$$

$\lim\limits_{x\to a} \frac{\cos x - \cos a}{x-a}$ $\qquad a \in R$

$$= \lim_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \sin\frac{x+a}{2}$$

$$= \left[\begin{array}{l|l} \text{Subst.} & z = \frac{x-a}{2} \\ x \to a & z \to 0 \end{array}\right] = -\lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \sin\frac{x+a}{2}$$

Riešené limity – 43, 44

$$\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\cos a \qquad a\in R$$

$$=\lim_{x\to a}\frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}}=\lim_{x\to a}\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}\cdot\lim_{x\to a}\cos\frac{x+a}{2}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & z=\frac{x-a}{2}\\ x\to a & z\to 0\end{array}\right]=\lim_{z\to 0}\frac{\sin z}{z}\cdot\lim_{x\to a}\cos\frac{x+a}{2}$$

$$=1\cdot\cos\frac{a+a}{2}=1\cdot\cos a=\cos a.$$

$$\lim_{x\to a}\frac{\cos x-\cos a}{x-a} \qquad a\in R$$

$$=\lim_{x\to a}\frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}}=-\lim_{x\to a}\frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}}\cdot\lim_{x\to a}\sin\frac{x+a}{2}$$

$$=\left[\begin{array}{l|l}\text{Subst.} & z=\frac{x-a}{2}\\ x\to a & z\to 0\end{array}\right]=-\lim_{z\to 0}\frac{\sin z}{z}\cdot\lim_{x\to a}\sin\frac{x+a}{2}$$

$$=-1\cdot\sin\frac{a+a}{2}$$

Riešené limity – 43, 44

$\lim\limits_{x\to a} \frac{\sin x - \sin a}{x-a} = \cos a \qquad a \in R$

$$= \lim_{x\to a} \frac{2\sin\frac{x-a}{2}\cos\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = \lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= \left[\begin{array}{c|c} \text{Subst.} & z = \frac{x-a}{2} \\ x\to a & z\to 0 \end{array} \right] = \lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \cos\frac{x+a}{2}$$

$$= 1 \cdot \cos\frac{a+a}{2} = 1 \cdot \cos a = \cos a.$$

$\lim\limits_{x\to a} \frac{\cos x - \cos a}{x-a} = -\sin a \qquad a \in R$

$$= \lim_{x\to a} \frac{-2\sin\frac{x-a}{2}\sin\frac{x+a}{2}}{2\cdot\frac{x-a}{2}} = -\lim_{x\to a} \frac{\sin\frac{x-a}{2}}{\frac{x-a}{2}} \cdot \lim_{x\to a} \sin\frac{x+a}{2}$$

$$= \left[\begin{array}{c|c} \text{Subst.} & z = \frac{x-a}{2} \\ x\to a & z\to 0 \end{array} \right] = -\lim_{z\to 0} \frac{\sin z}{z} \cdot \lim_{x\to a} \sin\frac{x+a}{2}$$

$$= -1 \cdot \sin\frac{a+a}{2} = -1 \cdot \sin a = -\sin a.$$