Matematická analýza 1

2018/2019

12. Neurčitý integrál
Riešené príklady

Obsah – príklady 001–100

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Zoznam integrálov – príklady 001–100

001. $\int \frac{dx}{\sin x}$ **002.** $\int \frac{dx}{\cos x}$ **003.** $\int \frac{dx}{1+\sin x}$ **004.** $\int \frac{dx}{1+\cos x}$ **005.** $\int \frac{dx}{\sinh x}$ **006.** $\int \frac{dx}{\cosh x}$ **007.** $\int \frac{dx}{1+\sinh x}$ **008.** $\int \frac{dx}{1+\cosh x}$ **009.** $\int \sin^2 x \, dx$ **010.** $\int \cos^2 x \, dx$ **011.** $\int \sin^3 x \, dx$

012. $\int \sin^{2n+1} x \, dx$ **013.** $\int \sin^n x \, dx$ **014.** $\int \cos^3 x \, dx$ **015.** $\int \cos^{2n+1} x \, dx$ **016.** $\int \cos^n x \, dx$ **017.** $\int \sinh^2 x \, dx$ **018.** $\int \cosh^2 x \, dx$ **019.** $\int \sinh^3 x \, dx$

020. $\int \sinh^{2n+1} x \, dx$ **021.** $\int \sinh^n x \, dx$ **022.** $\int \cosh^3 x \, dx$ **023.** $\int \cosh^{2n+1} x \, dx$ **024.** $\int \cosh^n x \, dx$ **025.** $\int \mathrm{tg}^2 x \, dx$ **026.** $\int \mathrm{tg}^3 x \, dx$ **027.** $\int \mathrm{cotg}^2 x \, dx$

028. $\int \mathrm{cotg}^3 x \, dx$ **029.** $\int \mathrm{tgh}^2 x \, dx$ **030.** $\int \mathrm{cotgh}^2 x \, dx$ **031.** $\int \frac{\cos x}{4+3\sin x} \, dx$ **032.** $\int \frac{1+3\sin x+2\cos x}{\sin 2x} \, dx$ **033.** $\int \frac{dx}{\sin^2 x \cos^2 x}$ **034.** $\int [\mathrm{tg}\, x + \mathrm{cotg}\, x] \, dx$

035. $\int \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ **036.** $\int \frac{dx}{a^2\cos^2 x-b^2\sin^2 x}$ **037.** $\int \frac{dx}{4\cos^2 x+\sin^2 x}$ **038.** $\int \frac{dx}{4\cos^2 x-\sin^2 x}$ **039.** $\int \frac{x^2\, dx}{\sin x^3}$ **040.** $\int x^2 \sin x^3 \, dx$ **041.** $\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}}$ **042.** $\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}}$

043. $\int \frac{\cos x-\sin x}{\cos x+\sin x} \, dx$ **044.** $\int \frac{\ln \cos x}{\sin^2 x} \, dx$ **045.** $\int \frac{dx}{\sin x \cos x}$ **046.** $\int \frac{dx}{\cos x+\sin x}$ **047.** $\int \frac{(\sin x-\cos x)\, dx}{\sqrt[4]{\sin x+\cos x}}$ **048.** $\int \frac{1-\mathrm{tg}\, x}{1+\mathrm{tg}\, x} \, dx$ **049.** $\int \cos^n ax \cdot \sin ax \, dx$ **050.** $\int \frac{\sin ax \, dx}{\cos^n ax}$

051. $\int \sin^n ax \cdot \cos ax \, dx$ **052.** $\int \frac{\cos ax \, dx}{\sin^n ax}$ **053.** $\int \sin ax \cdot \cos bx \, dx$ **054.** $\int \cos ax \cdot \cos bx \, dx$ **055.** $\int \sin ax \cdot \sin bx \, dx$ **056.** $\int \sin ax \cdot \cos ax \, dx$

057. $\int \cos^2 ax \, dx$ **058.** $\int \sin^2 ax \, dx$ **059.** $\int x \, \mathrm{tg}^2 x \, dx$ **060.** $\int x \, \mathrm{cotg}^2 x \, dx$ **061.** $\int x \sin ax \, dx$ **062.** $\int x \cos ax \, dx$ **063.** $\int x^2 \sinh ax \, dx$ **064.** $\int x^2 \sin ax \, dx$

065. $\int x^2 \cosh ax \, dx$ **066.** $\int x^2 \cos ax \, dx$ **067.** $\int x^n \sin ax \, dx$ **068.** $\int x^n \cos ax \, dx$ **069.** $\int x^3 \sin ax \, dx$ **070.** $\int x^3 \cos ax \, dx$ **071.** $\int \sqrt{1+\frac{1}{\sin x}} \, dx$

072. $\int \mathrm{arctg}\, x \, dx$ **073.** $\int x \, \mathrm{arctg}\, x \, dx$ **074.** $\int \ln x \, dx$ **075.** $\int \frac{\ln \mathrm{arctg}\, x}{x^2+1} \, dx$ **076.** $\int \ln (x-1)^5 \, dx$ **077.** $\int \frac{\ln x}{x} \, dx$ **078.** $\int \frac{dx}{x \ln x}$ **079.** $\int x^2 \ln \sqrt{1-x}$ **080.** $\int \frac{\ln x}{\sqrt{x}} \, dx$

081. $\int x \ln x \, dx$ **082.** $\int x^2 \ln x \, dx$ **083.** $\int x^n \ln x \, dx$ **084.** $\int (x+1)^2 \ln (x-1)^5 \, dx$ **085.** $\int x^x (\ln x+1) \, dx$ **086.** $\int x \ln^2 x \, dx$ **087.** $\int \ln (x^2+1) \, dx$

088. $\int \ln \left(\sqrt{1+x}+\sqrt{1-x}\right) dx$ **089.** $\int \ln \left(x+\sqrt{x^2+1}\right) dx$ **090.** $\int x(x-a)(x-b) \, dx$ **091.** $\int |x| \, dx$ **092.** $\int \min_{x\in(0,\infty)} \left\{1, \frac{1}{x}\right\} dx$ **093.** $\int \frac{dx}{5+4e^x}$

094. $\int \frac{dx}{\sqrt{5+4e^x}}$ **095.** $\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$ **096.** $\int \sqrt{\frac{1-e^x}{1+e^x}} \, dx$ **097.** $\int (x+1)e^x \, dx$ **098.** $\int x^2 e^{ax} \, dx$ **099.** $\int x^8 e^{ax} \, dx$ **100.** $\int x^n e^x \, dx$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in (-\pi + 2k\pi; 0 + 2k\pi),\ t \in (-\infty; 0) & \sin x \neq 0 \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2}{t^2+1} & x \in (0 + 2k\pi; \pi + 2k\pi),\ t \in (0; \infty) & x \neq k\pi,\ k \in Z \end{array} \right]$$

$$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \int \frac{\sin x\,dx}{\sin^2 x}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \text{tg}\,\frac{x}{2} & x \in (-\pi + 2k\pi; 0 + 2k\pi),\ t \in (-\infty; 0) & \sin x \neq 0 \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2}{t^2+1} & x \in (0 + 2k\pi; \pi + 2k\pi),\ t \in (0; \infty) & x \neq k\pi,\ k \in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}}$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t = \frac{x}{2} & x \in (0 + k\pi; \pi + k\pi) & \sin x \neq 0,\ k \in Z \\ dt = \frac{dx}{2} & t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) & x \neq k\pi,\ t \neq \frac{k\pi}{2} \end{array}\right]$$

$$= \int \frac{\sin x\, dx}{\sin^2 x} = \int \frac{\sin x\, dx}{1 - \cos^2 x}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg} \frac{x}{2} & x \in (-\pi + 2k\pi; 0 + 2k\pi),\ t \in (-\infty; 0) & \sin x \neq 0 \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2}{t^2+1} & x \in (0 + 2k\pi; \pi + 2k\pi),\ t \in (0; \infty) & x \neq k\pi,\ k \in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \int \frac{dx}{2 \sin \frac{x}{2} \cos \frac{x}{2}} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t = \frac{x}{2} & x \in (0 + k\pi; \pi + k\pi) & \sin x \neq 0,\ k \in Z \\ dt = \frac{dx}{2} & t \in (0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}) & x \neq k\pi,\ t \neq \frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t \cos t}$$

$$= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1 - \cos^2 x} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t = \cos x & x \in (0 + k\pi; \pi + k\pi) & \sin x \neq 0 \\ dt = -\sin x\,dx & t \in (-1; 1) & x \neq k\pi,\ k \in Z \end{array} \right]$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;0+2k\pi),\ t\in(-\infty;0) & \sin x\neq 0 \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2}{t^2+1} & x\in(0+2k\pi;\pi+2k\pi),\ t\in(0;\infty) & x\neq k\pi,\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \ln|t| + c_1$$

$$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0,\ k\in Z \\ dt=\frac{dx}{2} & t\in(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}) & x\neq k\pi,\ t\neq\frac{k\pi}{2} \end{array} \right] = \int \frac{dt}{\sin t\cos t}$$

$$= \int \frac{(\cos^2 t+\sin^2 t)\,dt}{\sin t\cos t}$$

$$= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1-\cos^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\cos x & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0 \\ dt=-\sin x\,dx & t\in(-1;1) & x\neq k\pi,\ k\in Z \end{array} \right] = \int \frac{-\,dt}{1-t^2}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;0+2k\pi),\ t\in(-\infty;0) & \sin x\neq 0 \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2}{t^2+1} & x\in(0+2k\pi;\pi+2k\pi),\ t\in(0;\infty) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \ln|t| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1,\ x\in R-\{k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0,\ k\in Z \\ dt=\frac{dx}{2} & t\in(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}) & x\neq k\pi,\ t\neq\frac{k\pi}{2} \end{array}\right] = \int \frac{dt}{\sin t\cos t}$$

$$= \int \frac{(\cos^2 t+\sin^2 t)\,dt}{\sin t\cos t} = \int \frac{\cos t\,dt}{\sin t} - \int \frac{-\sin t\,dt}{\cos t}$$

$$= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1-\cos^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\cos x & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0 \\ dt=-\sin x\,dx & t\in(-1;1) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1}$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;0+2k\pi),\ t\in(-\infty;0) & \sin x\neq 0 \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2}{t^2+1} & x\in(0+2k\pi;\pi+2k\pi),\ t\in(0;\infty) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \ln|t| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1,\ x\in R-\{k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0,\ k\in Z \\ dt=\frac{dx}{2} & t\in(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}) & x\neq k\pi,\ t\neq\frac{k\pi}{2} \end{array}\right] = \int \frac{dt}{\sin t\cos t}$$

$$= \int \frac{(\cos^2 t+\sin^2 t)\,dt}{\sin t\cos t} = \int \frac{\cos t\,dt}{\sin t} - \int \frac{-\sin t\,dt}{\cos t} = \ln|\sin t| - \ln|\cos t| + c_1$$

$$= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1-\cos^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\cos x & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0 \\ dt=-\sin x\,dx & t\in(-1;1) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1}$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;0+2k\pi),\ t\in(-\infty;0) & \sin x\neq 0 \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2}{t^2+1} & x\in(0+2k\pi;\pi+2k\pi),\ t\in(0;\infty) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \ln|t| + c_1 \ = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1,\ x\in R-\{k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0,\ k\in Z \\ \qquad dt=\frac{dx}{2} & t\in(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}) & x\neq k\pi,\ t\neq\frac{k\pi}{2} \end{array}\right] = \int \frac{dt}{\sin t\cos t}$$

$$= \int \frac{(\cos^2 t+\sin^2 t)\,dt}{\sin t\cos t} = \int \frac{\cos t\,dt}{\sin t} - \int \frac{-\sin t\,dt}{\cos t} = \ln|\sin t| - \ln|\cos t| + c_1$$

$$= \ln|\operatorname{tg} t| + c_1$$

$$= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1-\cos^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\cos x & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0 \\ \quad dt=-\sin x\,dx & t\in(-1;1) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{-\,dt}{1-t^2} = \int \frac{dt}{t^2-1}$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\frac{1-t}{1+t} + c_2$$

Riešené príklady – 001

$$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1 = \frac{1}{2}\ln\frac{1-\cos x}{1+\cos x} + c_2$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;0+2k\pi),\ t\in(-\infty;0) & \sin x\neq 0 \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2}{t^2+1} & x\in(0+2k\pi;\pi+2k\pi),\ t\in(0;\infty) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2t}{t^2+1}} = \int \frac{dt}{t}$$

$$= \ln|t| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1,\ x\in R-\{k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0,\ k\in Z \\ dt=\frac{dx}{2} & t\in(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}) & x\neq k\pi,\ t\neq\frac{k\pi}{2} \end{array}\right] = \int \frac{dt}{\sin t\cos t}$$

$$= \int \frac{(\cos^2 t+\sin^2 t)\,dt}{\sin t\cos t} = \int \frac{\cos t\,dt}{\sin t} - \int \frac{-\sin t\,dt}{\cos t} = \ln|\sin t| - \ln|\cos t| + c_1$$

$$= \ln|\operatorname{tg} t| + c_1 = \ln\left|\operatorname{tg}\frac{x}{2}\right| + c_1,\ x\in R-\{k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{\sin x\,dx}{\sin^2 x} = \int \frac{\sin x\,dx}{1-\cos^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\cos x & x\in(0+k\pi;\pi+k\pi) & \sin x\neq 0 \\ dt=-\sin x\,dx & t\in(-1;1) & x\neq k\pi,\ k\in Z \end{array}\right] = \int \frac{-dt}{1-t^2} = \int \frac{dt}{t^2-1}$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\frac{1-t}{1+t} + c_2 = \frac{1}{2}\ln\frac{1-\cos x}{1+\cos x} + c_2,$$

$$x\in R-\{k\pi,\ k\in Z\},\ c_2\in R.$$

Riešené príklady – 002

$$\int \frac{\mathrm{d}x}{\cos x}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t \in (-\infty; 1) & x \in \left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t \in (1; \infty) \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \in \left(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in (-1; 1) & \cos x \neq 0,\ x \neq \frac{\pi}{2}+k\pi,\ k \in Z,\ t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\cos x\, dx}{\cos^2 x}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t \in (-\infty; 1) & x \in \left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t \in (1; \infty) \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \in \left(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in (-1; 1) & \cos x \neq 0,\ x \neq \frac{\pi}{2}+k\pi,\ k \in Z,\ t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t \in (-\infty; 1) & x \in \left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t \in (1; \infty) \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \in \left(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in (-1; 1) & \cos x \neq 0,\ x \neq \frac{\pi}{2}+k\pi,\ k \in Z,\ t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = -\int \frac{2\,dt}{t^2-1}$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{l|l|l} \text{Subst.} \quad t = \sin x & x \in \left(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi\right) & \cos x \neq 0 \\ dt = \cos x\,dx & t \in (-1; 1) & x \neq \frac{\pi}{2}+k\pi,\ k \in Z \end{array} \right]$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;1) & x \in \left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t\in(1;\infty) \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \in \left(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t\in(-1;1) & \cos x \neq 0,\ x \neq \frac{\pi}{2}+k\pi,\ k\in Z,\ t\neq\pm1 \end{array} \right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2} \ln\left|\frac{t-1}{t+1}\right| + c_1$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t = \sin x & x \in \left(-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi\right) & \cos x \neq 0 \\ dt = \cos x\,dx & t \in (-1;1) & x \neq \frac{\pi}{2}+k\pi,\ k\in Z \end{array} \right]$$

$$= \int \frac{dt}{1-t^2}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;1) & x\in\left(\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(1;\infty) \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(-1;1) & \cos x \neq 0,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ t\neq\pm 1 \end{array}\right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t = \sin x & x\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & \cos x \neq 0 \\ dt = \cos x\,dx & t\in(-1;1) & x\neq\frac{\pi}{2}+k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dt}{1-t^2} = -\int \frac{dt}{t^2-1}$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;1) & x\in\left(\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(1;\infty) \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(-1;1) & \cos x\neq 0,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ t\neq\pm 1 \end{array}\right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1 = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1,$$

$$x\in R - \left\{\frac{\pi}{2}+k\pi,\ \pi+2k\pi,\ k\in Z\right\},\ c_1\in R.$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\sin x & x\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & \cos x\neq 0 \\ dt=\cos x\,dx & t\in(-1;1) & x\neq\frac{\pi}{2}+k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dt}{1-t^2} = -\int \frac{dt}{t^2-1} = -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg} \frac{x}{2} & x \in \left(-\pi + 2k\pi; -\frac{\pi}{2} + 2k\pi\right),\ t \in (-\infty; 1) & x \in \left(\frac{\pi}{2} + 2k\pi; \pi + 2k\pi\right),\ t \in (1; \infty) \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right),\ t \in (-1; 1) & \cos x \neq 0,\ x \neq \frac{\pi}{2} + k\pi,\ k \in Z,\ t \neq \pm 1 \end{array} \right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + c_1 = \ln \left| \frac{t+1}{t-1} \right| + c_1 = \ln \left| \frac{\operatorname{tg} \frac{x}{2} + 1}{\operatorname{tg} \frac{x}{2} - 1} \right| + c_1,$$

$$x \in R - \left\{ \frac{\pi}{2} + k\pi,\ \pi + 2k\pi,\ k \in Z \right\},\ c_1 \in R.$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1 - \sin^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t = \sin x & x \in \left(-\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi\right) & \cos x \neq 0 \\ \quad dt = \cos x\,dx & t \in (-1; 1) & x \neq \frac{\pi}{2} + k\pi,\ k \in Z \end{array} \right]$$

$$= \int \frac{dt}{1-t^2} = -\int \frac{dt}{t^2-1} = -\frac{1}{2} \ln \left| \frac{t-1}{t+1} \right| + c_2 = -\frac{1}{2} \ln \frac{1-t}{1+t} + c_2$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln \left| \frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1} \right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;1) & x\in\left(\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(1;\infty) \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(-1;1) & \cos x\neq 0,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ t\neq\pm 1 \end{array} \right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1 = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1,$$

$$x\in R - \left\{\frac{\pi}{2}+k\pi,\ \pi+2k\pi,\ k\in Z\right\},\ c_1\in R.$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{l|l|l} \text{Subst. } t=\sin x & x\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & \cos x\neq 0 \\ dt=\cos x\,dx & t\in(-1;1) & x\neq\frac{\pi}{2}+k\pi,\ k\in Z \end{array} \right]$$

$$= \int \frac{dt}{1-t^2} = -\int \frac{dt}{t^2-1} = -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = -\frac{1}{2}\ln\frac{1-t}{1+t} + c_2$$

$$= \frac{1}{2}\ln\frac{1+t}{1-t} + c_2$$

Riešené príklady – 002

$$\int \frac{dx}{\cos x} = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;1) & x\in\left(\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(1;\infty) \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(-1;1) & \cos x\neq 0,\ x\neq\frac{\pi}{2}+k\pi,\ k\in Z,\ t\neq\pm 1 \end{array}\right]$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}} = -\int \frac{2\,dt}{t^2-1} = -\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + c_1 = \ln\left|\frac{t+1}{t-1}\right| + c_1 = \ln\left|\frac{\operatorname{tg}\frac{x}{2}+1}{\operatorname{tg}\frac{x}{2}-1}\right| + c_1,$$

$$x\in R - \left\{\frac{\pi}{2}+k\pi,\ \pi+2k\pi,\ k\in Z\right\},\ c_1\in R.$$

$$= \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\sin x & x\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & \cos x\neq 0 \\ dt=\cos x\,dx & t\in(-1;1) & x\neq\frac{\pi}{2}+k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dt}{1-t^2} = -\int \frac{dt}{t^2-1} = -\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = -\frac{1}{2}\ln\frac{1-t}{1+t} + c_2$$

$$= \frac{1}{2}\ln\frac{1+t}{1-t} + c_2 = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} + c_2,\ x\in R - \left\{\frac{\pi}{2}+k\pi,\ k\in Z\right\},\ c_2\in R.$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

Riešené príklady – 003

$$\int \frac{\mathrm{d}x}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x\neq-1,\ k\in Z \\ \mathrm{d}x=\frac{2\,\mathrm{d}t}{t^2+1} & \sin x=\frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(-1;\infty) & x\neq-\frac{\pi}{2}+2k\pi \end{array} \right]$$

$$= \int \frac{(1-\sin x)\,\mathrm{d}x}{(1-\sin x)(1+\sin x)}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t \in (-\infty; -1) & \sin x \neq -1,\ k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} & x \in \left(-\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t \in (-1; \infty) & x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x\neq -1,\ k\in Z \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t\in(-1;\infty) & x\neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}}$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x\neq -1,\ k\in Z \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(-1;\infty) & x\neq-\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2}$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x\neq-1,\ k\in Z \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(-1;\infty) & x\neq-\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst.}\quad u=t+1 & t\in(-\infty;-1),\ u\in(-\infty;0) \\ \qquad\quad du=dt & x\in(-1;\infty),\ t\in(0;\infty) \end{array} \right]$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\cos x & x\in\left\langle 0+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle & x\in\left(\frac{\pi}{2}+2k\pi;\pi+2k\pi\right\rangle,\ t\in\langle-1;0) & \cos x\neq0 \\ \quad dt=-\sin x\,dx & x\in\left\langle\pi+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in\langle-1;0) & x\in\left(\frac{3\pi}{2}+2k\pi;\pi+2k\pi\right\rangle,\ t\in(0;1\rangle & x\neq\frac{\pi}{2}+k\pi,\ k\in Z \end{array} \right]$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x \neq -1,\ k\in Z \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t\in(-1;\infty) & x\neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst.}\quad u = t+1 & t\in(-\infty;-1),\ u\in(-\infty;0) \\ \qquad du = dt & x\in(-1;\infty),\ t\in(0;\infty) \end{array} \right] = 2\int \frac{du}{u^2}$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t = \cos x & x\in\left\langle 0+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle & x\in\left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t\in\langle -1;0) & \cos x\neq 0 \\ \quad dt = -\sin x\,dx & x\in\left\langle \pi+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t\in\langle -1;0) & x\in\left(\frac{3\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t\in(0;1\rangle & x\neq \frac{\pi}{2}+k\pi,\ k\in Z \end{array} \right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2}$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x\neq -1,\ k\in Z \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t\in(-1;\infty) & x\neq -\frac{\pi}{2}+2k\pi \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst.}\quad u=t+1 & t\in(-\infty;-1),\ u\in(-\infty;0) \\ \qquad du=dt & x\in(-1;\infty),\ t\in(0;\infty) \end{array}\right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\cos x & x\in\left\langle 0+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle & x\in\left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t\in\langle -1;0) & \cos x\neq 0 \\ \qquad dt=-\sin x\,dx & x\in\left\langle \pi+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t\in\langle -1;0) & x\in\left(\frac{3\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t\in(0;1\rangle & x\neq\frac{\pi}{2}+k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t \in (-\infty; -1) & \sin x \neq -1,\ k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} & x \in \left(-\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t \in (-1; \infty) & x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{ll|l} \text{Subst.} & u = t+1 & t \in (-\infty; -1),\ u \in (-\infty; 0) \\ & du = dt & x \in (-1; \infty),\ t \in (0; \infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$$

$$= 2\,\frac{u^{-1}}{-1} + c_1$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

$$= \left[\begin{array}{ll|l|l|l} \text{Subst.} & t = \cos x & x \in \left\langle 0+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in (0; 1\rangle & x \in \left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t \in \langle -1; 0) & \cos x \neq 0 \\ & dt = -\sin x\,dx & x \in \left\langle \pi+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in \langle -1; 0) & x \in \left(\frac{3\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t \in (0; 1\rangle & x \neq \frac{\pi}{2}+k\pi,\ k \in Z \end{array} \right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + c_2$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in \left(-\pi+2k\pi; -\frac{\pi}{2}+2k\pi\right),\ t \in (-\infty; -1) & \sin x \neq -1,\ k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \sin x = \frac{2t}{t^2+1} & x \in \left(-\frac{\pi}{2}+2k\pi; \pi+2k\pi\right),\ t \in (-1; \infty) & x \neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst.} \quad u = t+1 & t \in (-\infty; -1),\ u \in (-\infty; 0) \\ \qquad du = dt & x \in (-1; \infty),\ t \in (0; \infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$$

$$= 2\frac{u^{-1}}{-1} + c_1 = c_1 - \frac{2}{u}$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.} \quad t = \cos x & x \in \left\langle 0+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in (0; 1\rangle & x \in \left(\frac{\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t \in \langle -1; 0) & \cos x \neq 0 \\ \quad dt = -\sin x\,dx & x \in \left\langle \pi+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in \langle -1; 0) & x \in \left(\frac{3\pi}{2}+2k\pi; \pi+2k\pi\right\rangle,\ t \in (0; 1\rangle & x \neq \frac{\pi}{2}+k\pi,\ k \in Z \end{array} \right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x - \frac{1}{t} + c_2$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x\neq -1,\ k\in Z \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(-1;\infty) & x\neq -\frac{\pi}{2}+2k\pi \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst.}\quad u=t+1 & t\in(-\infty;-1),\ u\in(-\infty;0) \\ \qquad du=dt & x\in(-1;\infty),\ t\in(0;\infty) \end{array} \right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$$

$$= 2\frac{u^{-1}}{-1} + c_1 = c_1 - \frac{2}{u} = c_1 - \frac{2}{t+1}$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\cos x & x\in\left\langle 0+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle & x\in\left(\frac{\pi}{2}+2k\pi;\pi+2k\pi\right\rangle,\ t\in\langle -1;0) & \cos x\neq 0 \\ \quad dt=-\sin x\,dx & x\in\left\langle \pi+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in\langle -1;0) & x\in\left(\frac{3\pi}{2}+2k\pi;\pi+2k\pi\right\rangle,\ t\in(0;1\rangle & x\neq\frac{\pi}{2}+k\pi,\ k\in Z \end{array} \right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x - \frac{1}{t} + c_2$$

$$= \frac{\sin x}{\cos x} - \frac{1}{\cos x} + c_2$$

Riešené príklady – 003

$$\int \frac{dx}{1+\sin x} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}+1} = \frac{\sin x-1}{\cos x} + c_2$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right),\ t\in(-\infty;-1) & \sin x\neq -1,\ k\in Z \\ dx=\frac{2\,dt}{t^2+1} & \sin x=\frac{2t}{t^2+1} & x\in\left(-\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(-1;\infty) & x\neq -\frac{\pi}{2}+2k\pi \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{2t}{t^2+1}}$$

$$= \int \frac{\frac{2\,dt}{t^2+1}}{\frac{t^2+1+2t}{t^2+1}} = \int \frac{2\,dt}{(t+1)^2} = \left[\begin{array}{l|l} \text{Subst.}\quad u=t+1 & t\in(-\infty;-1),\ u\in(-\infty;0) \\ \qquad du=dt & x\in(-1;\infty),\ t\in(0;\infty) \end{array}\right] = 2\int \frac{du}{u^2} = 2\int u^{-2}\,du$$

$$= 2\frac{u^{-1}}{-1} + c_1 = c_1 - \frac{2}{u} = c_1 - \frac{2}{t+1} = c_1 - \frac{2}{\operatorname{tg}\frac{x}{2}+1},$$

$$x\in R-\left\{-\frac{\pi}{2}+2k\pi,\ \pi+2k\pi,\ k\in Z\right\},\ c_1\in R.$$

$$= \int \frac{(1-\sin x)\,dx}{(1-\sin x)(1+\sin x)} = \int \frac{(1-\sin x)\,dx}{1-\sin^2 x} = \int \frac{(1-\sin x)\,dx}{\cos^2 x} = \int \frac{dx}{\cos^2 x} + \int \frac{-\sin x\,dx}{\cos^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\cos x & x\in\left\langle 0+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle & x\in\left(\frac{\pi}{2}+2k\pi;\pi+2k\pi\right\rangle,\ t\in\langle -1;0) & \cos x\neq 0 \\ \quad dt=-\sin x\,dx & x\in\left\langle \pi+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in\langle -1;0) & x\in\left(\frac{3\pi}{2}+2k\pi;\pi+2k\pi\right\rangle,\ t\in(0;1\rangle & x\neq \frac{\pi}{2}+k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dt}{t^2} = \int \frac{dx}{\cos^2 x} + \int t^{-2}\,dt = \operatorname{tg} x + \frac{t^{-1}}{-1} + c_2 = \operatorname{tg} x - \frac{1}{t} + c_2$$

$$= \frac{\sin x}{\cos x} - \frac{1}{\cos x} + c_2 = \frac{\sin x-1}{\cos x} + c_2,\ x\in R - \left\{\frac{\pi}{2}+k\pi,\ k\in Z\right\},\ c_2\in R.$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{x}{2} & x \in (-\pi+2k\pi; \pi+2k\pi),\ t \in R \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & \cos x \neq -1,\ x \neq \pi+2k\pi,\ k \in Z \end{array} \right]$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}}$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq \pi+2k\pi,\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & x\neq \pi+2k\pi \end{array} \right]$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\text{UGS:} \;\; \begin{array}{l|l|l} dx = \frac{2\,dt}{t^2+1} & t = \operatorname{tg}\frac{x}{2} \\ & \cos x = \frac{1-t^2}{t^2+1} \end{array} \;\Bigg|\; \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi),\ t\in R \\ \cos x \neq -1,\ x \neq \pi+2k\pi,\ k \in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}}$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\text{Subst.} \;\; \begin{array}{r} t = \frac{x}{2} \\ dt = \frac{dx}{2} \end{array} \;\Bigg|\; \begin{array}{l} x \in (-\pi+2k\pi; \pi+2k\pi) \\ t \in (-\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi) \end{array} \;\Bigg|\; \begin{array}{l} \cos x \neq -1,\ k \in Z \\ x \neq \pi+2k\pi \end{array} \right] = \int \frac{dt}{\cos^2 t}$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq \pi+2k\pi,\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi) & x\neq \pi+2k\pi \end{array} \right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq \pi+2k\pi,\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi) & x\neq \pi+2k\pi \end{array} \right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg}\frac{x}{2} + c_1,\ x\in R - \{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\sin x & x\in(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\rangle,\ t\in\langle -1;0) & x\in\langle -\frac{\pi}{2}+2k\pi;0+2k\pi),\ t\in\langle -1;0) & \sin x\neq 0 \\ dt=\cos x\,dx & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi\rangle,\ t\in(0;1\rangle & x\in\langle \frac{\pi}{2}+2k\pi;\pi+2k\pi),\ t\in(0;1\rangle & x\neq k\pi,\ k\in Z \end{array} \right]$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq \pi+2k\pi,\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R-\{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & x\neq \pi+2k\pi \end{array} \right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg}\frac{x}{2} + c_1,\ x\in R-\{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$= \left[\begin{array}{r|l|l|l} \text{Subst.}\quad t=\sin x & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right\rangle,\ t\in\langle -1;0) & x\in\left\langle -\frac{\pi}{2}+2k\pi;0+2k\pi\right),\ t\in\langle -1;0) & \sin x\neq 0 \\ dt=\cos x\,dx & x\in\left(0+2k\pi;\frac{\pi}{2}+2k\pi\right\rangle,\ t\in(0;1\rangle & x\in\left\langle \frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(0;1\rangle & x\neq k\pi,\ k\in Z \end{array} \right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2}$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq\pi+2k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R-\{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & x\neq\pi+2k\pi \end{array}\right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg}\frac{x}{2} + c_1,\ x\in R-\{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\sin x & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right\rangle,\ t\in\langle -1;0) & x\in\left\langle -\frac{\pi}{2}+2k\pi;0+2k\pi\right),\ t\in\langle -1;0) & \sin x\neq 0 \\ dt=\cos x\,dx & x\in\left(0+2k\pi;\frac{\pi}{2}+2k\pi\right\rangle,\ t\in(0;1\rangle & x\in\left\langle\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(0;1\rangle & x\neq k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq \pi+2k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R - \{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi) & x\neq \pi+2k\pi \end{array}\right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg}\frac{x}{2} + c_1,\ x\in R - \{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$= \left[\begin{array}{r|l|l|l} \text{Subst.}\quad t=\sin x & x\in(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\rangle,\ t\in\langle -1;0) & x\in\langle -\frac{\pi}{2}+2k\pi;0+2k\pi),\ t\in\langle -1;0) & \sin x\neq 0 \\ dt=\cos x\,dx & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi\rangle,\ t\in(0;1\rangle & x\in\langle \frac{\pi}{2}+2k\pi;\pi+2k\pi),\ t\in(0;1\rangle & x\neq k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq\pi+2k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R - \{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & x\neq\pi+2k\pi \end{array}\right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg}\frac{x}{2} + c_1,\ x\in R - \{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\sin x & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right\rangle,\ t\in\langle -1;0) & x\in\left\langle -\frac{\pi}{2}+2k\pi;0+2k\pi\right),\ t\in\langle -1;0) & \sin x\neq 0 \\ dt=\cos x\,dx & x\in\left(0+2k\pi;\frac{\pi}{2}+2k\pi\right\rangle,\ t\in(0;1\rangle & x\in\left\langle\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(0;1\rangle & x\neq k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq\pi+2k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R - \{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi) & x\neq\pi+2k\pi \end{array}\right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg}\frac{x}{2} + c_1,\ x\in R - \{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\sin x & x\in(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\rangle,\ t\in\langle -1;0) & x\in\langle -\frac{\pi}{2}+2k\pi;0+2k\pi),\ t\in\langle -1;0) & \sin x\neq 0 \\ dt=\cos x\,dx & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi\rangle,\ t\in(0;1\rangle & x\in\langle\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ t\in(0;1\rangle & x\neq k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} + c_2$$

Riešené príklady – 004

$$\int \frac{dx}{1+\cos x} = \operatorname{tg}\frac{x}{2} + c_1 = \frac{1-\cos x}{\sin x} + c_2$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & \cos x\neq -1,\ x\neq\pi+2k\pi,\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{2}{t^2+1}} = \int dt$$

$$= t + c_1 = \operatorname{tg}\frac{x}{2} + c_1,\ x\in R-\{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{dx}{2\cos^2\frac{x}{2}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=\frac{x}{2} & x\in(-\pi+2k\pi;\pi+2k\pi) & \cos x\neq -1,\ k\in Z \\ dt=\frac{dx}{2} & t\in\left(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi\right) & x\neq\pi+2k\pi \end{array}\right] = \int \frac{dt}{\cos^2 t} = \operatorname{tg} t + c_1$$

$$= \operatorname{tg}\frac{x}{2} + c_1,\ x\in R-\{\pi+2k\pi,\ k\in Z\},\ c_1\in R.$$

$$= \int \frac{(1-\cos x)\,dx}{(1-\cos x)(1+\cos x)} = \int \frac{(1-\cos x)\,dx}{1-\cos^2 x} = \int \frac{(1-\cos x)\,dx}{\sin^2 x} = \int \frac{dx}{\sin^2 x} - \int \frac{\cos x\,dx}{\sin^2 x}$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=\sin x & x\in\left(-\pi+2k\pi;-\frac{\pi}{2}+2k\pi\right\rangle,\ t\in\langle -1;0) & x\in\left\langle-\frac{\pi}{2}+2k\pi;0+2k\pi\right),\ t\in\langle -1;0) & \sin x\neq 0 \\ dt=\cos x\,dx & x\in\left(0+2k\pi;\frac{\pi}{2}+2k\pi\right\rangle,\ t\in(0;1\rangle & x\in\left\langle\frac{\pi}{2}+2k\pi;\pi+2k\pi\right),\ t\in(0;1\rangle & x\neq k\pi,\ k\in Z \end{array}\right]$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dt}{t^2} = \int \frac{dx}{\sin^2 x} - \int t^{-2}\,dt = -\operatorname{cotg} x - \frac{t^{-1}}{-1} + c_2 = \frac{1}{t} - \operatorname{cotg} x + c_2$$

$$= \frac{1}{\sin x} - \frac{\cos x}{\sin x} + c_2 = \frac{1-\cos x}{\sin x} + c_2,\ x\in R-\{\pi+k\pi,\ k\in Z\},\ c_2\in R.$$

Riešené príklady – 005

$$\int \frac{\mathrm{d}x}{\sinh x}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0),\ t \in (-1; 0) & \sinh x \neq 0 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (0; \infty),\ t \in (0; 1) & x \neq 0 \end{array} \right]$$

$$= \int \frac{\sinh x\, dx}{\sinh^2 x}$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0),\ t \in (-1; 0) & \sinh x \neq 0 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (0; \infty),\ t \in (0; 1) & x \neq 0 \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2t}{1-t^2}}$$

$$= \int \frac{\sinh x\,dx}{\sinh^2 x} = \int \frac{\sinh x\,dx}{\cosh^2 x - 1}$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2\,dx}{e^x - e^{-x}}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0),\ t \in (-1; 0) & \sinh x \neq 0 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (0; \infty),\ t \in (0; 1) & x \neq 0 \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2t}{1-t^2}} = \int \frac{dt}{t}$$

$$= \int \frac{\sinh x\,dx}{\sinh^2 x} = \int \frac{\sinh x\,dx}{\cosh^2 x - 1} = \left[\begin{array}{l|l|l} \text{Subst. } t = \cosh x & x \in (-\infty; 0),\ t \in (1; \infty) & \sinh x \neq 0 \\ dt = \sinh x\,dx & x \in (0; \infty),\ t \in (1; \infty) & x \neq 0 \end{array} \right]$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2\,dx}{e^x - e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst. } t = e^x & x = \ln t & x \in (-\infty; 0),\ t \in (0; 1) & \sinh x \neq 0 \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & x \in (0; \infty),\ t \in (1; \infty) & x \neq 0 \end{array} \right]$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in (-\infty; 0),\ t \in (-1; 0) & \sinh x \neq 0 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (0; \infty),\ t \in (0; 1) & x \neq 0 \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2t}{1-t^2}} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \int \frac{\sinh x\,dx}{\sinh^2 x} = \int \frac{\sinh x\,dx}{\cosh^2 x - 1} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t = \cosh x & x \in (-\infty; 0),\ t \in (1; \infty) & \sinh x \neq 0 \\ dt = \sinh x\,dx & x \in (0; \infty),\ t \in (1; \infty) & x \neq 0 \end{array} \right] = \int \frac{dt}{t^2 - 1}$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2\,dx}{e^x - e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t = e^x & x = \ln t & x \in (-\infty; 0),\ t \in (0; 1) & \sinh x \neq 0 \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & x \in (0; \infty),\ t \in (1; \infty) & x \neq 0 \end{array} \right] = \int \frac{2\,dt}{t(t - \frac{1}{t})}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x} = \ln\left|\operatorname{tgh}\frac{x}{2}\right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in (-\infty; 0),\ t \in (-1; 0) & \sinh x \neq 0 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (0; \infty),\ t \in (0; 1) & x \neq 0 \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2t}{1-t^2}} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \ln\left|\operatorname{tgh}\frac{x}{2}\right| + c_1,\ x \in R - \{0\},\ c_1 \in R.$$

$$= \int \frac{\sinh x\,dx}{\sinh^2 x} = \int \frac{\sinh x\,dx}{\cosh^2 x - 1} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t = \cosh x & x \in (-\infty; 0),\ t \in (1; \infty) & \sinh x \neq 0 \\ dt = \sinh x\,dx & x \in (0; \infty),\ t \in (1; \infty) & x \neq 0 \end{array}\right] = \int \frac{dt}{t^2 - 1}$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2\,dx}{e^x - e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t = e^x & x = \ln t & x \in (-\infty; 0),\ t \in (0; 1) & \sinh x \neq 0 \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & x \in (0; \infty),\ t \in (1; \infty) & x \neq 0 \end{array}\right] = \int \frac{2\,dt}{t(t - \frac{1}{t})}$$

$$= \int \frac{2\,dt}{t^2 - 1}$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x} = \ln\left|\operatorname{tgh}\frac{x}{2}\right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in(-\infty;0),\ t\in(-1;0) & \sinh x\neq 0 \\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in(0;\infty),\ t\in(0;1) & x\neq 0 \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2t}{1-t^2}} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \ln\left|\operatorname{tgh}\frac{x}{2}\right| + c_1,\ x\in R-\{0\},\ c_1\in R.$$

$$= \int \frac{\sinh x\,dx}{\sinh^2 x} = \int \frac{\sinh x\,dx}{\cosh^2 x - 1} = \left[\begin{array}{r|l|l} \text{Subst.}\quad t=\cosh x & x\in(-\infty;0),\ t\in(1;\infty) & \sinh x\neq 0 \\ dt=\sinh x\,dx & x\in(0;\infty),\ t\in(1;\infty) & x\neq 0 \end{array}\right] = \int \frac{dt}{t^2-1}$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\frac{t-1}{t+1} + c_2$$

$$= \int \frac{dx}{\frac{e^x - e^{-x}}{2}} = \int \frac{2\,dx}{e^x - e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in(-\infty;0),\ t\in(0;1) & \sinh x\neq 0 \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in(0;\infty),\ t\in(1;\infty) & x\neq 0 \end{array}\right] = \int \frac{2\,dt}{t(t-\frac{1}{t})}$$

$$= \int \frac{2\,dt}{t^2-1} = \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_3$$

Riešené príklady – 005

$$\int \frac{dx}{\sinh x} = \ln\left|\operatorname{tgh}\frac{x}{2}\right| + c_1 = \frac{1}{2}\ln\frac{\cosh x-1}{\cosh x+1} + c_2 = \frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right| + c_3$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in(-\infty;0),\ t\in(-1;0) & \sinh x\neq 0 \\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in(0;\infty),\ t\in(0;1) & x\neq 0 \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2t}{1-t^2}} = \int \frac{dt}{t} = \ln t + c_1$$

$$= \ln\left|\operatorname{tgh}\frac{x}{2}\right| + c_1,\ x\in R-\{0\},\ c_1\in R.$$

$$= \int \frac{\sinh x\,dx}{\sinh^2 x} = \int \frac{\sinh x\,dx}{\cosh^2 x-1} = \left[\begin{array}{l|l|l} \text{Subst. } t=\cosh x & x\in(-\infty;0),\ t\in(1;\infty) & \sinh x\neq 0 \\ dt=\sinh x\,dx & x\in(0;\infty),\ t\in(1;\infty) & x\neq 0 \end{array}\right] = \int \frac{dt}{t^2-1}$$

$$= \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_2 = \frac{1}{2}\ln\frac{t-1}{t+1} + c_2 = \frac{1}{2}\ln\frac{\cosh x-1}{\cosh x+1} + c_2,\ x\in R-\{0\},\ c_2\in R.$$

$$= \int \frac{dx}{\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst. } t=e^x & x=\ln t & x\in(-\infty;0),\ t\in(0;1) & \sinh x\neq 0 \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in(0;\infty),\ t\in(1;\infty) & x\neq 0 \end{array}\right] = \int \frac{2\,dt}{t(t-\frac{1}{t})}$$

$$= \int \frac{2\,dt}{t^2-1} = \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + c_3 = \frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right| + c_3,\ x\in R-\{0\},\ c_3\in R.$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0\rangle,\ t \in (-1; 0\rangle \\ dx = \frac{2\,dt}{1-t^2} & \cos x = \frac{1+t^2}{1-t^2} & x \in \langle 0; \infty),\ t \in \langle 0; 1) \end{array} \right]$$

$$= \int \frac{\cosh x \, dx}{\cosh^2 x}$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0),\ t \in (-1; 0) \\ dx = \frac{2\,dt}{1-t^2} & \cos x = \frac{1+t^2}{1-t^2} & x \in \langle 0; \infty),\ t \in \langle 0; 1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1+t^2}{1-t^2}}$$

$$= \int \frac{\cosh x\,dx}{\cosh^2 x} = \int \frac{\cosh x\,dx}{\sinh^2 x + 1}$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2\,dx}{e^x + e^{-x}}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in (-\infty; 0\rangle,\ t \in (-1; 0\rangle \\ dx = \frac{2\,dt}{1-t^2} & \cos x = \frac{1+t^2}{1-t^2} & x \in \langle 0; \infty),\ t \in \langle 0; 1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1+t^2}{1-t^2}} = \int \frac{2\,dt}{t^2+1}$$

$$= \int \frac{\cosh x\,dx}{\cosh^2 x} = \int \frac{\cosh x\,dx}{\sinh^2 x + 1} = \left[\begin{array}{l|l} \text{Subst.}\quad t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array} \right]$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2\,dx}{e^x + e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t = e^x & x = \ln t & x \in R \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & t \in (0; \infty) \end{array} \right]$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in (-\infty; 0\rangle,\ t \in (-1; 0\rangle \\ dx = \frac{2\,dt}{1-t^2} & \cos x = \frac{1+t^2}{1-t^2} & x \in \langle 0; \infty),\ t \in \langle 0; 1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1+t^2}{1-t^2}} = \int \frac{2\,dt}{t^2+1} = 2 \operatorname{arctg} t + c_1$$

$$= \int \frac{\cosh x\,dx}{\cosh^2 x} = \int \frac{\cosh x\,dx}{\sinh^2 x + 1} = \left[\begin{array}{r|l} \text{Subst.}\quad t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array} \right] = \int \frac{dt}{t^2+1}$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2\,dx}{e^x + e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t = e^x & x = \ln t & x \in R \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & t \in (0; \infty) \end{array} \right] = \int \frac{2\,dt}{t(t+\frac{1}{t})}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x} = 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; 0\rangle,\ t \in (-1; 0\rangle \\ dx = \frac{2\,dt}{1-t^2} & \cos x = \frac{1+t^2}{1-t^2} & x \in \langle 0; \infty),\ t \in \langle 0; 1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1+t^2}{1-t^2}} = \int \frac{2\,dt}{t^2+1} = 2 \operatorname{arctg} t + c_1$$

$$= 2 \operatorname{arctg} \operatorname{tgh} \frac{x}{2} + c_1,\ x \in R,\ c_1 \in R.$$

$$= \int \frac{\cosh x\,dx}{\cosh^2 x} = \int \frac{\cosh x\,dx}{\sinh^2 x + 1} = \left[\begin{array}{l|l} \text{Subst.}\quad t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array} \right] = \int \frac{dt}{t^2+1} = \operatorname{arctg} t + c_2$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2\,dx}{e^x + e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t = e^x & x = \ln t & x \in R \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & t \in (0; \infty) \end{array} \right] = \int \frac{2\,dt}{t(t + \frac{1}{t})} = \int \frac{2\,dt}{t^2+1}$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x} = 2\,\mathrm{arctg}\,\mathrm{tgh}\,\frac{x}{2} + c_1 = \mathrm{arctg}\sinh x + c_2$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \mathrm{tgh}\,\frac{x}{2} & x \in (-\infty; 0\rangle,\ t \in (-1; 0\rangle \\ dx = \frac{2\,dt}{1-t^2} & \cos x = \frac{1+t^2}{1-t^2} & x \in \langle 0; \infty),\ t \in \langle 0; 1) \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1+t^2}{1-t^2}} = \int \frac{2\,dt}{t^2+1} = 2\,\mathrm{arctg}\,t + c_1$$

$$= 2\,\mathrm{arctg}\,\mathrm{tgh}\,\frac{x}{2} + c_1,\ x \in R,\ c_1 \in R.$$

$$= \int \frac{\cosh x\,dx}{\cosh^2 x} = \int \frac{\cosh x\,dx}{\sinh^2 x + 1} = \left[\begin{array}{l|l} \text{Subst.}\quad t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array}\right] = \int \frac{dt}{t^2+1} = \mathrm{arctg}\,t + c_2$$

$$= \mathrm{arctg}\sinh x + c_2,\ x \in R,\ c_2 \in R.$$

$$= \int \frac{dx}{\frac{e^x + e^{-x}}{2}} = \int \frac{2\,dx}{e^x + e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t = e^x & x = \ln t & x \in R \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & t \in (0; \infty) \end{array}\right] = \int \frac{2\,dt}{t(t+\frac{1}{t})} = \int \frac{2\,dt}{t^2+1}$$

$$= 2\,\mathrm{arctg}\,t + c_3$$

Riešené príklady – 006

$$\int \frac{dx}{\cosh x} = 2\,\mathrm{arctg}\,\mathrm{tgh}\,\frac{x}{2} + c_1 = \mathrm{arctg}\,\sinh x + c_2 = 2\,\mathrm{arctg}\,e^x + c_3$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t=\mathrm{tgh}\,\frac{x}{2} & x\in(-\infty;0\rangle,\ t\in(-1;0\rangle \\ dx=\frac{2\,dt}{1-t^2} & \cos x=\frac{1+t^2}{1-t^2} & x\in\langle 0;\infty),\ t\in\langle 0;1) \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1+t^2}{1-t^2}} = \int \frac{2\,dt}{t^2+1} = 2\,\mathrm{arctg}\,t + c_1$$

$$= 2\,\mathrm{arctg}\,\mathrm{tgh}\,\frac{x}{2} + c_1,\ x\in R,\ c_1\in R.$$

$$= \int \frac{\cosh x\,dx}{\cosh^2 x} = \int \frac{\cosh x\,dx}{\sinh^2 x + 1} = \left[\begin{array}{l|l} \text{Subst.}\quad t=\sinh x & x\in R \\ dt=\cosh x\,dx & t\in R \end{array}\right] = \int \frac{dt}{t^2+1} = \mathrm{arctg}\,t + c_2$$

$$= \mathrm{arctg}\,\sinh x + c_2,\ x\in R,\ c_2\in R.$$

$$= \int \frac{dx}{\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{e^x+e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in R \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{2\,dt}{t(t+\frac{1}{t})} = \int \frac{2\,dt}{t^2+1}$$

$$= 2\,\mathrm{arctg}\,t + c_3 = 2\,\mathrm{arctg}\,e^x + c_3,\ x\in R,\ c_3\in R.$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in (-\infty; \ln(\sqrt{2}-1)),\ t \in (-1; \sqrt{2}-1) & \sinh x \neq -1 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (\ln(\sqrt{2}-1); \infty),\ t \in (\sqrt{2}-1; 1) & x \neq \ln(\sqrt{2}-1) \end{array} \right]$$

$$= \int \frac{dx}{1+\frac{e^x - e^{-x}}{2}}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in \left(-\infty; \ln\left(\sqrt{2}-1\right)\right),\ t \in \left(-1; \sqrt{2}-1\right) & \sinh x \neq -1 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in \left(\ln\left(\sqrt{2}-1\right); \infty\right),\ t \in \left(\sqrt{2}-1; 1\right) & x \neq \ln\left(\sqrt{2}-1\right) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{dx}{1+\frac{e^x - e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x - e^{-x}}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in \left(-\infty; \ln\left(\sqrt{2}-1\right)\right),\ t \in \left(-1; \sqrt{2}-1\right) & \sinh x \neq -1 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in \left(\ln\left(\sqrt{2}-1\right); \infty\right),\ t \in \left(\sqrt{2}-1; 1\right) & x \neq \ln\left(\sqrt{2}-1\right) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}}$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst. } t = e^x & x = \ln t & x \in \left(-\infty; \ln\left(\sqrt{2}-1\right)\right),\ t \in \left(0; \sqrt{2}-1\right) & \sinh x \neq -1 \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & x \in \left(\ln\left(\sqrt{2}-1\right); \infty\right),\ t \in \left(\sqrt{2}-1; \infty\right) & x \neq \ln\left(\sqrt{2}-1\right) \end{array} \right]$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in\left(-\infty;\ln\left(\sqrt{2}-1\right)\right),\ t\in\left(-1;\sqrt{2}-1\right) & \sinh x\neq -1 \\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in\left(\ln\left(\sqrt{2}-1\right);\infty\right),\ t\in\left(\sqrt{2}-1;1\right) & x\neq \ln\left(\sqrt{2}-1\right) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2\,dt}{t^2-2t-1}$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in\left(-\infty;\ln\left(\sqrt{2}-1\right)\right),\ t\in\left(0;\sqrt{2}-1\right) & \sinh x\neq -1 \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in\left(\ln\left(\sqrt{2}-1\right);\infty\right),\ t\in\left(\sqrt{2}-1;\infty\right) & x\neq \ln\left(\sqrt{2}-1\right) \end{array} \right]$$

$$= \int \frac{2\,dt}{t(2+t-\frac{1}{t})}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in\left(-\infty;\ln\left(\sqrt{2}-1\right)\right),\ t\in\left(-1;\sqrt{2}-1\right) & \sinh x\neq -1 \\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in\left(\ln\left(\sqrt{2}-1\right);\infty\right),\ t\in\left(\sqrt{2}-1;1\right) & x\neq\ln\left(\sqrt{2}-1\right) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2\,dt}{t^2-2t-1} = \int \frac{-2\,dt}{(t-1)^2-2}$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in\left(-\infty;\ln\left(\sqrt{2}-1\right)\right),\ t\in\left(0;\sqrt{2}-1\right) & \sinh x\neq -1 \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in\left(\ln\left(\sqrt{2}-1\right);\infty\right),\ t\in\left(\sqrt{2}-1;\infty\right) & x\neq\ln\left(\sqrt{2}-1\right) \end{array} \right]$$

$$= \int \frac{2\,dt}{t(2+t-\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t-1}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in(-\infty;\ln(\sqrt{2}-1)),\ t\in(-1;\sqrt{2}-1) & \sinh x\neq -1 \\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in(\ln(\sqrt{2}-1);\infty),\ t\in(\sqrt{2}-1;1) & x\neq\ln(\sqrt{2}-1) \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2\,dt}{t^2-2t-1} = \int \frac{-2\,dt}{(t-1)^2-2} = -\int \frac{2\,dt}{(t-1)^2-(\sqrt{2})^2}$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in(-\infty;\ln(\sqrt{2}-1)),\ t\in(0;\sqrt{2}-1) & \sinh x\neq -1 \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in(\ln(\sqrt{2}-1);\infty),\ t\in(\sqrt{2}-1;\infty) & x\neq\ln(\sqrt{2}-1) \end{array}\right]$$

$$= \int \frac{2\,dt}{t(2+t-\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t-1} = \int \frac{2\,dt}{(t+1)^2-2}$$

Riešené príklady – 007

$$\int\frac{dx}{1+\sinh x}$$

$$=\left[\begin{array}{l|l|l|l}\text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in\left(-\infty;\ln\left(\sqrt{2}-1\right)\right),\ t\in\left(-1;\sqrt{2}-1\right) & \sinh x\neq -1\\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in\left(\ln\left(\sqrt{2}-1\right);\infty\right),\ t\in\left(\sqrt{2}-1;1\right) & x\neq\ln\left(\sqrt{2}-1\right)\end{array}\right]=\int\frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$=\int\frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}}=\int\frac{-2\,dt}{t^2-2t-1}=\int\frac{-2\,dt}{(t-1)^2-2}=-\int\frac{2\,dt}{(t-1)^2-(\sqrt{2})^2}=-\frac{2}{2\sqrt{2}}\ln\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right|+c_1$$

$$=\int\frac{dx}{1+\frac{e^x-e^{-x}}{2}}=\int\frac{2\,dx}{2+e^x-e^{-x}}=\left[\begin{array}{l|l|l|l}\text{Subst.}\quad t=e^x & x=\ln t & x\in\left(-\infty;\ln\left(\sqrt{2}-1\right)\right),\ t\in\left(0;\sqrt{2}-1\right) & \sinh x\neq -1\\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in\left(\ln\left(\sqrt{2}-1\right);\infty\right),\ t\in\left(\sqrt{2}-1;\infty\right) & x\neq\ln\left(\sqrt{2}-1\right)\end{array}\right]$$

$$=\int\frac{2\,dt}{t(2+t-\frac{1}{t})}=\int\frac{2\,dt}{t^2+2t-1}=\int\frac{2\,dt}{(t+1)^2-2}=\int\frac{2\,dt}{(t+1)^2-(\sqrt{2})^2}$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x}$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in(-\infty;\ln(\sqrt{2}-1)),\ t\in(-1;\sqrt{2}-1) & \sinh x\neq -1 \\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in(\ln(\sqrt{2}-1);\infty),\ t\in(\sqrt{2}-1;1) & x\neq \ln(\sqrt{2}-1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2\,dt}{t^2-2t-1} = \int \frac{-2\,dt}{(t-1)^2-2} = -\int \frac{2\,dt}{(t-1)^2-(\sqrt{2})^2} = -\frac{2}{2\sqrt{2}} \ln\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right| + c_1$$

$$= \frac{1}{\sqrt{2}} \ln\left|\frac{t-1+\sqrt{2}}{t-1-\sqrt{2}}\right| + c_1$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\ \ t=e^x & x=\ln t & x\in(-\infty;\ln(\sqrt{2}-1)),\ t\in(0;\sqrt{2}-1) & \sinh x\neq -1 \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in(\ln(\sqrt{2}-1);\infty),\ t\in(\sqrt{2}-1;\infty) & x\neq \ln(\sqrt{2}-1) \end{array} \right]$$

$$= \int \frac{2\,dt}{t(2+t-\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t-1} = \int \frac{2\,dt}{(t+1)^2-2} = \int \frac{2\,dt}{(t+1)^2-(\sqrt{2})^2} = \frac{2}{2\sqrt{2}} \ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c_2$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x} = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tgh} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tgh} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_1$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in (-\infty; \ln(\sqrt{2}-1)),\ t \in (-1; \sqrt{2}-1) & \sinh x \neq -1 \\ dx = \frac{2\,dt}{1-t^2} & \sinh x = \frac{2t}{1-t^2} & x \in (\ln(\sqrt{2}-1); \infty),\ t \in (\sqrt{2}-1; 1) & x \neq \ln(\sqrt{2}-1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2\,dt}{t^2-2t-1} = \int \frac{-2\,dt}{(t-1)^2-2} = -\int \frac{2\,dt}{(t-1)^2-(\sqrt{2})^2} = -\frac{2}{2\sqrt{2}} \ln \left| \frac{t-1-\sqrt{2}}{t-1+\sqrt{2}} \right| + c_1$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{t-1+\sqrt{2}}{t-1-\sqrt{2}} \right| + c_1 = \frac{\sqrt{2}}{2} \ln \left| \frac{\operatorname{tgh} \frac{x}{2} - 1 + \sqrt{2}}{\operatorname{tgh} \frac{x}{2} - 1 - \sqrt{2}} \right| + c_1,\ x \in R - \{\ln(\sqrt{2}-1)\},\ c_1 \in R.$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst. } t = e^x & x = \ln t & x \in (-\infty; \ln(\sqrt{2}-1)),\ t \in (0; \sqrt{2}-1) & \sinh x \neq -1 \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & x \in (\ln(\sqrt{2}-1); \infty),\ t \in (\sqrt{2}-1; \infty) & x \neq \ln(\sqrt{2}-1) \end{array} \right]$$

$$= \int \frac{2\,dt}{t(2+t-\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t-1} = \int \frac{2\,dt}{(t+1)^2-2} = \int \frac{2\,dt}{(t+1)^2-(\sqrt{2})^2} = \frac{2}{2\sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + c_2$$

$$= \frac{1}{\sqrt{2}} \ln \left| \frac{t+1-\sqrt{2}}{t+1+\sqrt{2}} \right| + c_2$$

Riešené príklady – 007

$$\int \frac{dx}{1+\sinh x} = \frac{\sqrt{2}}{2}\ln\left|\frac{\operatorname{tgh}\frac{x}{2}-1+\sqrt{2}}{\operatorname{tgh}\frac{x}{2}-1-\sqrt{2}}\right| + c_1 = \frac{\sqrt{2}}{2}\ln\left|\frac{e^x+1-\sqrt{2}}{e^x+1+\sqrt{2}}\right| + c_2$$

$$= \left[\begin{array}{l|l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in(-\infty;\ln(\sqrt{2}-1)),\ t\in(-1;\sqrt{2}-1) & \sinh x\neq -1 \\ dx=\frac{2\,dt}{1-t^2} & \sinh x=\frac{2t}{1-t^2} & x\in(\ln(\sqrt{2}-1);\infty),\ t\in(\sqrt{2}-1;1) & x\neq\ln(\sqrt{2}-1) \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{2t}{1-t^2}}$$

$$= \int \frac{\frac{2\,dt}{1-t^2}}{\frac{1-t^2+2t}{1-t^2}} = \int \frac{-2\,dt}{t^2-2t-1} = \int \frac{-2\,dt}{(t-1)^2-2} = -\int \frac{2\,dt}{(t-1)^2-(\sqrt{2})^2} = -\frac{2}{2\sqrt{2}}\ln\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right| + c_1$$

$$= \frac{1}{\sqrt{2}}\ln\left|\frac{t-1+\sqrt{2}}{t-1-\sqrt{2}}\right| + c_1 = \frac{\sqrt{2}}{2}\ln\left|\frac{\operatorname{tgh}\frac{x}{2}-1+\sqrt{2}}{\operatorname{tgh}\frac{x}{2}-1-\sqrt{2}}\right| + c_1,\ x\in R-\{\ln(\sqrt{2}-1)\},\ c_1\in R.$$

$$= \int \frac{dx}{1+\frac{e^x-e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x-e^{-x}} = \left[\begin{array}{l|l|l|l} \text{Subst.}\ t=e^x & x=\ln t & x\in(-\infty;\ln(\sqrt{2}-1)),\ t\in(0;\sqrt{2}-1) & \sinh x\neq -1 \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & x\in(\ln(\sqrt{2}-1);\infty),\ t\in(\sqrt{2}-1;\infty) & x\neq\ln(\sqrt{2}-1) \end{array}\right]$$

$$= \int \frac{2\,dt}{t(2+t-\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t-1} = \int \frac{2\,dt}{(t+1)^2-2} = \int \frac{2\,dt}{(t+1)^2-(\sqrt{2})^2} = \frac{2}{2\sqrt{2}}\ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c_2$$

$$= \frac{1}{\sqrt{2}}\ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c_2 = \frac{\sqrt{2}}{2}\ln\left|\frac{e^x+1-\sqrt{2}}{e^x+1+\sqrt{2}}\right| + c_2,\ x\in R-\{\ln(\sqrt{2}-1)\},\ c_2\in R.$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh} \frac{x}{2} & x \in R \\ dx = \frac{2\,dt}{1-t^2} & \cosh x = \frac{1+t^2}{1-t^2} & t \in (-1;1) \end{array} \right]$$

$$= \int \frac{dx}{2\cosh^2 \frac{x}{2}}$$

$$= \int \frac{\cosh x \, dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x}$$

$$= \int \frac{dx}{1+\frac{e^x + e^{-x}}{2}}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in R \\ dx=\frac{2\,dt}{1-t^2} & \cosh x=\frac{1+t^2}{1-t^2} & t\in(-1;1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}}$$

$$= \int \frac{dx}{2\cosh^2\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\frac{x}{2} & x\in R \\ dt=\frac{dx}{2} & t\in R \end{array} \right]$$

$$= \int \frac{\cosh x\,dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\sinh x & x\in(-\infty;0),\ t\in(-\infty;0) \\ dt=\cosh x\,dx & x\in(0;\infty),\ t\in(0;\infty) \end{array} \right]$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\text{UHS:} \;\; dx = \frac{2\,dt}{1-t^2} \;\middle|\; t = \operatorname{tgh}\frac{x}{2} \;\; \cosh x = \frac{1+t^2}{1-t^2} \;\middle|\; x \in R \;\; t \in (-1;1) \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}} = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2}{1-t^2}}$$

$$= \int \frac{dx}{2\cosh^2 \frac{x}{2}} = \left[\text{Subst.} \;\; t = \frac{x}{2} \;\; dt = \frac{dx}{2} \;\middle|\; x \in R \;\; t \in R \right] = \int \frac{dt}{\cosh^2 t}$$

$$= \int \frac{\cosh x\, dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\text{Subst.} \;\; t = \sinh x \;\; dt = \cosh x\, dx \;\middle|\; x \in (-\infty;0),\; t \in (-\infty;0) \;\; x \in (0;\infty),\; t \in (0;\infty) \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}} = \left[\text{Subst.} \;\; t = e^x \;\; e^{-x} = t^{-1} = \frac{1}{t} \;\middle|\; x = \ln t \;\; dx = \frac{dt}{t} \;\middle|\; x \in R \;\; t \in (0;\infty) \right]$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t = \operatorname{tgh}\frac{x}{2} & x \in R \\ dx = \frac{2\,dt}{1-t^2} & \cosh x = \frac{1+t^2}{1-t^2} & t \in (-1;1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}} = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2}{1-t^2}} = \int dt$$

$$= \int \frac{dx}{2\cosh^2 \frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.} \quad t = \frac{x}{2} & x \in R \\ dt = \frac{dx}{2} & t \in R \end{array} \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1$$

$$= \int \frac{\cosh x\,dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{r|l} \text{Subst.} \quad t = \sinh x & x \in (-\infty;0),\ t \in (-\infty;0) \\ dt = \cosh x\,dx & x \in (0;\infty),\ t \in (0;\infty) \end{array} \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \frac{1}{-t} - \operatorname{cotgh} x + c_2$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.} \quad t = e^x & x = \ln t & x \in R \\ e^{-x} = t^{-1} = \frac{1}{t} & dx = \frac{dt}{t} & t \in (0;\infty) \end{array} \right] = \int \frac{2\,dt}{t(2+t+\frac{1}{t})}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh} \frac{x}{2} + c_1$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in R \\ dx=\frac{2\,dt}{1-t^2} & \cosh x=\frac{1+t^2}{1-t^2} & t\in(-1;1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}} = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2}{1-t^2}} = \int dt = t + c_1$$

$$= \int \frac{dx}{2\cosh^2 \frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\frac{x}{2} & x\in R \\ dt=\frac{dx}{2} & t\in R \end{array} \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh}\frac{x}{2} + c_1,\ x\in R,\ c_1\in R.$$

$$= \int \frac{\cosh x\,dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\sinh x & x\in(-\infty;0),\ t\in(-\infty;0) \\ dt=\cosh x\,dx & x\in(0;\infty),\ t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x}$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in R \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{2\,dt}{t(2+t+\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t+1}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh}\frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in R \\ dx=\frac{2\,dt}{1-t^2} & \cosh x=\frac{1+t^2}{1-t^2} & t\in(-1;1) \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}} = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2}{1-t^2}} = \int dt = t+c_1 = \operatorname{tgh}\frac{x}{2}+c_1, \quad x\in R,\ c_1\in R.$$

$$= \int \frac{dx}{2\cosh^2\frac{x}{2}} = \left[\begin{array}{l|l} \text{Subst. } t=\frac{x}{2} & x\in R \\ dt=\frac{dx}{2} & t\in R \end{array}\right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh}\frac{x}{2}+c_1, \quad x\in R,\ c_1\in R.$$

$$= \int \frac{\cosh x\,dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{l|l} \text{Subst. } t=\sinh x & x\in(-\infty;0),\ t\in(-\infty;0) \\ dt=\cosh x\,dx & x\in(0;\infty),\ t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x}, \quad x\in R-\{0\},\ c_2\in R.$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst. } t=e^x & x=\ln t & x\in R \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{2\,dt}{t(2+t+\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{2\,dt}{(t+1)^2}$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh}\frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in R \\ dx=\frac{2\,dt}{1-t^2} & \cosh x=\frac{1+t^2}{1-t^2} & t\in(-1;1) \end{array} \right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}} = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2}{1-t^2}} = \int dt = t+c_1 = \operatorname{tgh}\frac{x}{2}+c_1, \quad x\in R,\ c_1\in R.$$

$$= \int \frac{dx}{2\cosh^2\frac{x}{2}} = \left[\begin{array}{l|l} \text{Subst. } t=\frac{x}{2} & x\in R \\ dt=\frac{dx}{2} & t\in R \end{array} \right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh}\frac{x}{2}+c_1, \quad x\in R,\ c_1\in R.$$

$$= \int \frac{\cosh x\,dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{l|l} \text{Subst. } t=\sinh x & x\in(-\infty;0),\ t\in(-\infty;0) \\ dt=\cosh x\,dx & x\in(0;\infty),\ t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$
$$= \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x}, \quad x\in R-\{0\},\ c_2\in R.$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst. } t=e^x & x=\ln t & x\in R \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{2\,dt}{t(2+t+\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t+1}$$
$$= \int \frac{2\,dt}{(t+1)^2} = \int 2(t+1)^{-2}\,dt$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh}\frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x}$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in R \\ dx=\frac{2\,dt}{1-t^2} & \cosh x=\frac{1+t^2}{1-t^2} & t\in(-1;1) \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}} = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2}{1-t^2}} = \int dt = t+c_1 = \operatorname{tgh}\frac{x}{2}+c_1,\quad x\in R,\ c_1\in R.$$

$$= \int \frac{dx}{2\cosh^2\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\frac{x}{2} & x\in R \\ dt=\frac{dx}{2} & t\in R \end{array}\right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh}\frac{x}{2}+c_1,\ x\in R,\ c_1\in R.$$

$$= \int \frac{\cosh x\,dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\sinh x & x\in(-\infty;0),\ t\in(-\infty;0) \\ dt=\cosh x\,dx & x\in(0;\infty),\ t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x},\ x\in R-\{0\},\ c_2\in R.$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in R \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{2\,dt}{t(2+t+\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{2\,dt}{(t+1)^2} = \int 2(t+1)^{-2}\,dt = \frac{2(t+1)^{-1}}{-1} + c_3$$

Riešené príklady – 008

$$\int \frac{dx}{1+\cosh x} = \operatorname{tgh}\frac{x}{2} + c_1 = c_2 - \frac{1+\cosh x}{\sinh x} = -\frac{2}{e^x+1} + c_3$$

$$= \left[\begin{array}{l|l|l} \text{UHS:} & t=\operatorname{tgh}\frac{x}{2} & x\in R \\ dx=\frac{2\,dt}{1-t^2} & \cosh x=\frac{1+t^2}{1-t^2} & t\in(-1;1) \end{array}\right] = \int \frac{\frac{2\,dt}{1-t^2}}{1+\frac{1+t^2}{1-t^2}} = \int \frac{\frac{2\,dt}{1-t^2}}{\frac{2}{1-t^2}} = \int dt = t+c_1 = \operatorname{tgh}\frac{x}{2}+c_1, \quad x\in R,\ c_1\in R.$$

$$= \int \frac{dx}{2\cosh^2\frac{x}{2}} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\frac{x}{2} & x\in R \\ dt=\frac{dx}{2} & t\in R \end{array}\right] = \int \frac{dt}{\cosh^2 t} = \operatorname{tgh} t + c_1 = \operatorname{tgh}\frac{x}{2}+c_1, \quad x\in R,\ c_1\in R.$$

$$= \int \frac{\cosh x\,dx}{\sinh^2 x} - \int \frac{dx}{\sinh^2 x} = \left[\begin{array}{r|l} \text{Subst.}\quad t=\sinh x & x\in(-\infty;0),\ t\in(-\infty;0) \\ dt=\cosh x\,dx & x\in(0;\infty),\ t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t^2} - \int \frac{dx}{\sinh^2 x}$$

$$= \frac{1}{-t} - \operatorname{cotgh} x + c_2 = c_2 - \frac{1}{\sinh x} - \frac{\cosh x}{\sinh x} = c_2 - \frac{1+\cosh x}{\sinh x}, \quad x\in R-\{0\},\ c_2\in R.$$

$$= \int \frac{dx}{1+\frac{e^x+e^{-x}}{2}} = \int \frac{2\,dx}{2+e^x+e^{-x}} = \left[\begin{array}{l|l|l} \text{Subst.}\quad t=e^x & x=\ln t & x\in R \\ e^{-x}=t^{-1}=\frac{1}{t} & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{2\,dt}{t(2+t+\frac{1}{t})} = \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{2\,dt}{(t+1)^2} = \int 2(t+1)^{-2}\,dt = \frac{2(t+1)^{-1}}{-1} + c_3 = -\frac{2}{e^x+1} + c_3, \quad x\in R,\ c_3\in R.$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx$$

$$= \int \frac{1-\cos 2x}{2} \, dx$$

$$= \int \sin x \cdot \sin x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \sin^2 x & v' = 2 \sin x \cos x = \sin 2x \end{array} \right]$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx$$

$$= \int \frac{1-\cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2}\right] dx$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{array}\right]$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \sin^2 x & v' = 2\sin x \cos x = \sin 2x \end{array}\right] = x \sin^2 x - \int x \sin 2x \, dx$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1-\cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2}\right] dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{array}\right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \sin^2 x & v' = 2 \sin x \cos x = \sin 2x \end{array}\right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array}\right]$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1-\cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= -\frac{\sin 2x}{2} + \int (1 - \sin^2 x) \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \sin^2 x & v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array} \right]$$

$$= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right]$$

Riešené príklady – 009

$$I=\int \sin^2 x\,dx = \frac{x}{2}-\frac{\sin 2x}{4}+c$$

$$=\int \frac{1-\cos 2x}{2}\,dx = \int\left[\frac{1}{2}-\frac{\cos 2x}{2}\right]dx = \frac{x}{2}-\frac{\sin 2x}{4}+c,\; x\in R,\; c\in R.$$

$$=\int \sin x\cdot\sin x\,dx = \left[\begin{array}{l|l} u=\sin x & u'=\cos x \\ v'=\sin x & v=-\cos x\end{array}\right] = -\sin x\cdot\cos x+\int \cos^2 x\,dx$$

$$=-\frac{\sin 2x}{2}+\int(1-\sin^2 x)\,dx = -\frac{\sin 2x}{2}+x-\int \sin^2 x\,dx$$

$$=\left[\begin{array}{l|l} u'=1 & u=x \\ v=\sin^2 x & v'=2\sin x\cos x=\sin 2x\end{array}\right] = x\sin^2 x-\int x\sin 2x\,dx = \left[\begin{array}{l|l} u'=\sin 2x & u=-\frac{\cos 2x}{2} \\ v=x & v'=1\end{array}\right]$$

$$=x\sin^2 x-\left[-\frac{x\cos 2x}{2}+\int\frac{\cos 2x}{2}\,dx\right] = \frac{2x\sin^2 x}{2}+\frac{x(\cos^2 x-\sin^2 x)}{2}-\frac{\sin 2x}{4}+c$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1-\cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2}\right] dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{array}\right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= -\frac{\sin 2x}{2} + \int (1-\sin^2 x) \, dx = -\frac{\sin 2x}{2} + x - \int \sin^2 x \, dx = -\frac{\sin 2x}{2} + x - I$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \sin^2 x & v' = 2 \sin x \cos x = \sin 2x \end{array}\right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array}\right]$$

$$= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx\right] = \frac{2x \sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c$$

$$= \frac{x \cos^2 x + x \sin^2 x}{2} - \frac{\sin 2x}{4} + c$$

Riešené príklady – 009

$$I = \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$= \int \frac{1 - \cos 2x}{2} \, dx = \int \left[\frac{1}{2} - \frac{\cos 2x}{2} \right] dx = \frac{x}{2} - \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \sin x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin x & u' = \cos x \\ v' = \sin x & v = -\cos x \end{array} \right] = -\sin x \cdot \cos x + \int \cos^2 x \, dx$$

$$= -\frac{\sin 2x}{2} + \int (1 - \sin^2 x) \, dx = -\frac{\sin 2x}{2} + x - \int \sin^2 x \, dx = -\frac{\sin 2x}{2} + x - I$$

$$= \left[\text{Rovnica } I = -\frac{\sin 2x}{2} + x - I \;\Rightarrow\; 2I = x - \frac{\sin 2x}{2} \right]$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \sin^2 x & v' = 2 \sin x \cos x = \sin 2x \end{array} \right] = x \sin^2 x - \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array} \right]$$

$$= x \sin^2 x - \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \sin^2 x}{2} + \frac{x(\cos^2 x - \sin^2 x)}{2} - \frac{\sin 2x}{4} + c$$

$$= \frac{x \cos^2 x + x \sin^2 x}{2} - \frac{\sin 2x}{4} + c = \frac{x}{2} - \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

Riešené príklady – 009

$$I=\int \sin^2 x\,dx = \frac{x}{2}-\frac{\sin 2x}{4}+c$$

$$=\int \frac{1-\cos 2x}{2}\,dx = \int\left[\frac{1}{2}-\frac{\cos 2x}{2}\right]dx = \frac{x}{2}-\frac{\sin 2x}{4}+c,\ x\in R,\ c\in R.$$

$$=\int \sin x\cdot \sin x\,dx = \left[\begin{array}{l|l} u=\sin x & u'=\cos x \\ v'=\sin x & v=-\cos x\end{array}\right] = -\sin x\cdot\cos x + \int \cos^2 x\,dx$$

$$=-\frac{\sin 2x}{2}+\int(1-\sin^2 x)\,dx = -\frac{\sin 2x}{2}+x-\int \sin^2 x\,dx = -\frac{\sin 2x}{2}+x-I$$

$$=\left[\text{Rovnica } I=-\frac{\sin 2x}{2}+x-I \ \Rightarrow\ 2I=x-\frac{\sin 2x}{2}\right] = \frac{x}{2}-\frac{\sin 2x}{4}+c,\ x\in R,\ c\in R.$$

$$=\left[\begin{array}{l|l} u'=1 & u=x \\ v=\sin^2 x & v'=2\sin x\cos x=\sin 2x\end{array}\right] = x\sin^2 x-\int x\sin 2x\,dx = \left[\begin{array}{l|l} u'=\sin 2x & u=-\frac{\cos 2x}{2} \\ v=x & v'=1\end{array}\right]$$

$$=x\sin^2 x-\left[-\frac{x\cos 2x}{2}+\int\frac{\cos 2x}{2}\,dx\right] = \frac{2x\sin^2 x}{2}+\frac{x(\cos^2 x-\sin^2 x)}{2}-\frac{\sin 2x}{4}+c$$

$$=\frac{x\cos^2 x+x\sin^2 x}{2}-\frac{\sin 2x}{4}+c = \frac{x}{2}-\frac{\sin 2x}{4}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx$$

$$= \int \frac{1+\cos 2x}{2} \, dx$$

$$= \int \cos x \cdot \cos x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \cos^2 x & v' = -2\cos x \sin x = -\sin 2x \end{array} \right]$$

Riešené príklady – 010

$I = \int \cos^2 x \, dx$

$= \int \frac{1+\cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] dx$

$= \int \cos x \cdot \cos x \, dx \; = \left[\begin{array}{l|l} u = \cos x & u' = -\sin x \\ v' = \cos x & v = \sin x \end{array} \right]$

$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \cos^2 x & v' = -2 \cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx$

Riešené príklady – 010

$$I=\int \cos^2 x\,dx = \frac{x}{2}+\frac{\sin 2x}{4}+c$$

$$=\int \frac{1+\cos 2x}{2}\,dx=\int\left[\frac{1}{2}+\frac{\cos 2x}{2}\right]dx=\frac{x}{2}+\frac{\sin 2x}{4}+c,\ x\in R,\ c\in R.$$

$$=\int \cos x\cdot\cos x\,dx = \left[\begin{array}{l|l} u=\cos x & u'=-\sin x \\ v'=\cos x & v=\sin x\end{array}\right]=\cos x\cdot\sin x+\int \sin^2 x\,dx$$

$$=\left[\begin{array}{l|l} u'=1 & u=x \\ v=\cos^2 x & v'=-2\cos x\sin x=-\sin 2x\end{array}\right]=x\cos^2 x+\int x\sin 2x\,dx=\left[\begin{array}{l|l} u'=\sin 2x & u=-\frac{\cos 2x}{2} \\ v=x & v'=1\end{array}\right]$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1+\cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos x & u' = -\sin x \\ v' = \cos x & v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \cos^2 x & v' = -2\cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array} \right]$$

$$= x \cos^2 x + \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right]$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1+\cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos x & u' = -\sin x \\ v' = \cos x & v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx = \frac{\sin 2x}{2} + x - \int \cos^2 x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \cos^2 x & v' = -2\cos x \sin x = -\sin 2x \end{array} \right] = x \cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array} \right]$$

$$= x \cos^2 x + \left[-\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x \cos^2 x}{2} - \frac{x(\cos^2 x - \sin^2 x)}{2} + \frac{\sin 2x}{4} + c$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1+\cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos x & u' = -\sin x \\ v' = \cos x & v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx = \frac{\sin 2x}{2} + x - \int \cos^2 x \, dx = \frac{\sin 2x}{2} + x - I$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \cos^2 x & v' = -2\cos x \sin x = -\sin 2x \end{array} \right] = x\cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array} \right]$$

$$= x\cos^2 x + \left[-\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x\cos^2 x}{2} - \frac{x(\cos^2 x - \sin^2 x)}{2} + \frac{\sin 2x}{4} + c$$

$$= \frac{x\cos^2 x + x\sin^2 x}{2} + \frac{\sin 2x}{4} + c$$

Riešené príklady – 010

$$I = \int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$$

$$= \int \frac{1+\cos 2x}{2} \, dx = \int \left[\frac{1}{2} + \frac{\cos 2x}{2} \right] dx = \frac{x}{2} + \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

$$= \int \cos x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos x & u' = -\sin x \\ v' = \cos x & v = \sin x \end{array} \right] = \cos x \cdot \sin x + \int \sin^2 x \, dx$$

$$= \frac{\sin 2x}{2} + \int (1 - \cos^2 x) \, dx = \frac{\sin 2x}{2} + x - \int \cos^2 x \, dx = \frac{\sin 2x}{2} + x - I$$

$$= \left[\text{Rovnica } I = \frac{\sin 2x}{2} + x - I \;\Rightarrow\; 2I = x + \frac{\sin 2x}{2} \right]$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \cos^2 x & v' = -2\cos x \sin x = -\sin 2x \end{array} \right] = x\cos^2 x + \int x \sin 2x \, dx = \left[\begin{array}{l|l} u' = \sin 2x & u = -\frac{\cos 2x}{2} \\ v = x & v' = 1 \end{array} \right]$$

$$= x\cos^2 x + \left[-\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx \right] = \frac{2x\cos^2 x}{2} - \frac{x(\cos^2 x - \sin^2 x)}{2} + \frac{\sin 2x}{4} + c$$

$$= \frac{x\cos^2 x + x\sin^2 x}{2} + \frac{\sin 2x}{4} + c = \frac{x}{2} + \frac{\sin 2x}{4} + c, \; x \in R, \; c \in R.$$

Riešené príklady – 010

$$I=\int \cos^2 x\,dx = \frac{x}{2}+\frac{\sin 2x}{4}+c$$

$$=\int \frac{1+\cos 2x}{2}\,dx=\int\left[\frac{1}{2}+\frac{\cos 2x}{2}\right]dx=\frac{x}{2}+\frac{\sin 2x}{4}+c,\ x\in R,\ c\in R.$$

$$=\int \cos x\cdot\cos x\,dx = \left[\begin{array}{l|l} u=\cos x & u'=-\sin x\\ v'=\cos x & v=\sin x\end{array}\right]=\cos x\cdot\sin x+\int \sin^2 x\,dx$$

$$=\frac{\sin 2x}{2}+\int(1-\cos^2 x)\,dx=\frac{\sin 2x}{2}+x-\int\cos^2 x\,dx=\frac{\sin 2x}{2}+x-I$$

$$=\left[\text{Rovnica } I=\frac{\sin 2x}{2}+x-I\ \Rightarrow\ 2I=x+\frac{\sin 2x}{2}\right]=\frac{x}{2}+\frac{\sin 2x}{4}+c,\ x\in R,\ c\in R.$$

$$=\left[\begin{array}{l|l} u'=1 & u=x\\ v=\cos^2 x & v'=-2\cos x\sin x=-\sin 2x\end{array}\right]=x\cos^2 x+\int x\sin 2x\,dx=\left[\begin{array}{l|l} u'=\sin 2x & u=-\frac{\cos 2x}{2}\\ v=x & v'=1\end{array}\right]$$

$$=x\cos^2 x+\left[-\frac{x\cos 2x}{2}+\int\frac{\cos 2x}{2}\,dx\right]=\frac{2x\cos^2 x}{2}-\frac{x(\cos^2 x-\sin^2 x)}{2}+\frac{\sin 2x}{4}+c$$

$$=\frac{x\cos^2 x+x\sin^2 x}{2}+\frac{\sin 2x}{4}+c=\frac{x}{2}+\frac{\sin 2x}{4}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{c|c} \text{Subst. } \quad t = \cos x & x \in \langle 0 + k\pi; \pi + k\pi \rangle \\ dt = -\sin x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1 - \cos^2 x)^n \cdot \sin x \, dx$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1-\cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{rl|l} \text{Subst.} & t = \cos x & x \in \langle 0+k\pi; \pi+k\pi \rangle \\ & dt = -\sin x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$

$$= -\int (1-t^2) \, dt$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1-\cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \cos x & x \in \langle 0+k\pi; \pi+k\pi \rangle \\ & dt = -\sin x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1-\cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{rl|l} \text{Subst.} & t = \cos x & x \in \langle 0+k\pi; \pi+k\pi \rangle \\ & dt = -\sin x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$

$$= -\int (1-t^2) \, dt = \int (t^2-1) \, dt$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1-\cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \cos x & x \in \langle 0+k\pi; \pi+k\pi \rangle \\ & dt = -\sin x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right] = -\int (1-t^2)^n \, dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1-\cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{c|c} \text{Subst.} \quad t = \cos x & x \in \langle 0+k\pi; \pi+k\pi \rangle \\ dt = -\sin x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$

$$= -\int (1-t^2) \, dt = \int (t^2-1) \, dt = \frac{t^3}{3} - t + c$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1-\cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t = \cos x & x \in \langle 0+k\pi; \pi+k\pi \rangle \\ dt = -\sin x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right] = -\int (1-t^2)^n \, dt = -\int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1-\cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{l|l} \text{Subst.} \quad t=\cos x & x\in\langle 0+k\pi; \pi+k\pi\rangle \\ dt=-\sin x \, dx & t\in\langle -1; 1\rangle, \ k\in Z \end{array} \right]$$

$$= -\int (1-t^2) \, dt = \int (t^2-1) \, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c, \ x\in R, \ c\in R.$$

$$\int \sin^{2n+1} x \, dx \qquad n\in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1-\cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\cos x & x\in\langle 0+k\pi; \pi+k\pi\rangle \\ dt=-\sin x \, dx & t\in\langle -1; 1\rangle, \ k\in Z \end{array} \right] = -\int (1-t^2)^n \, dt = -\int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= -\int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i t^{2i} \right] dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1-\cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{c|c} \text{Subst.} \quad t=\cos x & x\in\langle 0+k\pi ; \pi+k\pi \rangle \\ dt=-\sin x \, dx & t\in\langle -1;1 \rangle,\ k\in Z \end{array} \right]$$

$$= -\int (1-t^2) \, dt = \int (t^2-1) \, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c,\ x\in R,\ c\in R.$$

$$\int \sin^{2n+1} x \, dx \qquad n\in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1-\cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=\cos x & x\in\langle 0+k\pi ; \pi+k\pi \rangle \\ dt=-\sin x \, dx & t\in\langle -1;1 \rangle,\ k\in Z \end{array} \right] = -\int (1-t^2)^n \, dt = -\int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= -\int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i t^{2i} \right] dt = -\sum_{i=0}^{n} \binom{n}{i} (-1)^i \int t^{2i} \, dt$$

Riešené príklady – 011, 012

$$\int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x \, dx = \int (1-\cos^2 x) \cdot \sin x \, dx = \left[\begin{array}{c|c} \text{Subst.} \quad t=\cos x & x \in \langle 0+k\pi ; \pi+k\pi \rangle \\ dt=-\sin x \, dx & t \in \langle -1;1 \rangle ,\ k \in Z \end{array} \right]$$

$$= -\int (1-t^2) \, dt = \int (t^2-1) \, dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c, \ x \in R, \ c \in R.$$

$$\int \sin^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sin^{2n} x \cdot \sin x \, dx = \int (\sin^2 x)^n \cdot \sin x \, dx = \int (1-\cos^2 x)^n \cdot \sin x \, dx$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=\cos x & x \in \langle 0+k\pi ; \pi+k\pi \rangle \\ dt=-\sin x \, dx & t \in \langle -1;1 \rangle ,\ k \in Z \end{array} \right] = -\int (1-t^2)^n \, dt = -\int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$

$$= -\int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i t^{2i} \right] dt = -\sum_{i=0}^{n} \binom{n}{i} (-1)^i \int t^{2i} \, dt = \sum_{i=0}^{n} \binom{n}{i} (-1)^{i+1} \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 011, 012

$$\int \sin^3 x\,dx = \frac{\cos^3 x}{3} - \cos x + c$$

$$= \int \sin^2 x \cdot \sin x\,dx = \int (1-\cos^2 x)\cdot \sin x\,dx = \left[\begin{array}{l|l} \text{Subst.}\quad t=\cos x & x\in\langle 0+k\pi;\pi+k\pi\rangle \\ dt=-\sin x\,dx & t\in\langle -1;1\rangle,\ k\in Z \end{array}\right]$$

$$= -\int (1-t^2)\,dt = \int (t^2-1)\,dt = \frac{t^3}{3} - t + c = \frac{\cos^3 x}{3} - \cos x + c,\ x\in R,\ c\in R.$$

$$\int \sin^{2n+1} x\,dx = \sum_{i=0}^{n}\binom{n}{i}(-1)^{i+1}\frac{\cos^{2i+1} x}{2i+1} + c \qquad n\in N$$

$$= \int \sin^{2n} x\cdot \sin x\,dx = \int (\sin^2 x)^n \cdot \sin x\,dx = \int (1-\cos^2 x)^n\cdot \sin x\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=\cos x & x\in\langle 0+k\pi;\pi+k\pi\rangle \\ dt=-\sin x\,dx & t\in\langle -1;1\rangle,\ k\in Z \end{array}\right] = -\int (1-t^2)^n\,dt = -\int \left[\sum_{i=0}^{n}\binom{n}{i}1^{n-i}(-t^2)^i\right] dt$$

$$= -\int \left[\sum_{i=0}^{n}\binom{n}{i}(-1)^i t^{2i}\right] dt = -\sum_{i=0}^{n}\binom{n}{i}(-1)^i\int t^{2i}\,dt = \sum_{i=0}^{n}\binom{n}{i}(-1)^{i+1}\frac{t^{2i+1}}{2i+1} + c$$

$$= -\binom{n}{0}\cos x + \binom{n}{1}\frac{\cos^3 x}{3} - \binom{n}{2}\frac{\cos^5 x}{5} + \cdots + (-1)^{n+1}\binom{n}{n}\frac{\cos^{2n+1} x}{2n+1} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 013

$I_n = \int \sin^n x \, dx$ $\quad n = 2,3,4,5,\ldots$

Riešené príklady – 013

$I_n = \int \sin^n x \, dx$ $\qquad n = 2, 3, 4, 5, \ldots$

$$= \int \sin^{n-1} x \cdot \sin x \, dx$$

Riešené príklady – 013

$I_n = \int \sin^n x \, dx \qquad n = 2,3,4,5,\ldots$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin^{n-1} x & u' = (n-1)\sin^{n-2} x \cdot \cos x \\ v' = \sin x & v = -\cos x \end{array} \right]$$

Riešené príklady – 013

$I_n = \int \sin^n x \, dx$ $\qquad n = 2,3,4,5,\dots$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin^{n-1} x & u' = (n-1)\sin^{n-2} x \cdot \cos x \\ v' = \sin x & v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

Riešené príklady – 013

$I_n = \int \sin^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin^{n-1} x & u' = (n-1)\sin^{n-2} x \cdot \cos x \\ v' = \sin x & v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin^{n-1} x & u' = (n-1)\sin^{n-2} x \cdot \cos x \\ v' = \sin x & v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1-\sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

Riešené príklady – 013

$I_n = \int \sin^n x \, dx$ $\qquad n = 2,3,4,5,\dots$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin^{n-1} x & u' = (n-1)\sin^{n-2} x \cdot \cos x \\ v' = \sin x & v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

Riešené príklady – 013

$I_n = \int \sin^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

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$$= -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \quad \Rightarrow\ n I_n = I_n + (n-1) I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} \ \Rightarrow\ I_n = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 013

$$I_n = \int \sin^n x \, dx = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \sin^{n-1} x \cdot \sin x \, dx = \left[\begin{array}{l|l} u = \sin^{n-1} x & u' = (n-1)\sin^{n-2} x \cdot \cos x \\ v' = \sin x & v = -\cos x \end{array} \right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot \cos^2 x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \cdot (1 - \sin^2 x) \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \quad \Rightarrow \; nI_n = I_n + (n-1) I_n = -\cos x \cdot \sin^{n-1} x + (n-1) I_{n-2} \;\Rightarrow\; I_n = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \; x \in R,$$

Riešené príklady – 013

$$I_n = \int \sin^n x\,dx = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \sin^{n-1} x \cdot \sin x\,dx = \left[\begin{array}{l|l} u = \sin^{n-1} x & u' = (n-1)\sin^{n-2} x \cdot \cos x \\ v' = \sin x & v = -\cos x \end{array}\right]$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1)\int \sin^{n-2} x \cdot \cos^2 x\,dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1)\int \sin^{n-2} x \cdot (1-\sin^2 x)\,dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$$

$$= -\cos x \cdot \sin^{n-1} x + (n-1)I_{n-2} - (n-1)I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = -\cos x \cdot \sin^{n-1} x + (n-1)I_{n-2} - (n-1)I_n \\ \quad \Rightarrow\ nI_n = I_n + (n-1)I_n = -\cos x \cdot \sin^{n-1} x + (n-1)I_{n-2} \ \Rightarrow\ I_n = -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array}\right]$$

$$= -\frac{\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} I_{n-2},\ x \in R,$$

pričom $I_1 = \int \sin x\,dx = -\cos x + c,\ I_0 = \int \sin^0 x\,dx = \int dx = x + c,\ x \in R,\ c \in R.$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$\int \cos^{2n+1} x \, dx \qquad n \in N$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \, dx$$

$$\int \cos^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cos^{2n} x \cdot \cos x \, dx$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cdot \cos x \, dx$$

$$\int \cos^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cos^{2n} x \cdot \cos x \, dx = \int (\cos^2 x)^n \cdot \cos x \, dx$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cdot \cos x \, dx = \left[\begin{array}{rl|l} \text{Subst.} & t = \sin x & x \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ & dt = \cos x \, dx & t \in \langle -1; 1 \rangle, \; k \in Z \end{array} \right]$$

$$\int \cos^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cos^{2n} x \cdot \cos x \, dx = \int (\cos^2 x)^n \cdot \cos x \, dx = \int (1 - \sin^2 x)^n \cdot \cos x \, dx$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cdot \cos x \, dx = \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin x & x \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = \cos x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$

$$= \int (1 - t^2) \, dt$$

$$\int \cos^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cos^{2n} x \cdot \cos x \, dx = \int (\cos^2 x)^n \cdot \cos x \, dx = \int (1 - \sin^2 x)^n \cdot \cos x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin x & x \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = \cos x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx$$

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$$= \int (1 - t^2) \, dt = t - \frac{t^3}{3} + c$$

$$\int \cos^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cos^{2n} x \cdot \cos x \, dx = \int (\cos^2 x)^n \cdot \cos x \, dx = \int (1 - \sin^2 x)^n \cdot \cos x \, dx$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \sin x & x \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ & dt = \cos x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right] = \int (1 - t^2)^n \, dt$$

Riešené príklady – 014, 015

$$\int \cos^3 x\,dx = \sin x - \frac{\sin^3 x}{3} + c$$

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$$= \int (1-t^2)\,dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c,\ x\in R,\ c\in R.$$

$$\int \cos^{2n+1} x\,dx \qquad n\in N$$

$$= \int \cos^{2n} x \cdot \cos x\,dx = \int (\cos^2 x)^n \cdot \cos x\,dx = \int (1-\sin^2 x)^n \cdot \cos x\,dx$$

$$= \left[\begin{array}{rr|l} \text{Subst.} & t=\sin x & x\in\langle -\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi\rangle \\ & dt=\cos x\,dx & t\in\langle -1;1\rangle,\ k\in Z \end{array}\right] = \int (1-t^2)^n\,dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i}(-t^2)^i\right] dt$$

Riešené príklady – 014, 015

$$\int \cos^3 x\,dx = \sin x - \frac{\sin^3 x}{3} + c$$

$$= \int \cos^2 x \cdot \cos x\,dx = \int (1-\sin^2 x)\cdot \cos x\,dx = \left[\begin{array}{c|l} \text{Subst.}\quad t=\sin x & x\in\langle -\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi\rangle \\ dt=\cos x\,dx & t\in\langle -1;1\rangle,\ k\in Z \end{array}\right]$$

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$$= \left[\begin{array}{c|l} \text{Subst.}\quad t=\sin x & x\in\langle -\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi\rangle \\ dt=\cos x\,dx & t\in\langle -1;1\rangle,\ k\in Z \end{array}\right] = \int (1-t^2)^n\,dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i}(-t^2)^i\right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i t^{2i}\right] dt$$

Riešené príklady – 014, 015

$$\int \cos^3 x\,dx = \sin x - \frac{\sin^3 x}{3} + c$$

$$= \int \cos^2 x \cdot \cos x\,dx = \int (1-\sin^2 x)\cdot \cos x\,dx = \left[\begin{array}{c|c} \text{Subst. } t=\sin x & x\in\langle -\frac{\pi}{2}+k\pi;\ \frac{\pi}{2}+k\pi\rangle \\ dt=\cos x\,dx & t\in\langle -1;1\rangle,\ k\in Z \end{array}\right]$$

$$= \int (1-t^2)\,dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c,\ x\in R,\ c\in R.$$

$$\int \cos^{2n+1} x\,dx \qquad n\in N$$

$$= \int \cos^{2n} x\cdot \cos x\,dx = \int (\cos^2 x)^n \cdot \cos x\,dx = \int (1-\sin^2 x)^n\cdot \cos x\,dx$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=\sin x & x\in\langle -\frac{\pi}{2}+k\pi;\ \frac{\pi}{2}+k\pi\rangle \\ dt=\cos x\,dx & t\in\langle -1;1\rangle,\ k\in Z \end{array}\right] = \int (1-t^2)^n\,dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i}(-t^2)^i\right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i t^{2i}\right] dt = \sum_{i=0}^{n} \binom{n}{i}(-1)^i \int t^{2i}\,dt$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3} + c$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1-\sin^2 x) \cdot \cos x \, dx = \left[\begin{array}{cc|c} \text{Subst.} & t=\sin x & x\in\langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi\rangle \\ & dt=\cos x\, dx & t\in\langle -1;1\rangle,\ k\in Z \end{array} \right]$$

$$= \int (1-t^2)\, dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c,\ x\in R,\ c\in R.$$

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$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i t^{2i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} (-1)^i \int t^{2i}\, dt = \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 014, 015

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3} + c$$

$$= \int \cos^2 x \cdot \cos x \, dx = \int (1-\sin^2 x) \cdot \cos x \, dx = \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin x & x \in \langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi \rangle \\ dt = \cos x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right]$$
$$= \int (1-t^2) \, dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c, \ x \in R, \ c \in R.$$

$$\int \cos^{2n+1} x \, dx = \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{\sin^{2i+1} x}{2i+1} + c \qquad n \in N$$

$$= \int \cos^{2n} x \cdot \cos x \, dx = \int (\cos^2 x)^n \cdot \cos x \, dx = \int (1-\sin^2 x)^n \cdot \cos x \, dx$$
$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin x & x \in \langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi \rangle \\ dt = \cos x \, dx & t \in \langle -1; 1 \rangle, \ k \in Z \end{array} \right] = \int (1-t^2)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} 1^{n-i} (-t^2)^i \right] dt$$
$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i t^{2i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} (-1)^i \int t^{2i} \, dt = \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$
$$= \binom{n}{0} \sin x - \binom{n}{1} \frac{\sin^3 x}{3} + \binom{n}{2} \frac{\sin^5 x}{5} - \cdots + (-1)^n \binom{n}{n} \frac{\sin^{2n+1} x}{2n+1} + c, \ x \in R, \ c \in R.$$

Riešené príklady – 016

$I_n = \int \cos^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

Riešené príklady – 016

$I_n = \int \cos^n x \, dx$ $\qquad n = 2, 3, 4, 5, \ldots$

$$= \int \cos^{n-1} x \cdot \cos x \, dx$$

Riešené príklady – 016

$I_n = \int \cos^n x \, dx$ $\quad n = 2,3,4,5,\ldots$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos^{n-1} x & u' = -(n-1)\cos^{n-2} x \cdot \sin x \\ v' = \cos x & v = \sin x \end{array} \right]$$

Riešené príklady – 016

$I_n = \int \cos^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos^{n-1} x & u' = -(n-1)\cos^{n-2} x \cdot \sin x \\ v' = \cos x & v = \sin x \end{array} \right]$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

Riešené príklady – 016

$I_n = \int \cos^n x \, dx \qquad n = 2,3,4,5,\ldots$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos^{n-1} x & u' = -(n-1)\cos^{n-2} x \cdot \sin x \\ v' = \cos x & v = \sin x \end{array} \right]$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

Riešené príklady – 016

$I_n = \int \cos^n x \,dx$ $\qquad n = 2,3,4,5,\ldots$

$$= \int \cos^{n-1} x \cdot \cos x \,dx = \left[\begin{array}{l|l} u = \cos^{n-1} x & u' = -(n-1)\cos^{n-2} x \cdot \sin x \\ v' = \cos x & v = \sin x \end{array} \right]$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \,dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1-\cos^2 x) \,dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \,dx - (n-1) \int \cos^n x \,dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

Riešené príklady – 016

$I_n = \int \cos^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

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$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \quad \Rightarrow \ nI_n = I_n + (n-1) I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} \ \Rightarrow \ I_n = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx \qquad n = 2,3,4,5,\dots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos^{n-1} x & u' = -(n-1)\cos^{n-2} x \cdot \sin x \\ v' = \cos x & v = \sin x \end{array} \right]$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \quad \Rightarrow\ n I_n = I_n + (n-1) I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} \ \Rightarrow\ I_n = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \ x \in R,$$

Riešené príklady – 016

$$I_n = \int \cos^n x \, dx = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \cos^{n-1} x \cdot \cos x \, dx = \left[\begin{array}{l|l} u = \cos^{n-1} x & u' = -(n-1)\cos^{n-2} x \cdot \sin x \\ v' = \cos x & v = \sin x \end{array} \right]$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \cdot (1 - \cos^2 x) \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$= \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} - (n-1) I_n \\ \quad \Rightarrow \; n I_n = I_n + (n-1) I_n = \sin x \cdot \cos^{n-1} x + (n-1) I_{n-2} \; \Rightarrow \; I_n = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \; x \in R,$$

pričom $I_1 = \int \cos x \, dx = \sin x + c, \; I_0 = \int \cos^0 x \, dx = \int dx = x + c, \; x \in R, \; c \in R.$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx$$

$$= \int \sinh x \cdot \sinh x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] dx$$

$$= \int \sinh x \cdot \sinh x \, dx = \left[\begin{array}{l|l} u = \sinh x & u' = \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} - 2\, e^x\, e^{-x} + e^{-2x}}{4} \, dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \; x \in R, \; c \in R.$$

$$= \int \sinh x \cdot \sinh x \, dx = \left[\begin{array}{c|c} u = \sinh x & u' = \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right] = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} - 2 e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int \left[e^{2x} - 2 + e^{-2x} \right] dx$$

Riešené príklady – 017

$$I = \int \sinh^2 x \, dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x - 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} - \frac{1}{2} \right] dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c, \; x \in R, \; c \in R.$$

$$= \int \sinh x \cdot \sinh x \, dx = \left[\begin{array}{c|c} u = \sinh x & u' = \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right] = \sinh x \cdot \cosh x - \int \cosh^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\sinh^2 x + 1) \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} - 2 e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int \left[e^{2x} - 2 + e^{-2x} \right] dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2} \right] + c$$

Riešené príklady – 017

$$I=\int \sinh^2 x\,dx=\frac{\sinh 2x}{4}-\frac{x}{2}+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c$$

$$=\int \frac{\cosh 2x-1}{2}\,dx=\int\left[\frac{\cosh 2x}{2}-\frac{1}{2}\right]dx=\frac{\sinh 2x}{4}-\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int \sinh x\cdot\sinh x\,dx=\left[\begin{array}{l|l} u=\sinh x & u'=\cosh x\\ v'=\sinh x & v=\cosh x\end{array}\right]=\sinh x\cdot\cosh x-\int\cosh^2 x\,dx$$

$$=\frac{\sinh 2x}{2}-\int(\sinh^2 x+1)\,dx=\frac{\sinh 2x}{2}-\int\sinh^2 x\,dx-x$$

$$=\int\left[\frac{e^{x}-e^{-x}}{2}\right]^2 dx=\int\frac{e^{2x}-2\,e^{x}\,e^{-x}+e^{-2x}}{4}\,dx=\frac{1}{4}\int\left[e^{2x}-2+e^{-2x}\right]dx$$

$$=\frac{1}{4}\left[\frac{e^{2x}}{2}-2x+\frac{e^{-2x}}{-2}\right]+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c$$

Riešené príklady – 017

$$I=\int \sinh^2 x\,dx = \frac{\sinh 2x}{4}-\frac{x}{2}+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c$$

$$=\int \frac{\cosh 2x-1}{2}\,dx=\int\left[\frac{\cosh 2x}{2}-\frac{1}{2}\right]dx=\frac{\sinh 2x}{4}-\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int \sinh x\cdot\sinh x\,dx=\left[\begin{array}{l|l} u=\sinh x & u'=\cosh x\\ v'=\sinh x & v=\cosh x\end{array}\right]=\sinh x\cdot\cosh x-\int \cosh^2 x\,dx$$

$$=\frac{\sinh 2x}{2}-\int(\sinh^2 x+1)\,dx=\frac{\sinh 2x}{2}-\int \sinh^2 x\,dx-x=\frac{\sinh 2x}{2}-I-x$$

$$=\int\left[\frac{e^x-e^{-x}}{2}\right]^2dx=\int\frac{e^{2x}-2e^xe^{-x}+e^{-2x}}{4}\,dx=\frac{1}{4}\int\left[e^{2x}-2+e^{-2x}\right]dx$$

$$=\frac{1}{4}\left[\frac{e^{2x}}{2}-2x+\frac{e^{-2x}}{-2}\right]+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c=\frac{1}{4}\cdot\frac{e^{2x}-e^{-2x}}{2}-\frac{x}{2}+c$$

Riešené príklady – 017

$$I=\int \sinh^2 x\,dx=\frac{\sinh 2x}{4}-\frac{x}{2}+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c$$

$$=\int\frac{\cosh 2x-1}{2}\,dx=\int\left[\frac{\cosh 2x}{2}-\frac{1}{2}\right]dx=\frac{\sinh 2x}{4}-\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int \sinh x\cdot\sinh x\,dx=\left[\begin{array}{l|l} u=\sinh x & u'=\cosh x\\ v'=\sinh x & v=\cosh x\end{array}\right]=\sinh x\cdot\cosh x-\int\cosh^2 x\,dx$$

$$=\frac{\sinh 2x}{2}-\int(\sinh^2 x+1)\,dx=\frac{\sinh 2x}{2}-\int\sinh^2 x\,dx-x=\frac{\sinh 2x}{2}-I-x$$

$$=\left[\text{Rovnica } I=\tfrac{\sinh 2x}{2}-I-x\ \Rightarrow\ 2I=\tfrac{\sinh 2x}{2}-x\right]$$

$$=\int\left[\frac{e^x-e^{-x}}{2}\right]^2dx=\int\frac{e^{2x}-2e^xe^{-x}+e^{-2x}}{4}\,dx=\frac{1}{4}\int\left[e^{2x}-2+e^{-2x}\right]dx$$

$$=\frac{1}{4}\left[\frac{e^{2x}}{2}-2x+\frac{e^{-2x}}{-2}\right]+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c=\frac{1}{4}\cdot\frac{e^{2x}-e^{-2x}}{2}-\frac{x}{2}+c$$

$$=\frac{\sinh 2x}{4}-\frac{x}{2}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 017

$$I=\int \sinh^2 x\,dx=\frac{\sinh 2x}{4}-\frac{x}{2}+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c$$

$$=\int\frac{\cosh 2x-1}{2}\,dx=\int\left[\frac{\cosh 2x}{2}-\frac{1}{2}\right]dx=\frac{\sinh 2x}{4}-\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int \sinh x\cdot\sinh x\,dx=\left[\begin{array}{c|c} u=\sinh x & u'=\cosh x\\ v'=\sinh x & v=\cosh x\end{array}\right]=\sinh x\cdot\cosh x-\int\cosh^2 x\,dx$$

$$=\frac{\sinh 2x}{2}-\int(\sinh^2 x+1)\,dx=\frac{\sinh 2x}{2}-\int\sinh^2 x\,dx-x=\frac{\sinh 2x}{2}-I-x$$

$$=\left[\text{Rovnica } I=\frac{\sinh 2x}{2}-I-x\ \Rightarrow\ 2I=\frac{\sinh 2x}{2}-x\right]=\frac{\sinh 2x}{4}-\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int\left[\frac{e^{x}-e^{-x}}{2}\right]^2 dx=\int\frac{e^{2x}-2e^{x}e^{-x}+e^{-2x}}{4}\,dx=\frac{1}{4}\int\left[e^{2x}-2+e^{-2x}\right]dx$$

$$=\frac{1}{4}\left[\frac{e^{2x}}{2}-2x+\frac{e^{-2x}}{-2}\right]+c=\frac{e^{2x}-e^{-2x}-4x}{8}+c=\frac{1}{4}\cdot\frac{e^{2x}-e^{-2x}}{2}-\frac{x}{2}+c$$

$$=\frac{\sinh 2x}{4}-\frac{x}{2}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx$$

$$= \int \cosh x \cdot \cosh x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] dx$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh x & u' = \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} + 2 e^x e^{-x} + e^{-2x}}{4} \, dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \; x \in R, \; c \in R.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{c|c} u = \cosh x & u' = \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sin^2 x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} + 2 e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int \left[e^{2x} + 2 + e^{-2x} \right] dx$$

Riešené príklady – 018

$$I = \int \cosh^2 x \, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c$$

$$= \int \frac{\cosh 2x + 1}{2} \, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c, \; x \in R, \; c \in R.$$

$$= \int \cosh x \cdot \cosh x \, dx = \left[\begin{array}{c|c} u = \cosh x & u' = \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sin^2 x \, dx$$

$$= \frac{\sinh 2x}{2} - \int (\cosh^2 x - 1) \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} + 2 e^x e^{-x} + e^{-2x}}{4} \, dx = \frac{1}{4} \int \left[e^{2x} + 2 + e^{-2x} \right] dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right] + c$$

Riešené príklady – 018

$$I=\int \cosh^2 x\,dx = \frac{\sinh 2x}{4}+\frac{x}{2}+c = \frac{e^{2x}-e^{-2x}+4x}{8}+c$$

$$=\int \frac{\cosh 2x+1}{2}\,dx = \int \left[\frac{\cosh 2x}{2}+\frac{1}{2}\right]dx = \frac{\sinh 2x}{4}+\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int \cosh x\cdot\cosh x\,dx = \left[\begin{array}{c|c} u=\cosh x & u'=\sinh x \\ v'=\cosh x & v=\sinh x\end{array}\right] = \cosh x\cdot\sinh x - \int \sin^2 x\,dx$$

$$=\frac{\sinh 2x}{2}-\int(\cosh^2 x-1)\,dx = \frac{\sinh 2x}{2}-\int\cosh^2 x\,dx + x$$

$$=\int\left[\frac{e^x+e^{-x}}{2}\right]^2 dx = \int\frac{e^{2x}+2e^x e^{-x}+e^{-2x}}{4}\,dx = \frac{1}{4}\int\left[e^{2x}+2+e^{-2x}\right]dx$$

$$=\frac{1}{4}\left[\frac{e^{2x}}{2}+2x+\frac{e^{-2x}}{-2}\right]+c = \frac{e^{2x}-e^{-2x}+4x}{8}+c$$

Riešené príklady – 018

$$I=\int \cosh^2 x\,dx = \frac{\sinh 2x}{4}+\frac{x}{2}+c=\frac{e^{2x}-e^{-2x}+4x}{8}+c$$

$$=\int \frac{\cosh 2x+1}{2}\,dx=\int\left[\frac{\cosh 2x}{2}+\frac{1}{2}\right]dx=\frac{\sinh 2x}{4}+\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int \cosh x\cdot\cosh x\,dx=\left[\begin{array}{c|c} u=\cosh x & u'=\sinh x\\ v'=\cosh x & v=\sinh x\end{array}\right]=\cosh x\cdot\sinh x-\int \sin^2 x\,dx$$

$$=\frac{\sinh 2x}{2}-\int(\cosh^2 x-1)\,dx=\frac{\sinh 2x}{2}-\int \cosh^2 x\,dx+x=\frac{\sinh 2x}{2}-I+x$$

$$=\int\left[\frac{e^{x}+e^{-x}}{2}\right]^2 dx=\int\frac{e^{2x}+2e^{x}e^{-x}+e^{-2x}}{4}\,dx=\frac{1}{4}\int\left[e^{2x}+2+e^{-2x}\right]dx$$

$$=\frac{1}{4}\left[\frac{e^{2x}}{2}+2x+\frac{e^{-2x}}{-2}\right]+c=\frac{e^{2x}-e^{-2x}+4x}{8}+c=\frac{1}{4}\cdot\frac{e^{2x}-e^{-2x}}{2}+\frac{x}{2}+c$$

Riešené príklady – 018

$$I = \int \cosh^2 x\, dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c$$

$$= \int \frac{\cosh 2x + 1}{2}\, dx = \int \left[\frac{\cosh 2x}{2} + \frac{1}{2} \right] dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c,\ x \in R,\ c \in R.$$

$$= \int \cosh x \cdot \cosh x\, dx = \left[\begin{array}{c|c} u = \cosh x & u' = \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right] = \cosh x \cdot \sinh x - \int \sin^2 x\, dx$$

$$= \frac{\sinh 2x}{2} - \int (\cosh^2 x - 1)\, dx = \frac{\sinh 2x}{2} - \int \cosh^2 x\, dx + x = \frac{\sinh 2x}{2} - I + x$$

$$= \left[\text{Rovnica } I = \frac{\sinh 2x}{2} - I + x \ \Rightarrow\ 2I = \frac{\sinh 2x}{2} + x \right]$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^2 dx = \int \frac{e^{2x} + 2\, e^x\, e^{-x} + e^{-2x}}{4}\, dx = \frac{1}{4} \int \left[e^{2x} + 2 + e^{-2x} \right] dx$$

$$= \frac{1}{4} \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right] + c = \frac{e^{2x} - e^{-2x} + 4x}{8} + c = \frac{1}{4} \cdot \frac{e^{2x} - e^{-2x}}{2} + \frac{x}{2} + c$$

$$= \frac{\sinh 2x}{4} + \frac{x}{2} + c,\ x \in R,\ c \in R.$$

Riešené príklady – 018

$$I=\int \cosh^2 x\,dx = \frac{\sinh 2x}{4}+\frac{x}{2}+c=\frac{e^{2x}-e^{-2x}+4x}{8}+c$$

$$=\int \frac{\cosh 2x+1}{2}\,dx=\int\left[\frac{\cosh 2x}{2}+\frac{1}{2}\right]dx=\frac{\sinh 2x}{4}+\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int \cosh x\cdot\cosh x\,dx=\left[\begin{array}{l|l} u=\cosh x & u'=\sinh x\\ v'=\cosh x & v=\sinh x\end{array}\right]=\cosh x\cdot\sinh x-\int \sin^2 x\,dx$$

$$=\frac{\sinh 2x}{2}-\int(\cosh^2 x-1)\,dx=\frac{\sinh 2x}{2}-\int\cosh^2 x\,dx+x=\frac{\sinh 2x}{2}-I+x$$

$$=\left[\text{Rovnica } I=\tfrac{\sinh 2x}{2}-I+x\ \Rightarrow\ 2I=\tfrac{\sinh 2x}{2}+x\right]=\frac{\sinh 2x}{4}+\frac{x}{2}+c,\ x\in R,\ c\in R.$$

$$=\int\left[\frac{e^x+e^{-x}}{2}\right]^2dx=\int\frac{e^{2x}+2e^xe^{-x}+e^{-2x}}{4}\,dx=\frac{1}{4}\int\left[e^{2x}+2+e^{-2x}\right]dx$$

$$=\frac{1}{4}\left[\frac{e^{2x}}{2}+2x+\frac{e^{-2x}}{-2}\right]+c=\frac{e^{2x}-e^{-2x}+4x}{8}+c=\frac{1}{4}\cdot\frac{e^{2x}-e^{-2x}}{2}+\frac{x}{2}+c$$

$$=\frac{\sinh 2x}{4}+\frac{x}{2}+c,\ x\in R,\ c\in R.$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3\,e^{2x}\,e^{-x} + 3\,e^x\,e^{-2x} - e^{-3x}}{8} \, dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{r|l} \text{Subst. } t = \cosh x & x \in (-\infty; 0\rangle, \; t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty), \; t \in \langle 1; \infty) \end{array} \right]$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3\,e^{2x}\,e^{-x} + 3\,e^x\,e^{-2x} - e^{-3x}}{8} \, dx$$

$$= \int \frac{e^{3x} - 3\,e^x + 3\,e^{-x} - e^{-3x}}{8} \, dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{r|l} \text{Subst. } t = \cosh x & x \in (-\infty; 0\rangle, \ t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty), \ t \in \langle 1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x} e^{-x} + 3e^x e^{-2x} - e^{-3x}}{8} \, dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} \, dx = \frac{1}{8} \int \left[e^{3x} - 3e^x + 3e^{-x} - e^{-3x} \right] dx$$

Riešené príklady – 019

$$\int \sinh^3 x \, dx$$

$$= \int \sinh^2 x \cdot \sinh x \, dx = \int (\cosh^2 x - 1) \cdot \sinh x \, dx = \left[\begin{array}{r|l} \text{Subst. } t = \cosh x & x \in (-\infty; 0\rangle, \ t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty), \ t \in \langle 1; \infty) \end{array} \right]$$

$$= \int (t^2 - 1) \, dt = \frac{t^3}{3} - t + c$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} - 3e^{2x} e^{-x} + 3e^x e^{-2x} - e^{-3x}}{8} \, dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8} \, dx = \frac{1}{8} \int \left[e^{3x} - 3e^x + 3e^{-x} - e^{-3x} \right] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3} \right] + c$$

Riešené príklady – 019

$$\int \sinh^3 x\,dx = \frac{\cosh^3 x}{3} - \cosh x + c$$

$$= \int \sinh^2 x \cdot \sinh x\,dx = \int (\cosh^2 x - 1)\cdot \sinh x\,dx = \left[\begin{array}{r|l} \text{Subst. } t=\cosh x & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \\ dt=\sinh x\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right]$$

$$= \int (t^2-1)\,dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c,\ x\in R,\ c\in R.$$

$$= \int \left[\frac{e^x - e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^x e^{-2x} - e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} - 3e^x + 3e^{-x} - e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} - 3e^x + 3e^{-x} - e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^x + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^x - 3e^{-x} + \frac{e^{-3x}}{3}\right] + c$$

Riešené príklady – 019

$$\int \sinh^3 x\,dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x\,dx = \int (\cosh^2 x - 1)\cdot \sinh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t = \cosh x & x \in (-\infty; 0\rangle,\ t \in \langle 1; \infty) \\ dt = \sinh x\,dx & x \in \langle 0; \infty),\ t \in \langle 1; \infty) \end{array}\right]$$

$$= \int (t^2 - 1)\,dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c,\ x \in R,\ c \in R.$$

$$= \int \left[\frac{e^x - e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^{x}e^{-2x} - e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c$$

Riešené príklady – 019

$$\int \sinh^3 x\,dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x\,dx = \int (\cosh^2 x - 1)\cdot \sinh x\,dx = \left[\begin{array}{c|l} \text{Subst. } t = \cosh x & x \in (-\infty; 0\rangle,\ t \in \langle 1; \infty) \\ dt = \sinh x\,dx & x \in \langle 0; \infty),\ t \in \langle 1; \infty) \end{array}\right]$$

$$= \int (t^2 - 1)\,dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c,\ x \in R,\ c \in R.$$

$$= \int \left[\frac{e^{x} - e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^{x}e^{-2x} - e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3e^{x} - 12e^{x} + 3e^{-x} - 12e^{-x} + e^{-3x}}{24} + c$$

Riešené príklady – 019

$$\int \sinh^3 x\,dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9\,e^{x} - 9\,e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x\,dx = \int (\cosh^2 x - 1)\cdot \sinh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t=\cosh x & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \\ dt=\sinh x\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right]$$

$$= \int (t^2-1)\,dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c,\ x\in R,\ c\in R.$$

$$= \int \left[\frac{e^{x} - e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} - 3\,e^{2x}\,e^{-x} + 3\,e^{x}\,e^{-2x} - e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} - 3\,e^{x} + 3\,e^{-x} - e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} - 3\,e^{x} + 3\,e^{-x} - e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} - 3\,e^{x} + \frac{3\,e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} - 3\,e^{x} - 3\,e^{-x} + \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} - 9\,e^{x} - 9\,e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3\,e^{x} - 12\,e^{x} + 3\,e^{-x} - 12\,e^{-x} + e^{-3x}}{24} + c$$

$$= \frac{e^{3x} + 3\,e^{x} + 3\,e^{-x} + e^{-3x}}{3\cdot 8} - \frac{12\,e^{x} + 12\,e^{-x}}{24} + c$$

Riešené príklady – 019

$$\int \sinh^3 x\,dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x\,dx = \int (\cosh^2 x - 1)\cdot \sinh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t=\cosh x & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \\ dt=\sinh x\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right]$$

$$= \int (t^2-1)\,dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c,\ x\in R,\ c\in R.$$

$$= \int \left[\frac{e^x - e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^{x}e^{-2x} - e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3e^{x} - 12e^{x} + 3e^{-x} - 12e^{-x} + e^{-3x}}{24} + c$$

$$= \frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{3\cdot 8} - \frac{12e^{x} + 12e^{-x}}{24} + c = \frac{1}{3}\cdot\left[\frac{e^{2x} + e^{-2x}}{2}\right]^3 - \frac{e^{x} + e^{-x}}{2} + c$$

Riešené príklady – 019

$$\int \sinh^3 x\,dx = \frac{\cosh^3 x}{3} - \cosh x + c = \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c$$

$$= \int \sinh^2 x \cdot \sinh x\,dx = \int (\cosh^2 x - 1)\cdot \sinh x\,dx = \left[\begin{array}{r|l} \text{Subst. } t=\cosh x & x\in(-\infty;0\rangle,\ t\in\langle 1;\infty) \\ dt=\sinh x\,dx & x\in\langle 0;\infty),\ t\in\langle 1;\infty) \end{array}\right]$$

$$= \int (t^2-1)\,dt = \frac{t^3}{3} - t + c = \frac{\cosh^3 x}{3} - \cosh x + c,\ x\in R,\ c\in R.$$

$$= \int \left[\frac{e^{x} - e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} - 3e^{2x}e^{-x} + 3e^{x}e^{-2x} - e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} + \frac{3e^{-x}}{-1} - \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} - 9e^{x} - 9e^{-x} + e^{-3x}}{24} + c = \frac{e^{3x} + 3e^{x} - 12e^{x} + 3e^{-x} - 12e^{-x} + e^{-3x}}{24} + c$$

$$= \frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{3\cdot 8} - \frac{12e^{x} + 12e^{-x}}{24} + c = \frac{1}{3}\cdot\left[\frac{e^{2x} + e^{-2x}}{2}\right]^3 - \frac{e^{x} + e^{-x}}{2} + c$$

$$= \frac{\cosh^3 x}{3} - \cosh x + c,\ x\in R,\ c\in R.$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx \qquad n \in N$$

Riešené príklady – 020

$\int \sinh^{2n+1} x \, dx$ $\qquad n \in N$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx$$

Riešené príklady – 020

$\int \sinh^{2n+1} x \, dx$ $\qquad n \in N$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0\rangle \,,\; t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty) \,,\; t \in \langle 1; \infty) \end{array} \right]$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0\rangle\,,\ t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty)\,,\ t \in \langle 1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt$$

Riešené príklady – 020

$\int \sinh^{2n+1} x \, dx$ $\qquad n \in N$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0\rangle, \; t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty), \; t \in \langle 1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

Riešené príklady – 020

$\int \sinh^{2n+1} x \, dx$ $\qquad n \in N$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0) \,,\ t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty) \,,\ t \in \langle 1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0\rangle\,,\ t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty)\,,\ t \in \langle 1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} \, dt$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x\,\mathrm{d}x \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x\,\mathrm{d}x = \int (\sinh^2 x)^n \cdot \sinh x\,\mathrm{d}x = \int (\cosh^2 x - 1)^n \cdot \sinh x\,\mathrm{d}x$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \cosh x & x \in (-\infty; 0\rangle\,,\ t \in \langle 1; \infty) \\ \mathrm{d}t = \sinh x\,\mathrm{d}x & x \in \langle 0; \infty)\,,\ t \in \langle 1; \infty) \end{array}\right] = \int (t^2 - 1)^n\,\mathrm{d}t = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i}\right] \mathrm{d}t$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i}\right] \mathrm{d}t = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i}\,\mathrm{d}t = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0),\ t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty),\ t \in \langle 1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} \, dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\frac{1}{(-1)^i} = \frac{1}{(-1)^i} \cdot \frac{(-1)^i}{(-1)^i} = \frac{(-1)^i}{(-1)^{2i}} = (-1)^i \right]$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x \, dx = \int (\sinh^2 x)^n \cdot \sinh x \, dx = \int (\cosh^2 x - 1)^n \cdot \sinh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0\rangle, \ t \in \langle 1; \infty) \\ dt = \sinh x \, dx & x \in \langle 0; \infty), \ t \in \langle 1; \infty) \end{array} \right] = \int (t^2 - 1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i} \, dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\tfrac{1}{(-1)^i} = \tfrac{1}{(-1)^i} \cdot \tfrac{(-1)^i}{(-1)^i} = \tfrac{(-1)^i}{(-1)^{2i}} = (-1)^i \right] = (-1)^n \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x\,dx = (-1)^n \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x\,dx = \int (\sinh^2 x)^n \cdot \sinh x\,dx = \int (\cosh^2 x - 1)^n \cdot \sinh x\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cosh x & x \in (-\infty; 0\rangle,\ t \in \langle 1; \infty) \\ dt = \sinh x\,dx & x \in \langle 0; \infty),\ t \in \langle 1; \infty) \end{array} \right] = \int (t^2 - 1)^n\,dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i}\,dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\frac{1}{(-1)^i} = \frac{1}{(-1)^i} \cdot \frac{(-1)^i}{(-1)^i} = \frac{(-1)^i}{(-1)^{2i}} = (-1)^i \right] = (-1)^n \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

$$= (-1)^n \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c$$

Riešené príklady – 020

$$\int \sinh^{2n+1} x\,dx = (-1)^n \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c \qquad n \in N$$

$$= \int \sinh x^{2n} \cdot \sinh x\,dx = \int (\sinh^2 x)^n \cdot \sinh x\,dx = \int (\cosh^2 x - 1)^n \cdot \sinh x\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \cosh x & x \in (-\infty; 0)\,,\ t \in \langle 1; \infty) \\ dt = \sinh x\,dx & x \in \langle 0; \infty)\,,\ t \in \langle 1; \infty) \end{array}\right] = \int (t^2-1)^n\,dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i (-1)^{n-i}\right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \frac{(-1)^n}{(-1)^i}\right] dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \int t^{2i}\,dt = \sum_{i=0}^{n} \binom{n}{i} \frac{(-1)^n}{(-1)^i} \frac{t^{2i+1}}{2i+1} + c$$

$$= \left[\frac{1}{(-1)^i} = \frac{1}{(-1)^i} \cdot \frac{(-1)^i}{(-1)^i} = \frac{(-1)^i}{(-1)^{2i}} = (-1)^i\right] = (-1)^n \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{t^{2i+1}}{2i+1} + c$$

$$= (-1)^n \sum_{i=0}^{n} \binom{n}{i} (-1)^i \frac{\cosh^{2i+1} x}{2i+1} + c$$

$$= (-1)^n \left[\binom{n}{0} \cosh x - \binom{n}{1} \frac{\cosh^3 x}{3} + \binom{n}{2} \frac{\cosh^5 x}{5} + \cdots + (-1)^n \binom{n}{n} \frac{\cosh^{2n+1} x}{2n+1}\right] + c,$$

$$x \in R,\ c \in R.$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\quad n = 2,3,4,5,\ldots$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\quad n = 2,3,4,5,\ldots$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx \qquad n = 2,3,4,5,\ldots$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx \qquad n = 2,3,4,5,\ldots$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx \qquad n = 2,3,4,5,\ldots$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx \qquad n = 2,3,4,5,\dots$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n} x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2}$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \sinh^{n-1} x \cdot \sinh x \, dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1) \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x \, dx - (n-1) \int \sinh^{n-2} x \, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2} \\ \quad \Rightarrow \ n I_n = I_n + (n-1) I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_{n-2} \ \Rightarrow \ I_n = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 021

$$I_n = \int \sinh^n x\,dx = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} \int \sinh^{n-2} x\,dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \sinh^{n-1} x \cdot \sinh x\,dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x\,dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1)\,dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x\,dx - (n-1) \int \sinh^{n-2} x\,dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2} \\ \quad \Rightarrow\ n I_n = I_n + (n-1) I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_{n-2} \ \Rightarrow\ I_n = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2},\ x \in R,$$

Riešené príklady – 021

$$I_n = \int \sinh^n x\, dx = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} \int \sinh^{n-2} x\, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \sinh^{n-1} x \cdot \sinh x\, dx = \left[\begin{array}{l|l} u = \sinh^{n-1} x & u' = (n-1)\sinh^{n-2} x \cdot \cosh x \\ v' = \sinh x & v = \cosh x \end{array} \right]$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot \cosh^2 x\, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^{n-2} x \cdot (\sinh^2 x + 1)\, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) \int \sinh^n x\, dx - (n-1) \int \sinh^{n-2} x\, dx$$

$$= \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_n - (n-1) I_{n-2} \\ \quad \Rightarrow\; n I_n = I_n + (n-1) I_n = \cosh x \cdot \sinh^{n-1} x - (n-1) I_{n-2} \;\Rightarrow\; I_n = \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\cosh x \cdot \sinh^{n-1} x}{n} - \frac{n-1}{n} I_{n-2},\; x \in R,$$

pričom $I_1 = \int \sinh x\, dx = \cosh x + c,\; I_0 = \int \sinh^0 x\, dx = \int dx = x + c,\; x \in R,\; c \in R.$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\qquad n \in N$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\qquad n \in N$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\qquad n \in N$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\qquad n \in N$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

Riešené príklady – 021

$I_n = \int \sinh^n x \, dx$ $\qquad n \in N$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx \qquad n \in N$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (- e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx \qquad n \in N$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \qquad n \in N$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \ x \in R,$$

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \qquad n \in N$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \; x \in R,$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} \, dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in R$.

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \qquad n \in N$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \; x \in R,$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} \, dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in R$.

$n = 2k+1, \; k = 0,1,2,\ldots$ (nepárne)

$n = 2k, \; k = 1,2,3,\ldots$ (párne)

Riešené príklady – 021

$$I_n = \int \sinh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right] \qquad n \in N$$

$$= \int \left[\frac{e^x - e^{-x}}{2} \right]^n dx = \int \frac{(e^x - e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x - e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (-e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} (-1)^i e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (-1)^i e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[(-1)^i \binom{n}{i} \int e^{(n-2i)x} \, dx \right], \; x \in R,$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} \, dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in R$.

$n = 2k+1, \; k = 0,1,2, \ldots$ (nepárne)

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} - n \frac{e^{(n-2)x}}{n-2} + \cdots + (-1)^k \binom{n}{k} e^x + (-1)^{k+1} \binom{n}{k+1} \frac{e^{-x}}{-1} + \cdots + n \frac{e^{(2-n)x}}{2-n} - \frac{e^{-nx}}{-n} \right] + c.$$

$n = 2k, \; k = 1,2,3, \ldots$ (párne)

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} - n \frac{e^{(n-2)x}}{n-2} + \cdots + (-1)^{k-1} \binom{n}{k-1} \frac{e^{2x}}{2} + (-1)^k \binom{n}{k} x + (-1)^{k+1} \binom{n}{k+1} \frac{e^{-2x}}{-2} + \cdots - n \frac{e^{(2-n)x}}{2-n} + \frac{e^{-nx}}{-n} \right] + c.$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3\, e^{2x}\, e^{-x} + 3\, e^x\, e^{-2x} + e^{-3x}}{8} \, dx$$

Riešené príklady – 022

$\int \cosh^3 x \, dx$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{c|c} \text{Subst. } t = \sinh x & x \in R \\ dt = \cosh x \, dx & t \in R \end{array} \right]$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3 e^{2x} e^{-x} + 3 e^x e^{-2x} + e^{-3x}}{8} \, dx$$

$$= \int \frac{e^{3x} + 3 e^x + 3 e^{-x} + e^{-3x}}{8} \, dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{c|c} \text{Subst. } t = \sinh x & x \in R \\ dt = \cosh x \, dx & t \in R \end{array} \right]$$

$$= \int (t^2 + 1) \, dt$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^x e^{-2x} + e^{-3x}}{8} \, dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8} \, dx = \frac{1}{8} \int \left[e^{3x} + 3e^x + 3e^{-x} + e^{-3x} \right] dx$$

Riešené príklady – 022

$$\int \cosh^3 x \, dx$$

$$= \int \cosh^2 x \cdot \cosh x \, dx = \int (\sinh^2 x + 1) \cdot \cosh x \, dx = \left[\begin{array}{c|c} \text{Subst. } t = \sinh x & x \in R \\ dt = \cosh x \, dx & t \in R \end{array} \right]$$

$$= \int (t^2 + 1) \, dt = \frac{t^3}{3} + t + c$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^3 dx = \int \frac{e^{3x} + 3 e^{2x} e^{-x} + 3 e^x e^{-2x} + e^{-3x}}{8} \, dx$$

$$= \int \frac{e^{3x} + 3 e^x + 3 e^{-x} + e^{-3x}}{8} \, dx = \frac{1}{8} \int \left[e^{3x} + 3 e^x + 3 e^{-x} + e^{-3x} \right] dx$$

$$= \frac{1}{8} \left[\frac{e^{3x}}{3} + 3 e^x + \frac{3 e^{-x}}{-1} + \frac{e^{-3x}}{-3} \right] + c$$

Riešené príklady – 022

$$\int \cosh^3 x\,dx = \frac{\sinh^3 x}{3} + \sinh x + c$$

$$= \int \cosh^2 x \cdot \cosh x\,dx = \int (\sinh^2 x + 1)\cdot \cosh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t=\sinh x & x\in R \\ dt=\cosh x\,dx & t\in R \end{array}\right]$$

$$= \int (t^2+1)\,dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c,\ x\in R,\ c\in R.$$

$$= \int \left[\frac{e^x + e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^x e^{-2x} + e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} + 3e^x + 3e^{-x} + e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} + 3e^x + 3e^{-x} + e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^x + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^x - 3e^{-x} - \frac{e^{-3x}}{3}\right] + c$$

Riešené príklady – 022

$$\int \cosh^3 x\,dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x\,dx = \int (\sinh^2 x + 1)\cdot \cosh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t=\sinh x & x\in R \\ dt=\cosh x\,dx & t\in R \end{array}\right]$$

$$= \int (t^2+1)\,dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c,\ x\in R,\ c\in R.$$

$$= \int \left[\frac{e^{x}+e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^{x}e^{-2x} + e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} - 3e^{-x} - \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c$$

Riešené príklady – 022

$$\int \cosh^3 x\,dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x\,dx = \int (\sinh^2 x + 1)\cdot \cosh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array}\right]$$

$$= \int (t^2+1)\,dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c,\ x \in R,\ c \in R.$$

$$= \int \left[\frac{e^{x} + e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^{x}e^{-2x} + e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} - 3e^{-x} - \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3e^{x} + 12e^{x} + 3e^{-x} - 12e^{-x} - e^{-3x}}{24} + c$$

Riešené príklady – 022

$$\int \cosh^3 x\,dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x\,dx = \int (\sinh^2 x + 1)\cdot \cosh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array} \right]$$

$$= \int (t^2+1)\,dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c,\ x \in R,\ c \in R.$$

$$= \int \left[\frac{e^{x} + e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^{x}e^{-2x} + e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x} \right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} - 3e^{-x} - \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3e^{x} + 12e^{x} + 3e^{-x} - 12e^{-x} - e^{-3x}}{24} + c$$

$$= \frac{e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}}{3\cdot 8} + \frac{12e^{x} - 12e^{-x}}{24} + c$$

Riešené príklady – 022

$$\int \cosh^3 x\,dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9\,e^{x} - 9\,e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x\,dx = \int (\sinh^2 x + 1) \cdot \cosh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array}\right]$$

$$= \int (t^2 + 1)\,dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c,\ x \in R,\ c \in R.$$

$$= \int \left[\frac{e^{x} + e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} + 3\,e^{2x}\,e^{-x} + 3\,e^{x}\,e^{-2x} + e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} + 3\,e^{x} + 3\,e^{-x} + e^{-3x}}{8}\,dx = \frac{1}{8} \int \left[e^{3x} + 3\,e^{x} + 3\,e^{-x} + e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} + 3\,e^{x} + \frac{3\,e^{-x}}{-1} + \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} + 3\,e^{x} - 3\,e^{-x} - \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} + 9\,e^{x} - 9\,e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3\,e^{x} + 12\,e^{x} + 3\,e^{-x} - 12\,e^{-x} - e^{-3x}}{24} + c$$

$$= \frac{e^{3x} - 3\,e^{x} + 3\,e^{-x} - e^{-3x}}{3 \cdot 8} + \frac{12\,e^{x} - 12\,e^{-x}}{24} + c = \frac{1}{3} \cdot \left[\frac{e^{2x} - e^{-2x}}{2}\right]^3 + \frac{e^{x} - e^{-x}}{2} + c$$

Riešené príklady – 022

$$\int \cosh^3 x\,dx = \frac{\sinh^3 x}{3} + \sinh x + c = \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c$$

$$= \int \cosh^2 x \cdot \cosh x\,dx = \int (\sinh^2 x + 1)\cdot \cosh x\,dx = \left[\begin{array}{c|c} \text{Subst. } t = \sinh x & x \in R \\ dt = \cosh x\,dx & t \in R \end{array}\right]$$

$$= \int (t^2 + 1)\,dt = \frac{t^3}{3} + t + c = \frac{\sinh^3 x}{3} + \sinh x + c,\ x \in R,\ c \in R.$$

$$= \int \left[\frac{e^{x} + e^{-x}}{2}\right]^3 dx = \int \frac{e^{3x} + 3e^{2x}e^{-x} + 3e^{x}e^{-2x} + e^{-3x}}{8}\,dx$$

$$= \int \frac{e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}}{8}\,dx = \frac{1}{8}\int \left[e^{3x} + 3e^{x} + 3e^{-x} + e^{-3x}\right] dx$$

$$= \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} + \frac{3e^{-x}}{-1} + \frac{e^{-3x}}{-3}\right] + c = \frac{1}{8}\left[\frac{e^{3x}}{3} + 3e^{x} - 3e^{-x} - \frac{e^{-3x}}{3}\right] + c$$

$$= \frac{e^{3x} + 9e^{x} - 9e^{-x} - e^{-3x}}{24} + c = \frac{e^{3x} - 3e^{x} + 12e^{x} + 3e^{-x} - 12e^{-x} - e^{-3x}}{24} + c$$

$$= \frac{e^{3x} - 3e^{x} + 3e^{-x} - e^{-3x}}{3\cdot 8} + \frac{12e^{x} - 12e^{-x}}{24} + c = \frac{1}{3}\cdot\left[\frac{e^{2x} - e^{-2x}}{2}\right]^3 + \frac{e^{x} - e^{-x}}{2} + c$$

$$= \frac{\sinh^3 x}{3} + \sinh x + c,\ x \in R,\ c \in R.$$

Riešené príklady – 023

$\int \cosh^{2n+1} x \, dx$ $\qquad n \in N$

Riešené príklady – 023

$\int \cosh^{2n+1} x \, dx$ $\quad n \in N$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx$$

Riešené príklady – 023

$\int \cosh^{2n+1} x \, dx$ $\qquad n \in N$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx$$

Riešené príklady – 023

$\int \cosh^{2n+1} x \, dx$ $\quad n \in N$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sinh x & x \in R \\ dt = \cosh x \, dx & t \in R \end{array} \right]$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sinh x & x \in R \\ dt = \cosh x \, dx & t \in R \end{array} \right] = \int (t^2 + 1)^n \, dt$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \sinh x & x \in R \\ & dt = \cosh x \, dx & t \in R \end{array} \right] = \int (t^2+1)^n \, dt = \int \Big[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i \cdot 1^{n-i} \Big] \, dt$$

Riešené príklady – 023

$\int \cosh^{2n+1} x \, dx$ $\quad n \in N$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{rr|r} \text{Subst.} & t = \sinh x & x \in R \\ & dt = \cosh x \, dx & t \in R \end{array} \right] = \int (t^2+1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i \cdot 1^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \right] dt$$

Riešené príklady – 023

$\int \cosh^{2n+1} x \, dx$ $\qquad n \in N$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sinh x & x \in R \\ dt = \cosh x \, dx & t \in R \end{array} \right] = \int (t^2+1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i \cdot 1^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \int t^{2i} \, dt$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx \qquad n \in N$$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sinh x & x \in R \\ dt = \cosh x \, dx & t \in R \end{array} \right] = \int (t^2+1)^n \, dt = \int \Big[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i \cdot 1^{n-i} \Big] \, dt$$

$$= \int \Big[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \Big] \, dt = \sum_{i=0}^{n} \binom{n}{i} \int t^{2i} \, dt = \sum_{i=0}^{n} \binom{n}{i} \frac{t^{2i+1}}{2i+1} + c$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx = \sum_{i=0}^{n} \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c \qquad n \in N$$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{rr|c} \text{Subst.} & t = \sinh x & x \in R \\ & dt = \cosh x \, dx & t \in R \end{array} \right] = \int (t^2+1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i \cdot 1^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \int t^{2i} \, dt = \sum_{i=0}^{n} \binom{n}{i} \frac{t^{2i+1}}{2i+1} + c = \sum_{i=0}^{n} \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x \, dx = \sum_{i=0}^{n} \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c \qquad n \in N$$

$$= \int \cosh x^{2n} \cdot \cosh x \, dx = \int (\cosh^2 x)^n \cdot \cosh x \, dx = \int (\sinh^2 x + 1)^n \cdot \cosh x \, dx$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \sinh x & x \in R \\ & dt = \cosh x \, dx & t \in R \end{array} \right] = \int (t^2+1)^n \, dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i \cdot 1^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \int t^{2i} \, dt = \sum_{i=0}^{n} \binom{n}{i} \frac{t^{2i+1}}{2i+1} + c = \sum_{i=0}^{n} \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c$$

$$= \binom{n}{0} \sinh x - \binom{n}{1} \frac{\sinh^3 x}{3} + \binom{n}{2} \frac{\sinh^5 x}{5} + \cdots$$

$$+ (-1)^{n-1} \binom{n}{n-1} \frac{\sinh^{2n-1} x}{2n-1} + (-1)^n \binom{n}{n} \frac{\sinh^{2n+1} x}{2n+1} + c$$

Riešené príklady – 023

$$\int \cosh^{2n+1} x\,dx = \sum_{i=0}^{n} \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c \qquad n \in N$$

$$= \int \cosh x^{2n} \cdot \cosh x\,dx = \int (\cosh^2 x)^n \cdot \cosh x\,dx = \int (\sinh^2 x + 1)^n \cdot \cosh x\,dx$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \sinh x & x \in R \\ & dt = \cosh x\,dx & t \in R \end{array} \right] = \int (t^2+1)^n\,dt = \int \left[\sum_{i=0}^{n} \binom{n}{i} (t^2)^i \cdot 1^{n-i} \right] dt$$

$$= \int \left[\sum_{i=0}^{n} \binom{n}{i} t^{2i} \right] dt = \sum_{i=0}^{n} \binom{n}{i} \int t^{2i}\,dt = \sum_{i=0}^{n} \binom{n}{i} \frac{t^{2i+1}}{2i+1} + c = \sum_{i=0}^{n} \binom{n}{i} \frac{\sinh^{2i+1} x}{2i+1} + c$$

$$= \binom{n}{0} \sinh x - \binom{n}{1} \frac{\sinh^3 x}{3} + \binom{n}{2} \frac{\sinh^5 x}{5} + \cdots + (-1)^{n-1} \binom{n}{n-1} \frac{\sinh^{2n-1} x}{2n-1} + (-1)^n \binom{n}{n} \frac{\sinh^{2n+1} x}{2n+1} + c$$

$$= \sinh x - \frac{n \cdot \sinh^3 x}{3} + \binom{n}{2} \frac{\sinh^5 x}{5} + \cdots + (-1)^{n-1} \cdot \frac{n \sinh^{2n-1} x}{2n-1} + (-1)^n \cdot \frac{\sinh^{2n+1} x}{2n+1} + c,$$

$$x \in R,\ c \in R.$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx \qquad n = 2, 3, 4, 5, \ldots$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx \qquad n = 2,3,4,5,\dots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $\qquad n = 2,3,4,5,\ldots$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^n x \, dx + (n-1) \int \cosh^{n-2} x \, dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx \qquad n = 2,3,4,5,\dots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^n x \, dx + (n-1) \int \cosh^{n-2} x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx \qquad n = 2,3,4,5,\dots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^n x \, dx + (n-1) \int \cosh^{n-2} x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2} \\ \quad \Rightarrow \; n I_n = I_n + (n-1) I_n = \sinh x \cdot \cosh^{n-1} x + (n-1) I_{n-2} \;\Rightarrow\; I_n = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} \int \cosh^{n-2} x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^n x \, dx + (n-1) \int \cosh^{n-2} x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2} \\ \quad \Rightarrow nI_n = I_n + (n-1) I_n = \sinh x \cdot \cosh^{n-1} x + (n-1) I_{n-2} \Rightarrow I_n = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \; x \in R,$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} \int \cosh^{n-2} x \, dx \qquad n = 2,3,4,5,\ldots$$

$$= \int \cosh^{n-1} x \cdot \cosh x \, dx = \left[\begin{array}{l|l} u = \cosh^{n-1} x & u' = (n-1)\cosh^{n-2} x \cdot \sinh x \\ v' = \cosh x & v = \sinh x \end{array} \right]$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot \sinh^2 x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n-2} x \cdot (\cosh^2 x - 1) \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) \int \cosh^{n} x \, dx + (n-1) \int \cosh^{n-2} x \, dx$$

$$= \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2}$$

$$= \left[\begin{array}{l} \text{Rovnica } I_n = \sinh x \cdot \cosh^{n-1} x - (n-1) I_n + (n-1) I_{n-2} \\ \quad \Rightarrow \; n I_n = I_n + (n-1) I_n = \sinh x \cdot \cosh^{n-1} x + (n-1) I_{n-2} \; \Rightarrow \; I_n = \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2} \end{array} \right]$$

$$= \frac{\sinh x \cdot \cosh^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \; x \in R,$$

pričom $I_1 = \int \cosh x \, dx = \sinh x + c, \; I_0 = \int \cosh^0 x \, dx = \int dx = x + c, \; x \in R, \; c \in R.$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $\quad n \in N$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $n \in N$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx \qquad n \in N$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $\qquad n \in N$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x + e^{-x} \right)^n dx$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $\qquad n \in N$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x + e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx$ $\qquad n \in N$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x + e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} \, e^{nx-ix} \, e^{-ix} \right] dx$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx \qquad n \in N$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x + e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} \, e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-2ix} \right] dx$$

Riešené príklady – 024

$I_n = \int \cosh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right]$ $\quad n \in N$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x + e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} \, e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right], \; x \in R,$$

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right] \qquad n \in N$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x + e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} \, e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right], \; x \in R,$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} \, dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in R$.

Riešené príklady – 024

$$I_n=\int\cosh^n x\,dx=\frac{1}{2^n}\sum_{i=0}^{n}\left[\binom{n}{i}\int e^{(n-2i)x}\,dx\right] \qquad n\in N$$

$$=\int\left[\frac{e^x+e^{-x}}{2}\right]^n dx=\int\frac{(e^x+e^{-x})^n}{2^n}\,dx=\frac{1}{2^n}\int\left(e^x+e^{-x}\right)^n dx$$

$$=\frac{1}{2^n}\int\left[\sum_{i=0}^{n}\binom{n}{i}(e^x)^{n-i}(e^{-x})^i\right]dx=\frac{1}{2^n}\int\left[\sum_{i=0}^{n}\binom{n}{i}\,e^{nx-ix}\,e^{-ix}\right]dx$$

$$=\frac{1}{2^n}\int\left[\sum_{i=0}^{n}\binom{n}{i}\,e^{nx-2ix}\right]dx=\frac{1}{2^n}\sum_{i=0}^{n}\left[\binom{n}{i}\int e^{(n-2i)x}\,dx\right],\ x\in R,$$

pričom $\int dx=x+c$ pre $i=\frac{n}{2}$, $\int e^{(n-2i)x}\,dx=\frac{e^{(n-2i)x}}{n-2i}+c$ pre $i\neq\frac{n}{2}$, $c\in R$.

$n=2k+1,\ k=0,1,2,\dots$ (nepárne)

$n=2k,\ k=1,2,3,\dots$ (párne)

Riešené príklady – 024

$$I_n = \int \cosh^n x \, dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right] \qquad n \in N$$

$$= \int \left[\frac{e^x + e^{-x}}{2} \right]^n dx = \int \frac{(e^x + e^{-x})^n}{2^n} \, dx = \frac{1}{2^n} \int \left(e^x + e^{-x} \right)^n dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} (e^x)^{n-i} (e^{-x})^i \right] dx = \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-ix} e^{-ix} \right] dx$$

$$= \frac{1}{2^n} \int \left[\sum_{i=0}^{n} \binom{n}{i} e^{nx-2ix} \right] dx = \frac{1}{2^n} \sum_{i=0}^{n} \left[\binom{n}{i} \int e^{(n-2i)x} \, dx \right], \; x \in R,$$

pričom $\int dx = x + c$ pre $i = \frac{n}{2}$, $\int e^{(n-2i)x} \, dx = \frac{e^{(n-2i)x}}{n-2i} + c$ pre $i \neq \frac{n}{2}$, $c \in R$.

$n = 2k+1, \; k = 0,1,2,\ldots$ (nepárne)

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} + n \frac{e^{(n-2)x}}{n-2} + \cdots + \binom{n}{k} e^x + \binom{n}{k+1} \frac{e^{-x}}{-1} + \cdots + n \frac{e^{(2-n)x}}{2-n} + \frac{e^{-nx}}{-n} \right] + c.$$

$n = 2k, \; k = 1,2,3,\ldots$ (párne)

$$I_n = \frac{1}{2^n} \left[\frac{e^{nx}}{n} + n \frac{e^{(n-2)x}}{n-2} + \cdots + \binom{n}{k-1} \frac{e^{2x}}{2} + \binom{n}{k} x + \binom{n}{k+1} \frac{e^{-2x}}{-2} + \cdots + n \frac{e^{(2-n)x}}{2-n} + \frac{e^{-nx}}{-n} \right] + c.$$

Riešené príklady – 025, 026

$\int \operatorname{tg}^2 x \, dx$

$\int \operatorname{tg}^3 x \, dx$

Riešené príklady – 025, 026

$$\int \mathrm{tg}^2 x \, \mathrm{d}x$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x$$

$$\int \mathrm{tg}^3 x \, \mathrm{d}x$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \mathrm{tg}\, x & x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi) \\ x = \mathrm{arctg}\, t & \mathrm{d}x = \frac{\mathrm{d}t}{t^2+1}, \quad t \in R, \ k \in Z \end{array} \right]$$

Riešené príklady – 025, 026

$$\int \mathrm{tg}^2 x \, \mathrm{d}x$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^2 x} \, \mathrm{d}x$$

$$\int \mathrm{tg}^3 x \, \mathrm{d}x$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \mathrm{tg}\, x & x \in (-\frac{\pi}{2} + 2k\pi ; \frac{\pi}{2} + 2k\pi) \\ x = \mathrm{arctg}\, t & \mathrm{d}x = \frac{\mathrm{d}t}{t^2+1}, \quad t \in R, \ k \in Z \end{array} \right] = \int \frac{t^3 \, \mathrm{d}t}{t^2+1}$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1-\cos^2 x}{\cos^2 x} \, dx = \int \Big[\frac{1}{\cos^2 x} - 1\Big] dx$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \operatorname{tg} x & x \in (-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi) \\ x = \operatorname{arctg} t & dx = \frac{dt}{t^2+1}, \quad t \in R, \ k \in Z \end{array} \right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3+t-t \, dt}{t^2+1}$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, \mathrm{d}x = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^2 x} \, \mathrm{d}x = \int \left[\frac{1}{\cos^2 x} - 1 \right] \mathrm{d}x = \operatorname{tg} x - x + c,$$

$$x \in R - \left\{ \frac{\pi}{2} + k\pi, \ k \in Z \right\}, \ c \in R.$$

$$\int \operatorname{tg}^3 x \, \mathrm{d}x$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \operatorname{tg} x & x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t & \mathrm{d}x = \frac{\mathrm{d}t}{t^2+1}, \quad t \in R, \ k \in Z \end{array} \right] = \int \frac{t^3 \, \mathrm{d}t}{t^2+1} = \int \frac{t^3+t-t \, \mathrm{d}t}{t^2+1} = \int t \, \mathrm{d}t - \frac{1}{2} \int \frac{2t \, \mathrm{d}t}{t^2+1}$$

Riešené príklady – 025, 026

$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1-\cos^2 x}{\cos^2 x} \, dx = \int \Big[\frac{1}{\cos^2 x} - 1\Big] dx = \operatorname{tg} x - x + c,$$

$$x \in R - \Big\{\frac{\pi}{2} + k\pi,\ k \in Z\Big\},\ c \in R.$$

$\int \operatorname{tg}^3 x \, dx$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \operatorname{tg} x & x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi) \\ x = \operatorname{arctg} t & dx = \frac{dt}{t^2+1},\quad t \in R,\ k \in Z \end{array}\right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1}$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1-\cos^2 x}{\cos^2 x} \, dx = \int \Big[\frac{1}{\cos^2 x} - 1\Big] dx = \operatorname{tg} x - x + c,$$

$$x \in R - \Big\{\frac{\pi}{2} + k\pi, \ k \in Z\Big\}, \ c \in R.$$

$$\int \operatorname{tg}^3 x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \operatorname{tg} x & x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi) \\ x = \operatorname{arctg} t & dx = \frac{dt}{t^2+1}, \quad t \in R, \ k \in Z \end{array}\right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln\left(t^2+1\right) + c$$

Riešené príklady – 025, 026

$$\int \mathrm{tg}^2 x \, dx = \mathrm{tg}\, x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \frac{1-\cos^2 x}{\cos^2 x} \, dx = \int \Big[\frac{1}{\cos^2 x} - 1\Big] dx = \mathrm{tg}\, x - x + c,$$

$$x \in R - \Big\{\frac{\pi}{2} + k\pi,\ k \in Z\Big\},\ c \in R.$$

$$\int \mathrm{tg}^3 x \, dx = \frac{\mathrm{tg}^2 x}{2} - \ln\left(\mathrm{tg}^2 x + 1\right) + c$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \mathrm{tg}\, x & x \in (-\frac{\pi}{2} + 2k\pi;\ \frac{\pi}{2} + 2k\pi) \\ x = \mathrm{arctg}\, t & dx = \frac{dt}{t^2+1},\quad t \in R,\ k \in Z \end{array}\right] = \int \frac{t^3 \, dt}{t^2+1} = \int \frac{t^3 + t - t \, dt}{t^2+1} = \int t \, dt - \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \int t \, dt - \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} = \frac{t^2}{2} - \ln\left(t^2+1\right) + c = \frac{\mathrm{tg}^2 x}{2} - \ln\left(\mathrm{tg}^2 x + 1\right) + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x \, \mathrm{d}x = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x} \, \mathrm{d}x = \int \frac{1-\cos^2 x}{\cos^2 x} \, \mathrm{d}x = \int \Big[\frac{1}{\cos^2 x} - 1\Big] \mathrm{d}x = \operatorname{tg} x - x + c,$$

$$x \in R - \Big\{\frac{\pi}{2} + k\pi,\ k \in Z\Big\},\ c \in R.$$

$$\int \operatorname{tg}^3 x \, \mathrm{d}x = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \operatorname{tg} x & x \in \left(-\frac{\pi}{2} + 2k\pi;\ \frac{\pi}{2} + 2k\pi\right) \\ x = \operatorname{arctg} t & \mathrm{d}x = \frac{\mathrm{d}t}{t^2+1}, \quad t \in R,\ k \in Z \end{array}\right] = \int \frac{t^3 \, \mathrm{d}t}{t^2+1} = \int \frac{t^3 + t - t \, \mathrm{d}t}{t^2+1} = \int t \, \mathrm{d}t - \frac{1}{2} \int \frac{2t \, \mathrm{d}t}{t^2+1}$$

$$= \int t \, \mathrm{d}t - \frac{1}{2} \int \frac{(t^2+1)' \, \mathrm{d}t}{t^2+1} = \frac{t^2}{2} - \ln\left(t^2+1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x\,dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \left[\frac{1}{\cos^2 x} - 1\right] dx = \operatorname{tg} x - x + c,$$

$$x \in R - \left\{\frac{\pi}{2} + k\pi,\ k \in Z\right\},\ c \in R.$$

$$\int \operatorname{tg}^3 x\,dx = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \operatorname{tg} x & x \in \left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right) \\ x = \operatorname{arctg} t & dx = \frac{dt}{t^2+1},\quad t \in R,\ k \in Z \end{array}\right] = \int \frac{t^3\,dt}{t^2+1} = \int \frac{t^3+t-t\,dt}{t^2+1} = \int t\,dt - \frac{1}{2}\int \frac{2t\,dt}{t^2+1}$$

$$= \int t\,dt - \frac{1}{2}\int \frac{(t^2+1)'\,dt}{t^2+1} = \frac{t^2}{2} - \ln\left(t^2+1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln \frac{\sin^2 x + \cos^2 x}{\cos^2 x} + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x\,dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \Big[\frac{1}{\cos^2 x} - 1\Big]dx = \operatorname{tg} x - x + c,$$
$$x \in R - \big\{\tfrac{\pi}{2} + k\pi,\ k\in Z\big\},\ c \in R.$$

$$\int \operatorname{tg}^3 x\,dx = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \operatorname{tg} x & x \in \left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right) \\ & x = \operatorname{arctg} t & dx = \frac{dt}{t^2+1},\quad t\in R,\ k\in Z \end{array}\right] = \int \frac{t^3\,dt}{t^2+1} = \int \frac{t^3+t-t\,dt}{t^2+1} = \int t\,dt - \frac{1}{2}\int \frac{2t\,dt}{t^2+1}$$

$$= \int t\,dt - \frac{1}{2}\int \frac{(t^2+1)'\,dt}{t^2+1} = \frac{t^2}{2} - \ln\left(t^2+1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln\frac{\sin^2 x + \cos^2 x}{\cos^2 x} + c = \frac{\operatorname{tg}^2 x}{2} - \ln\frac{1}{\cos^2 x} + c$$

Riešené príklady – 025, 026

$$\int \operatorname{tg}^2 x\,dx = \operatorname{tg} x - x + c$$

$$= \int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx = \int \Big[\frac{1}{\cos^2 x} - 1\Big]dx = \operatorname{tg} x - x + c,$$
$$x \in R - \Big\{\frac{\pi}{2} + k\pi,\ k \in Z\Big\},\ c \in R.$$

$$\int \operatorname{tg}^3 x\,dx = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c = \frac{\operatorname{tg}^2 x}{2} + \ln\cos^2 x + c$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t = \operatorname{tg} x & x \in (-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi) \\ \quad x = \operatorname{arctg} t & dx = \frac{dt}{t^2+1},\quad t \in R,\ k \in Z \end{array}\right] = \int \frac{t^3\,dt}{t^2+1} = \int \frac{t^3+t-t\,dt}{t^2+1} = \int t\,dt - \frac{1}{2}\int \frac{2t\,dt}{t^2+1}$$

$$= \int t\,dt - \frac{1}{2}\int \frac{(t^2+1)'\,dt}{t^2+1} = \frac{t^2}{2} - \ln\left(t^2+1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln\left(\operatorname{tg}^2 x + 1\right) + c$$

$$= \frac{\operatorname{tg}^2 x}{2} - \ln\left(\frac{\sin^2 x}{\cos^2 x} + 1\right) + c = \frac{\operatorname{tg}^2 x}{2} - \ln\frac{\sin^2 x + \cos^2 x}{\cos^2 x} + c = \frac{\operatorname{tg}^2 x}{2} - \ln\frac{1}{\cos^2 x} + c$$

$$= \frac{\operatorname{tg}^2 x}{2} + \ln\cos^2 x + c,\ x \in R - \Big\{\frac{\pi}{2} + k\pi,\ k \in Z\Big\},\ c \in R.$$

Riešené príklady – 027, 028

$\int \text{cotg}^2 x \, dx$

$\int \text{cotg}^3 x \, dx$

Riešené príklady – 027, 028

$\int \operatorname{cotg}^2 x \, dx$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx$$

$\int \operatorname{cotg}^3 x \, dx$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1}, \quad t \in R, \ k \in Z \end{array} \right]$$

Riešené príklady – 027, 028

$\int \operatorname{cotg}^2 x \, dx$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx$$

$\int \operatorname{cotg}^3 x \, dx$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1}, \ t \in R, \ k \in Z \end{array} \right] = -\int \frac{t^3 \, dt}{t^2+1}$$

Riešené príklady – 027, 028

$\int \operatorname{cotg}^2 x \, dx$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \Big[\frac{1}{\sin^2 x} - 1\Big] dx$$

$\int \operatorname{cotg}^3 x \, dx$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1}, \ t \in R, \ k \in Z \end{array} \right] = -\int \frac{t^3 \, dt}{t^2+1} = -\int \frac{t^3+t-t \, dt}{t^2+1}$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x}\, dx = \int \frac{1-\sin^2 x}{\sin^2 x}\, dx = \int \Big[\frac{1}{\sin^2 x} - 1\Big] dx = -\operatorname{cotg} x - x + c,$$

$$x \in R - \{k\pi,\ k \in Z\},\ c \in R.$$

$$\int \operatorname{cotg}^3 x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1},\ t \in R,\ k \in Z \end{array}\right] = -\int \frac{t^3\, dt}{t^2+1} = -\int \frac{t^3+t-t\, dt}{t^2+1} = -\int t\, dt + \frac{1}{2}\int \frac{2t\, dt}{t^2+1}$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x\, dx = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x}\, dx = \int \frac{1-\sin^2 x}{\sin^2 x}\, dx = \int \Big[\frac{1}{\sin^2 x} - 1\Big] dx = -\operatorname{cotg} x - x + c,$$
$$x \in R - \{k\pi,\ k \in Z\},\ c \in R.$$

$$\int \operatorname{cotg}^3 x\, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1},\ t \in R,\ k \in Z \end{array} \right] = -\int \frac{t^3\, dt}{t^2+1} = -\int \frac{t^3+t-t\, dt}{t^2+1} = -\int t\, dt + \frac{1}{2}\int \frac{2t\, dt}{t^2+1}$$
$$= \frac{1}{2}\int \frac{(t^2+1)'\, dt}{t^2+1} - \int t\, dt$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x}\, dx = \int \frac{1-\sin^2 x}{\sin^2 x}\, dx = \int \Big[\frac{1}{\sin^2 x} - 1\Big] dx = -\operatorname{cotg} x - x + c,$$
$$x \in R - \{k\pi,\ k \in Z\},\ c \in R.$$

$$\int \operatorname{cotg}^3 x \, dx$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1},\ \ t \in R,\ k \in Z \end{array} \right] = -\int \frac{t^3\, dt}{t^2+1} = -\int \frac{t^3+t-t\, dt}{t^2+1} = -\int t\, dt + \frac{1}{2}\int \frac{2t\, dt}{t^2+1}$$
$$= \frac{1}{2}\int \frac{(t^2+1)'\, dt}{t^2+1} - \int t\, dt = \ln\left(t^2+1\right) - \frac{t^2}{2} + c$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \Big[\frac{1}{\sin^2 x} - 1\Big] dx = -\operatorname{cotg} x - x + c,$$
$$x \in R - \{k\pi,\ k \in Z\},\ c \in R.$$

$$\int \operatorname{cotg}^3 x \, dx = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1},\ t \in R,\ k \in Z \end{array}\right] = -\int \frac{t^3 \, dt}{t^2+1} = -\int \frac{t^3+t-t \, dt}{t^2+1} = -\int t \, dt + \frac{1}{2}\int \frac{2t \, dt}{t^2+1}$$

$$= \frac{1}{2}\int \frac{(t^2+1)' \, dt}{t^2+1} - \int t \, dt = \ln\left(t^2+1\right) - \frac{t^2}{2} + c = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x \,\mathrm{d}x = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x}\,\mathrm{d}x = \int \frac{1-\sin^2 x}{\sin^2 x}\,\mathrm{d}x = \int \Big[\frac{1}{\sin^2 x} - 1\Big]\mathrm{d}x = -\operatorname{cotg} x - x + c,$$

$$x \in R - \{k\pi,\ k \in Z\},\ c \in R.$$

$$\int \operatorname{cotg}^3 x \,\mathrm{d}x = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & \mathrm{d}x = -\frac{\mathrm{d}t}{t^2+1},\ t \in R,\ k \in Z \end{array}\right] = -\int \frac{t^3\,\mathrm{d}t}{t^2+1} = -\int \frac{t^3+t-t\,\mathrm{d}t}{t^2+1} = -\int t\,\mathrm{d}t + \frac{1}{2}\int \frac{2t\,\mathrm{d}t}{t^2+1}$$

$$= \frac{1}{2}\int \frac{(t^2+1)'\,\mathrm{d}t}{t^2+1} - \int t\,\mathrm{d}t = \ln\left(t^2+1\right) - \frac{t^2}{2} + c = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \ln\left(\frac{\cos^2 x}{\sin^2 x} + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \left[\frac{1}{\sin^2 x} - 1\right] dx = -\operatorname{cotg} x - x + c,$$

$$x \in R - \{k\pi, \ k \in Z\}, \ c \in R.$$

$$\int \operatorname{cotg}^3 x \, dx = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } \ t = \operatorname{cotg} x & x \in (0+2k\pi; \pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1}, \ t \in R, \ k \in Z \end{array} \right] = -\int \frac{t^3 \, dt}{t^2+1} = -\int \frac{t^3+t-t \, dt}{t^2+1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} - \int t \, dt = \ln\left(t^2+1\right) - \frac{t^2}{2} + c = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \ln\left(\frac{\cos^2 x}{\sin^2 x} + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c = \ln \frac{\cos^2 x + \sin^2 x}{\sin^2 x} - \frac{\operatorname{cotg}^2 x}{2} + c$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x} \, dx = \int \frac{1-\sin^2 x}{\sin^2 x} \, dx = \int \Big[\frac{1}{\sin^2 x} - 1\Big] dx = -\operatorname{cotg} x - x + c,$$

$$x \in R - \{k\pi,\ k \in Z\},\ c \in R.$$

$$\int \operatorname{cotg}^3 x \, dx = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t = \operatorname{cotg} x & x \in (0+2k\pi;\pi+2k\pi) \\ x = \operatorname{arccotg} t & dx = -\frac{dt}{t^2+1},\ \ t \in R,\ k \in Z \end{array}\right] = -\int \frac{t^3 \, dt}{t^2+1} = -\int \frac{t^3+t-t \, dt}{t^2+1} = -\int t \, dt + \frac{1}{2} \int \frac{2t \, dt}{t^2+1}$$

$$= \frac{1}{2} \int \frac{(t^2+1)' \, dt}{t^2+1} - \int t \, dt = \ln\left(t^2+1\right) - \frac{t^2}{2} + c = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \ln\left(\frac{\cos^2 x}{\sin^2 x} + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c = \ln \frac{\cos^2 x + \sin^2 x}{\sin^2 x} - \frac{\operatorname{cotg}^2 x}{2} + c = \ln \frac{1}{\sin^2 x} - \frac{\operatorname{cotg}^2 x}{2} + c$$

Riešené príklady – 027, 028

$$\int \operatorname{cotg}^2 x\,dx = -\operatorname{cotg} x - x + c$$

$$= \int \frac{\cos^2 x}{\sin^2 x}\,dx = \int \frac{1-\sin^2 x}{\sin^2 x}\,dx = \int \Big[\frac{1}{\sin^2 x} - 1\Big]dx = -\operatorname{cotg} x - x + c,$$
$$x \in R - \{k\pi,\ k\in Z\},\ c\in R.$$

$$\int \operatorname{cotg}^3 x\,dx = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c = -\ln\sin^2 x - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=\operatorname{cotg} x & x\in(0+2k\pi;\pi+2k\pi) \\ x=\operatorname{arccotg} t & dx=-\frac{dt}{t^2+1},\ t\in R,\ k\in Z \end{array}\right] = -\int \frac{t^3\,dt}{t^2+1} = -\int \frac{t^3+t-t\,dt}{t^2+1} = -\int t\,dt + \frac{1}{2}\int \frac{2t\,dt}{t^2+1}$$

$$= \frac{1}{2}\int \frac{(t^2+1)'\,dt}{t^2+1} - \int t\,dt = \ln\left(t^2+1\right) - \frac{t^2}{2} + c = \ln\left(\operatorname{cotg}^2 x + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= \ln\left(\frac{\cos^2 x}{\sin^2 x} + 1\right) - \frac{\operatorname{cotg}^2 x}{2} + c = \ln\frac{\cos^2 x + \sin^2 x}{\sin^2 x} - \frac{\operatorname{cotg}^2 x}{2} + c = \ln\frac{1}{\sin^2 x} - \frac{\operatorname{cotg}^2 x}{2} + c$$

$$= -\ln\sin^2 x - \frac{\operatorname{cotg}^2 x}{2} + c,\ x\in R - \{k\pi,\ k\in Z\},\ c\in R.$$

Riešené príklady – 029

$\int \operatorname{tgh}^2 x \, dx$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx$$

$$= \int \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)^2 dx$$

Riešené príklady – 029

$$\int \mathrm{tgh}^2 x \, \mathrm{d}x$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, \mathrm{d}x = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, \mathrm{d}x$$

$$= \int \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right)^2 \mathrm{d}x = \left[\text{Subst. } t = e^{2x},\ \mathrm{d}x = \frac{\mathrm{d}t}{2t} \;\middle|\; 2x = \ln t,\ t \in (0; \infty) \;\middle|\; x = \frac{\ln t}{2},\ x \in R \right]$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x}\right] dx$$

$$= \int \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0; \infty) & x \in R \end{array}\right] = \int \left(\frac{t-1}{t+1}\right)^2 \frac{dt}{2t}$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x}\right] dx = x - \operatorname{tgh} x + c_1,$$

$$x \in R, \; c_1 \in R.$$

$$= \int \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0; \infty) & x \in R \end{array}\right] = \int \left(\frac{t-1}{t+1}\right)^2 \frac{dt}{2t} = \int \frac{(t^2 - 2t + 1) \, dt}{2t(t^2 + 2t + 1)}$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \Big[1 - \frac{1}{\cosh^2 x}\Big] dx = x - \operatorname{tgh} x + c_1, \quad x \in R, \ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x} - 1}{e^{2x} + 1}\Big)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0; \infty) & x \in R \end{array}\right] = \int \Big(\frac{t-1}{t+1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2 - 2t + 1)\, dt}{2t(t^2 + 2t + 1)}$$

$$= \int \frac{(t^2 + 2t + 1 - 4t)\, dt}{2t(t^2 + 2t + 1)}$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \Big[1 - \frac{1}{\cosh^2 x}\Big] dx = x - \operatorname{tgh} x + c_1,$$

$$x \in R, \ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x} - 1}{e^{2x} + 1}\Big)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0; \infty) & x \in R \end{array}\right] = \int \Big(\frac{t-1}{t+1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2 - 2t + 1)\, dt}{2t(t^2 + 2t + 1)}$$

$$= \int \frac{(t^2 + 2t + 1 - 4t)\, dt}{2t(t^2 + 2t + 1)} = \int \frac{(t^2 + 2t + 1)\, dt}{2t(t^2 + 2t + 1)} - \int \frac{4t\, dt}{2t(t^2 + 2t + 1)}$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \Big[1 - \frac{1}{\cosh^2 x}\Big] dx = x - \operatorname{tgh} x + c_1, \quad x \in R, \ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x} - 1}{e^{2x} + 1}\Big)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0; \infty) & x \in R \end{array}\right] = \int \Big(\frac{t-1}{t+1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2 - 2t + 1)\, dt}{2t(t^2 + 2t + 1)}$$

$$= \int \frac{(t^2 + 2t + 1 - 4t)\, dt}{2t(t^2 + 2t + 1)} = \int \frac{(t^2 + 2t + 1)\, dt}{2t(t^2 + 2t + 1)} - \int \frac{4t \, dt}{2t(t^2 + 2t + 1)} = \int \frac{dt}{2t} - \int \frac{2 \, dt}{t^2 + 2t + 1}$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x}\right] dx = x - \operatorname{tgh} x + c_1, \quad x \in R, \ c_1 \in R.$$

$$= \int \left(\frac{e^{2x}-1}{e^{2x}+1}\right)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \left(\frac{t-1}{t+1}\right)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2+2t+1)}$$

$$= \int \frac{(t^2+2t+1-4t)\,dt}{2t(t^2+2t+1)} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2+2t+1)} - \int \frac{4t\,dt}{2t(t^2+2t+1)} = \int \frac{dt}{2t} - \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{dt}{2t} - \int \frac{2\,dt}{(t+1)^2}$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \Big[1 - \frac{1}{\cosh^2 x}\Big] dx = x - \operatorname{tgh} x + c_1, \quad x \in R, \; c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}-1}{e^{2x}+1}\Big)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \Big(\frac{t-1}{t+1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2+2t+1)}$$

$$= \int \frac{(t^2+2t+1-4t)\,dt}{2t(t^2+2t+1)} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2+2t+1)} - \int \frac{4t\,dt}{2t(t^2+2t+1)} = \int \frac{dt}{2t} - \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{dt}{2t} - \int \frac{2\,dt}{(t+1)^2} = \frac{1}{2}\int \frac{dt}{t} - 2\int (t+1)^{-2}\,dt$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \Big[1 - \frac{1}{\cosh^2 x}\Big] dx = x - \operatorname{tgh} x + c_1,$$

$$x \in R, \; c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}-1}{e^{2x}+1}\Big)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \Big(\frac{t-1}{t+1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2+2t+1)}$$

$$= \int \frac{(t^2+2t+1-4t)\,dt}{2t(t^2+2t+1)} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2+2t+1)} - \int \frac{4t\,dt}{2t(t^2+2t+1)} = \int \frac{dt}{2t} - \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{dt}{2t} - \int \frac{2\,dt}{(t+1)^2} = \frac{1}{2}\int \frac{dt}{t} - 2\int (t+1)^{-2}\,dt = \frac{1}{2}\ln t - 2\frac{(t+1)^{-1}}{-1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x}\right] dx = x - \operatorname{tgh} x + c_1,$$

$$x \in R, \; c_1 \in R.$$

$$= \int \left(\frac{e^{2x}-1}{e^{2x}+1}\right)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \left(\frac{t-1}{t+1}\right)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1)\, dt}{2t(t^2+2t+1)}$$

$$= \int \frac{(t^2+2t+1-4t)\, dt}{2t(t^2+2t+1)} = \int \frac{(t^2+2t+1)\, dt}{2t(t^2+2t+1)} - \int \frac{4t\, dt}{2t(t^2+2t+1)} = \int \frac{dt}{2t} - \int \frac{2\, dt}{t^2+2t+1}$$

$$= \int \frac{dt}{2t} - \int \frac{2\, dt}{(t+1)^2} = \frac{1}{2}\int \frac{dt}{t} - 2\int (t+1)^{-2}\, dt = \frac{1}{2} \ln t - 2\frac{(t+1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \Big[1 - \frac{1}{\cosh^2 x}\Big] dx = x - \operatorname{tgh} x + c_1, \quad x \in R,\ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}-1}{e^{2x}+1}\Big)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \Big(\frac{t-1}{t+1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1)\, dt}{2t(t^2+2t+1)}$$

$$= \int \frac{(t^2+2t+1-4t)\, dt}{2t(t^2+2t+1)} = \int \frac{(t^2+2t+1)\, dt}{2t(t^2+2t+1)} - \int \frac{4t\, dt}{2t(t^2+2t+1)} = \int \frac{dt}{2t} - \int \frac{2\, dt}{t^2+2t+1}$$

$$= \int \frac{dt}{2t} - \int \frac{2\, dt}{(t+1)^2} = \frac{1}{2}\int \frac{dt}{t} - 2\int (t+1)^{-2}\, dt = \frac{1}{2} \ln t - 2\frac{(t+1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2} \ln t + \frac{2}{t+1} + c_2 = \frac{1}{2} \ln e^{2x} + \frac{2}{e^{2x}+1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x\,dx = x - \operatorname{tgh} x + c_1$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x}\,dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x}\,dx = \int \Big[1 - \frac{1}{\cosh^2 x}\Big]dx = x - \operatorname{tgh} x + c_1,$$

$$x \in R,\ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}-1}{e^{2x}+1}\Big)^2 dx = \left[\begin{array}{r|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \Big(\frac{t-1}{t+1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2+2t+1)}$$

$$= \int \frac{(t^2+2t+1-4t)\,dt}{2t(t^2+2t+1)} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2+2t+1)} - \int \frac{4t\,dt}{2t(t^2+2t+1)} = \int \frac{dt}{2t} - \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{dt}{2t} - \int \frac{2\,dt}{(t+1)^2} = \frac{1}{2}\int \frac{dt}{t} - 2\int (t+1)^{-2}\,dt = \frac{1}{2}\ln t - 2\frac{(t+1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t + \frac{2}{t+1} + c_2 = \frac{1}{2}\ln e^{2x} + \frac{2}{e^{2x}+1} + c_2 = \frac{2x}{2}\ln e + \frac{2}{e^{2x}+1} + c_2$$

Riešené príklady – 029

$$\int \operatorname{tgh}^2 x \, dx = x - \operatorname{tgh} x + c_1 = x + \frac{2}{e^{2x}+1} + c_2$$

$$= \int \frac{\sinh^2 x}{\cosh^2 x} \, dx = \int \frac{\cosh^2 x - 1}{\cosh^2 x} \, dx = \int \left[1 - \frac{1}{\cosh^2 x}\right] dx = x - \operatorname{tgh} x + c_1,$$

$$x \in R, \ c_1 \in R.$$

$$= \int \left(\frac{e^{2x}-1}{e^{2x}+1}\right)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \left(\frac{t-1}{t+1}\right)^2 \frac{dt}{2t} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2+2t+1)}$$

$$= \int \frac{(t^2+2t+1-4t)\,dt}{2t(t^2+2t+1)} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2+2t+1)} - \int \frac{4t\,dt}{2t(t^2+2t+1)} = \int \frac{dt}{2t} - \int \frac{2\,dt}{t^2+2t+1}$$

$$= \int \frac{dt}{2t} - \int \frac{2\,dt}{(t+1)^2} = \frac{1}{2}\int \frac{dt}{t} - 2\int (t+1)^{-2}\,dt = \frac{1}{2}\ln t - 2\frac{(t+1)^{-1}}{-1} + c_2$$

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Riešené príklady – 029

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$$= \left[\begin{array}{c} c_1 = c_2 + 1 \\ c_1 \in R, \ c_2 \in R \end{array}\right]$$

Riešené príklady – 029

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$$= \left[\begin{array}{c} c_1 = c_2 + 1 \\ c_1 \in R,\ c_2 \in R \end{array}\right] = x - \frac{e^{2x}-1}{e^{2x}+1} + c_1$$

Riešené príklady – 029

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$$= \left[\begin{array}{c} c_1 = c_2 + 1 \\ c_1 \in R,\ c_2 \in R \end{array}\right] = x - \frac{e^{2x}-1}{e^{2x}+1} + c_1 = x - \operatorname{tgh} x + c_1,\ x \in R,\ c_1, c_2 \in R.$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx$$

$$= \int \left(\frac{e^{2x} + 1}{e^{2x} - 1} \right)^2 dx$$

Riešené príklady – 030

$\int \operatorname{cotgh}^2 x \, dx$

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Riešené príklady – 030

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Riešené príklady – 030

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Riešené príklady – 030

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Riešené príklady – 030

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Riešené príklady – 030

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Riešené príklady – 030

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Riešené príklady – 030

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Riešené príklady – 030

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Riešené príklady – 030

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$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sin^2 x} \, dx = \int \Big[1 + \frac{1}{\sinh^2 x}\Big] dx = x - \operatorname{cotgh} x + c_1,$$

$$x \in R, \ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}+1}{e^{2x}-1}\Big)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array} \right] = \int \Big(\frac{t+1}{t-1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\, dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\, dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\, dt}{2t(t^2-2t+1)} + \int \frac{4t\, dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\, dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\, dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\, dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x\,dx = x - \operatorname{cotgh} x + c_1$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x}\,dx = \int \frac{\sinh^2 x+1}{\sin^2 x}\,dx = \int \Big[1 + \frac{1}{\sinh^2 x}\Big]dx = x - \operatorname{cotgh} x + c_1,$$

$$x \in R,\ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}+1}{e^{2x}-1}\Big)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t=e^{2x} & 2x=\ln t & x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} & t\in(0;\infty) & x\in R \end{array}\right] = \int \Big(\frac{t+1}{t-1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\,dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2-2t+1)} + \int \frac{4t\,dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\,dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\,dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\,dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2}\ln e - \frac{2}{e^{2x}-1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x\,dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x}\,dx = \int \frac{\sinh^2 x + 1}{\sin^2 x}\,dx = \int \left[1 + \frac{1}{\sinh^2 x}\right] dx = x - \operatorname{cotgh} x + c_1,$$
$$x \in R,\ c_1 \in R.$$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1}\right)^2 dx = \left[\begin{array}{r|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \left(\frac{t+1}{t-1}\right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\,dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2-2t+1)} + \int \frac{4t\,dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\,dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\,dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\,dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2}\ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x\,dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x}\,dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x}\,dx = \int \Big[1 + \frac{1}{\sinh^2 x}\Big]dx = x - \operatorname{cotgh} x + c_1,$$

$$x \in R,\ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}+1}{e^{2x}-1}\Big)^2 dx = \left[\begin{array}{l|l|l} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \Big(\frac{t+1}{t-1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\,dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2-2t+1)} + \int \frac{4t\,dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\,dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\,dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\,dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2}\ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x\,dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x}\,dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x}\,dx = \int \Big[1 + \frac{1}{\sinh^2 x}\Big]dx = x - \operatorname{cotgh} x + c_1,$$

$$x \in R,\ c_1 \in R.$$

$$= \int \Big(\frac{e^{2x}+1}{e^{2x}-1}\Big)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \Big(\frac{t+1}{t-1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\,dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2-2t+1)} + \int \frac{4t\,dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\,dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\,dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\,dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2}\ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x \, dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x} \, dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x} \, dx = \int \left[1 + \frac{1}{\sinh^2 x}\right] dx = x - \operatorname{cotgh} x + c_1, \quad x \in R,\ c_1 \in R.$$

$$= \int \left(\frac{e^{2x}+1}{e^{2x}-1}\right)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t = e^{2x} & 2x = \ln t & x = \frac{\ln t}{2} \\ dx = \frac{dt}{2t} & t \in (0;\infty) & x \in R \end{array}\right] = \int \left(\frac{t+1}{t-1}\right)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\,dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2-2t+1)} + \int \frac{4t\,dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\,dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\,dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\,dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2}\ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

$$= \left[\begin{array}{c} c_1 = c_2 + 1 \\ c_1 \in R,\ c_2 \in R \end{array}\right]$$

Riešené príklady – 030

$$\int \operatorname{cotgh}^2 x\,dx = x - \operatorname{cotgh} x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x}\,dx = \int \frac{\sinh^2 x + 1}{\sinh^2 x}\,dx = \int \Big[1 + \frac{1}{\sinh^2 x}\Big]dx = x - \operatorname{cotgh} x + c_1, \quad x\in R,\ c_1\in R.$$

$$= \int \Big(\frac{e^{2x}+1}{e^{2x}-1}\Big)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t=e^{2x} & 2x=\ln t & x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} & t\in(0;\infty) & x\in R \end{array}\right] = \int \Big(\frac{t+1}{t-1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\,dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2-2t+1)} + \int \frac{4t\,dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\,dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\,dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\,dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2}\ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

$$= \left[\begin{array}{c} c_1 = c_2 + 1 \\ c_1\in R,\ c_2\in R \end{array}\right] = x - \frac{e^{2x}+1}{e^{2x}-1} + c_1$$

Riešené príklady – 030

$$\int \coth^2 x\,dx = x - \coth x + c_1 = x - \frac{2}{e^{2x}-1} + c_2$$

$$= \int \frac{\cosh^2 x}{\sinh^2 x}\,dx = \int \frac{\sinh^2 x+1}{\sin^2 x}\,dx = \int \Big[1+\frac{1}{\sinh^2 x}\Big]dx = x - \coth x + c_1,$$
$$x\in R,\ c_1\in R.$$

$$= \int \Big(\frac{e^{2x}+1}{e^{2x}-1}\Big)^2 dx = \left[\begin{array}{c|c|c} \text{Subst. } t=e^{2x} & 2x=\ln t & x=\frac{\ln t}{2} \\ dx=\frac{dt}{2t} & t\in(0;\infty) & x\in R \end{array}\right] = \int \Big(\frac{t+1}{t-1}\Big)^2 \frac{dt}{2t} = \int \frac{(t^2+2t+1)\,dt}{2t(t^2-2t+1)}$$

$$= \int \frac{(t^2-2t+1+4t)\,dt}{2t(t^2-2t+1)} = \int \frac{(t^2-2t+1)\,dt}{2t(t^2-2t+1)} + \int \frac{4t\,dt}{2t(t^2-2t+1)} = \int \frac{dt}{2t} + \int \frac{2\,dt}{t^2-2t+1}$$

$$= \int \frac{dt}{2t} + \int \frac{2\,dt}{(t-1)^2} = \frac{1}{2}\int \frac{dt}{t} + 2\int (t-1)^{-2}\,dt = \frac{1}{2}\ln t + 2\frac{(t-1)^{-1}}{-1} + c_2$$

$$= \frac{1}{2}\ln t - \frac{2}{t-1} + c_2 = \frac{1}{2}\ln e^{2x} - \frac{2}{e^{2x}-1} + c_2 = \frac{2x}{2}\ln e - \frac{2}{e^{2x}-1} + c_2$$

$$= x - \frac{2}{e^{2x}-1} + c_2 = x + \frac{-2}{e^{2x}-1} - 1 + c_2 + 1 = x + \frac{-2-e^{2x}+1}{e^{2x}-1} + c_2 + 1$$

$$= \left[\begin{array}{c} c_1 = c_2+1 \\ c_1\in R,\ c_2\in R \end{array}\right] = x - \frac{e^{2x}+1}{e^{2x}-1} + c_1 = x - \coth x + c_1,\ x\in R,\ c_1,c_2\in R.$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t=4+3\sin x & x\in\left\langle -\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi\right\rangle \\ dt=3\cos x\,dx & t\in\langle 1;7\rangle,\ \ k\in Z \end{array} \right]$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3\sin x+2\cos x}{2\sin x\cos x}\,dx$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t=4+3\sin x & x\in\left\langle -\frac{\pi}{2}+k\pi;\ \frac{\pi}{2}+k\pi \right\rangle \\ dt=3\cos x\,dx & t\in\langle 1;7\rangle,\ \ k\in Z \end{array} \right] = \frac{1}{3}\int \frac{dt}{t}$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx$$

$$= \int \frac{dx}{\sin 2x} + \int \frac{3\sin x+2\cos x}{2\sin x\cos x}\,dx = \left[\begin{array}{r|l|l} \text{Subst.}\quad u=2x & x\in\left(0+\frac{k\pi}{2};\ \frac{\pi}{2}+\frac{k\pi}{2}\right) = \left(\frac{k\pi}{2};\ (k+1)\frac{\pi}{2}\right) & x\neq\frac{k\pi}{2} \\ du=2\,dx & u\in(0+k\pi;\ \pi+k\pi) = (k\pi;\ (k+1)\pi) & k\in Z \end{array} \right]$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t=4+3\sin x & x\in\left\langle -\frac{\pi}{2}+k\pi;\ \frac{\pi}{2}+k\pi\right\rangle \\ dt=3\cos x\,dx & t\in\langle 1;7\rangle,\ \ k\in Z \end{array} \right] = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln t + c$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx$$

$$= \int\frac{dx}{\sin 2x} + \int\frac{3\sin x+2\cos x}{2\sin x\cos x}\,dx = \left[\begin{array}{r|l|l} \text{Subst.}\quad u=2x & x\in\left(0+\frac{k\pi}{2};\ \frac{\pi}{2}+\frac{k\pi}{2}\right) = \left(\frac{k\pi}{2};\ (k+1)\frac{\pi}{2}\right) & x\neq\frac{k\pi}{2} \\ du=2\,dx & u\in(0+k\pi;\ \pi+k\pi) = (k\pi;\ (k+1)\pi) & k\in Z \end{array} \right]$$

$$= \frac{1}{2}\int\frac{du}{\sin u} + \frac{3}{2}\int\frac{dx}{\cos x} + \frac{dx}{\sin x}$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx = \frac{1}{3}\ln(4+3\sin x)+c$$

$$= \left[\begin{array}{r|l|} \text{Subst. } t=4+3\sin x & x\in\langle -\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi\rangle \\ dt=3\cos x\,dx & t\in\langle 1;7\rangle,\ k\in Z \end{array}\right] = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln t + c$$

$$= \frac{1}{3}\ln(4+3\sin x)+c,\ x\in R,\ c\in R.$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx = \frac{1}{2}\ln|\operatorname{tg} x| - \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}-1\right| + \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}+1\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$= \int\frac{dx}{\sin 2x} + \int\frac{3\sin x+2\cos x}{2\sin x\cos x}\,dx = \left[\begin{array}{r|l|l} \text{Subst. } u=2x & x\in(0+\frac{k\pi}{2};\, \frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2};\,(k+1)\frac{\pi}{2}) & x\neq\frac{k\pi}{2} \\ du=2\,dx & u\in(0+k\pi;\, \pi+k\pi) = (k\pi;\,(k+1)\pi) & k\in Z \end{array}\right]$$

$$= \frac{1}{2}\int\frac{du}{\sin u} + \frac{3}{2}\int\frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l} \text{Pr. } 01 \\ \text{Pr. } 02 \end{array}\right]$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx = \frac{1}{3}\ln(4+3\sin x)+c$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t=4+3\sin x & x\in\left\langle -\frac{\pi}{2}+k\pi;\ \frac{\pi}{2}+k\pi\right\rangle \\ dt=3\cos x\,dx & t\in\langle 1;7\rangle,\ \ k\in Z \end{array}\right] = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln t + c$$

$$= \frac{1}{3}\ln(4+3\sin x)+c,\ x\in R,\ c\in R.$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx = \frac{1}{2}\ln|\operatorname{tg} x| - \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}-1\right| + \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}+1\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$= \int\frac{dx}{\sin 2x} + \int\frac{3\sin x+2\cos x}{2\sin x\cos x}\,dx = \left[\begin{array}{r|l|l} \text{Subst.}\quad u=2x & x\in\left(0+\frac{k\pi}{2};\ \frac{\pi}{2}+\frac{k\pi}{2}\right)=\left(\frac{k\pi}{2};(k+1)\frac{\pi}{2}\right) & x\neq\frac{k\pi}{2} \\ du=2\,dx & u\in(0+k\pi;\ \pi+k\pi)=(k\pi;(k+1)\pi) & k\in Z \end{array}\right]$$

$$= \frac{1}{2}\int\frac{du}{\sin u} + \frac{3}{2}\int\frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l}\text{Pr. 01}\\ \text{Pr. 02}\end{array}\right] = \frac{1}{2}\ln\left|\operatorname{tg}\frac{u}{2}\right| - \frac{3}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx = \frac{1}{3}\ln(4+3\sin x)+c$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t=4+3\sin x & x\in\langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi\rangle \\ dt=3\cos x\,dx & t\in\langle 1;7\rangle,\ k\in Z \end{array} \right] = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln t + c$$

$$= \frac{1}{3}\ln(4+3\sin x)+c,\ x\in R,\ c\in R.$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx = \frac{1}{2}\ln|\operatorname{tg} x| - \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}-1\right| + \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}+1\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$= \int\frac{dx}{\sin 2x} + \int\frac{3\sin x+2\cos x}{2\sin x\cos x}\,dx = \left[\begin{array}{r|l|l} \text{Subst.}\quad u=2x & x\in(0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) & x\neq\frac{k\pi}{2} \\ du=2\,dx & u\in(0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) & k\in Z \end{array} \right]$$

$$= \frac{1}{2}\int\frac{du}{\sin u} + \frac{3}{2}\int\frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l} \text{Pr. } 01 \\ \text{Pr. } 02 \end{array} \right] = \frac{1}{2}\ln\left|\operatorname{tg}\frac{u}{2}\right| - \frac{3}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$= \frac{1}{2}\ln|\operatorname{tg} x| - \frac{3}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

Riešené príklady – 031, 032

$$\int \frac{\cos x}{4+3\sin x}\,dx = \frac{1}{3}\ln(4+3\sin x)+c$$

$$= \left[\begin{array}{r|l} \text{Subst.}\quad t=4+3\sin x & x\in\langle -\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi\rangle \\ dt=3\cos x\,dx & t\in\langle 1;7\rangle,\ \ k\in Z \end{array}\right] = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln t + c$$

$$= \frac{1}{3}\ln(4+3\sin x)+c,\ x\in R,\ c\in R.$$

$$\int \frac{1+3\sin x+2\cos x}{\sin 2x}\,dx = \frac{1}{2}\ln|\operatorname{tg} x| - \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}-1\right| + \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}+1\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$= \int\frac{dx}{\sin 2x} + \int\frac{3\sin x+2\cos x}{2\sin x\cos x}\,dx = \left[\begin{array}{r|l|l} \text{Subst.}\quad u=2x & x\in(0+\frac{k\pi}{2};\, \frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2};\, (k+1)\frac{\pi}{2}) & x\neq\frac{k\pi}{2} \\ du=2\,dx & u\in(0+k\pi;\, \pi+k\pi) = (k\pi;\, (k+1)\pi) & k\in Z \end{array}\right]$$

$$= \frac{1}{2}\int\frac{du}{\sin u} + \frac{3}{2}\int\frac{dx}{\cos x} + \frac{dx}{\sin x} = \left[\begin{array}{l} \text{Pr. 01} \\ \text{Pr. 02} \end{array}\right] = \frac{1}{2}\ln\left|\operatorname{tg}\frac{u}{2}\right| - \frac{3}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$= \frac{1}{2}\ln|\operatorname{tg} x| - \frac{3}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-1}{\operatorname{tg}\frac{x}{2}+1}\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c$$

$$= \frac{1}{2}\ln|\operatorname{tg} x| - \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}-1\right| + \frac{3}{2}\ln\left|\operatorname{tg}\frac{x}{2}+1\right| + \ln\left|\operatorname{tg}\frac{x}{2}\right| + c,$$

$$x\in R-\left\{\frac{k\pi}{2},\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$\int [\mathrm{tg}\, x + \mathrm{cotg}\, x]\, dx$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{4\,dx}{(2\sin x\cos x)^2}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x]\,dx$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{4\,dx}{(2\sin x\cos x)^2} = \int \frac{2\cdot 2\,dx}{\sin^2 2x}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x}$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x]\,dx$$

$$= \int \Big(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\Big)dx = \int \Big(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\Big)dx$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$= \int \frac{4\,dx}{(2\sin x\cos x)^2} = \int \frac{2\cdot 2\,dx}{\sin^2 2x} = \left[\begin{array}{r|l|l} \text{Subst. } t=2x & x\in(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2};(k+1)\frac{\pi}{2}) & x\neq\frac{k\pi}{2} \\ dt=2\,dx & t\in(0+k\pi;\pi+k\pi) = (k\pi;(k+1)\pi) & k\in Z \end{array} \right]$$

$$= \int \frac{\sin^2 x+\cos^2 x}{\sin^2 x\cos^2 x}\,dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x]\,dx$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right)dx = \ln|\sin x| - \ln|\cos x| + c$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$= \int \frac{4\,dx}{(2\sin x\cos x)^2} = \int \frac{2\cdot 2\,dx}{\sin^2 2x} = \left[\begin{array}{c|c|c} \text{Subst. } t=2x & x \in (0+\frac{k\pi}{2}; \frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2}; (k+1)\frac{\pi}{2}) & x \neq \frac{k\pi}{2} \\ dt = 2\,dx & t \in (0+k\pi; \pi+k\pi) = (k\pi; (k+1)\pi) & k \in Z \end{array} \right]$$

$$= \int \frac{2\,dt}{\sin^2 t}$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x]\,dx$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right) dx = \ln|\sin x| - \ln|\cos x| + c$$

$$= \ln\left|\frac{\sin x}{\cos x}\right| + c$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c$$

$$= \int \frac{4\,dx}{(2\sin x \cos x)^2} = \int \frac{2 \cdot 2\,dx}{\sin^2 2x} = \left[\begin{array}{l|l|l} \text{Subst. } t = 2x & x \in \left(0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}\right) = \left(\frac{k\pi}{2}; (k+1)\frac{\pi}{2}\right) & x \neq \frac{k\pi}{2} \\ dt = 2\,dx & t \in (0 + k\pi; \pi + k\pi) = (k\pi; (k+1)\pi) & k \in Z \end{array} \right]$$

$$= \int \frac{2\,dt}{\sin^2 t} = -2 \operatorname{cotg} t + c$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$$

$$= 2\,\frac{\sin^2 x - \cos^2 x}{2 \sin x \cos x} + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x]\,dx = \ln |\operatorname{tg} x| + c$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right) dx = \ln |\sin x| - \ln |\cos x| + c$$

$$= \ln \left| \frac{\sin x}{\cos x} \right| + c = \ln |\operatorname{tg} x| + c, \; x \in R - \left\{ \frac{k\pi}{2}, k \in Z \right\}, \; c \in R.$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = -2\operatorname{cotg} 2x + c$$

$$= \int \frac{4\,dx}{(2\sin x\cos x)^2} = \int \frac{2\cdot 2\,dx}{\sin^2 2x} = \left[\begin{array}{c|c|c} \text{Subst.}\quad t=2x & x\in(0+\frac{k\pi}{2};\frac{\pi}{2}+\frac{k\pi}{2}) = (\frac{k\pi}{2};(k+1)\frac{\pi}{2}) & x\neq\frac{k\pi}{2} \\ dt=2\,dx & t\in(0+k\pi;\pi+k\pi)=(k\pi;(k+1)\pi) & k\in Z \end{array} \right]$$

$$= \int \frac{2\,dt}{\sin^2 t} = -2\operatorname{cotg} t + c = -2\operatorname{cotg} 2x + c,\ x\in R-\left\{\frac{k\pi}{2}, k\in Z\right\},\ c\in R.$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x\cos^2 x}\,dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$$

$$= 2\frac{\sin^2 x - \cos^2 x}{2\sin x\cos x} + c = 2\frac{-\cos 2x}{\sin 2x} + c$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x]\,dx = \ln|\operatorname{tg} x| + c$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x}\right) dx = \ln|\sin x| - \ln|\cos x| + c$$

$$= \ln\left|\frac{\sin x}{\cos x}\right| + c = \ln|\operatorname{tg} x| + c,\ x\in R-\left\{\frac{k\pi}{2}, k\in Z\right\},\ c\in R.$$

Riešené príklady – 033, 034

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = -2 \operatorname{cotg} 2x + c$$

$$= \int \frac{4\,dx}{(2 \sin x \cos x)^2} = \int \frac{2 \cdot 2\,dx}{\sin^2 2x} = \left[\begin{array}{c|c|c} \text{Subst. } t = 2x & x \in \left(0 + \frac{k\pi}{2}; \frac{\pi}{2} + \frac{k\pi}{2}\right) = \left(\frac{k\pi}{2}; (k+1)\frac{\pi}{2}\right) & x \neq \frac{k\pi}{2} \\ dt = 2\,dx & t \in (0 + k\pi; \pi + k\pi) = (k\pi; (k+1)\pi) & k \in Z \end{array} \right]$$

$$= \int \frac{2\,dt}{\sin^2 t} = -2 \operatorname{cotg} t + c = -2 \operatorname{cotg} 2x + c, \ x \in R - \left\{ \frac{k\pi}{2}, k \in Z \right\}, \ c \in R.$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx = \int \frac{dx}{\sin^2 x} + \int \frac{dx}{\cos^2 x} = \operatorname{tg} x - \operatorname{cotg} x + c = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} + c$$

$$= 2 \frac{\sin^2 x - \cos^2 x}{2 \sin x \cos x} + c = 2 \frac{-\cos 2x}{\sin 2x} + c = c - 2 \operatorname{cotg} 2x, \ x \in R - \left\{ \frac{k\pi}{2}, k \in Z \right\}, \ c \in R.$$

$$\int [\operatorname{tg} x + \operatorname{cotg} x]\,dx = \ln |\operatorname{tg} x| + c$$

$$= \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left(\frac{(\sin x)'}{\sin x} - \frac{(\cos x)'}{\cos x} \right) dx = \ln |\sin x| - \ln |\cos x| + c$$

$$= \ln \left| \frac{\sin x}{\cos x} \right| + c = \ln |\operatorname{tg} x| + c, \ x \in R - \left\{ \frac{k\pi}{2}, k \in Z \right\}, \ c \in R.$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad a > 0,\ b > 0$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c} \text{Pre všetky } x\in R \\ a^2\cos^2 x + b^2\sin^2 x > 0 \end{array} \right]$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R \\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right]$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} + b^2\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} + b^2\frac{1-\cos 2x}{2}} = \begin{bmatrix} \text{Pre všetky } x\in R \\ a^2\cos^2 x + b^2\sin^2 x > 0 \end{bmatrix} = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right]$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} + b^2\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x>0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2}+b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x+b^2\sin^2 x>0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\,\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} + b^2\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du = 2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u = \frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|c}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|c}\text{Subst. } u=2x & x\in R\\ du = 2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2} = \int \frac{dt}{b^2t^2+a^2}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} + b^2 \frac{1-\cos 2x}{2}} = \left[\begin{matrix} \text{Pre všetky } x\in R \\ a^2\cos^2 x + b^2\sin^2 x > 0 \end{matrix} \right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{matrix} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{matrix} \right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{matrix} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u = \frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{matrix} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2} = \int \frac{dt}{b^2t^2+a^2} = \frac{1}{b^2}\int \frac{dt}{t^2+\frac{a^2}{b^2}}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2} = \int \frac{dt}{b^2t^2+a^2} = \frac{1}{b^2}\int \frac{dt}{t^2+\frac{a^2}{b^2}} = \frac{1}{b^2}\int \frac{dt}{t^2+(\frac{a}{b})^2}$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c} \text{Pre všetky } x\in R \\ a^2\cos^2 x + b^2\sin^2 x > 0 \end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|c} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2} = \int \frac{dt}{b^2t^2+a^2} = \frac{1}{b^2}\int \frac{dt}{t^2+\frac{a^2}{b^2}} = \frac{1}{b^2}\int \frac{dt}{t^2+(\frac{a}{b})^2}$$

$$= \frac{1}{b^2}\cdot\frac{b}{a}\operatorname{arctg}\frac{t}{\frac{a}{b}} + c$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} + b^2\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|c}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2} = \int \frac{dt}{b^2t^2+a^2} = \frac{1}{b^2}\int \frac{dt}{t^2+\frac{a^2}{b^2}} = \frac{1}{b^2}\int \frac{dt}{t^2+(\frac{a}{b})^2}$$

$$= \frac{1}{b^2}\cdot\frac{b}{a}\operatorname{arctg}\frac{t}{\frac{a}{b}} + c = \frac{1}{ab}\operatorname{arctg}\left(\frac{b}{a}t\right) + c$$

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Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} + b^2\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x > 0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du = 2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2} = \int \frac{dt}{b^2t^2+a^2} = \frac{1}{b^2}\int \frac{dt}{t^2+\frac{a^2}{b^2}} = \frac{1}{b^2}\int \frac{dt}{t^2+(\frac{a}{b})^2}$$

$$= \frac{1}{b^2}\cdot\frac{b}{a}\operatorname{arctg}\frac{t}{\frac{a}{b}} + c = \frac{1}{ab}\operatorname{arctg}\left(\frac{b}{a}t\right) + c = \frac{1}{ab}\operatorname{arctg}\left(\frac{b}{a}\operatorname{tg}\frac{u}{2}\right) + c$$

Riešené príklady – 035

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{1}{ab}\,\text{arctg}\left(\frac{b}{a}\,\text{tg}\, x\right) + c \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} + b^2\,\frac{1-\cos 2x}{2}} = \left[\begin{array}{c}\text{Pre všetky } x\in R\\ a^2\cos^2 x + b^2\sin^2 x>0\end{array}\right] = \int \frac{2\,dx}{(a^2+b^2)+(a^2-b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right] = \int \frac{du}{(a^2+b^2)+(a^2-b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l}\text{UGS:} & t=\text{tg}\,\frac{u}{2}=\text{tg}\,x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2+b^2)+(a^2-b^2)\,\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2+b^2)(t^2+1)+(a^2-b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2+b^2t^2+b^2)+(a^2-a^2t^2-b^2+b^2t^2)}$$

$$= \int \frac{2\,dt}{2b^2t^2+2a^2} = \int \frac{dt}{b^2t^2+a^2} = \frac{1}{b^2}\int \frac{dt}{t^2+\frac{a^2}{b^2}} = \frac{1}{b^2}\int \frac{dt}{t^2+(\frac{a}{b})^2}$$

$$= \frac{1}{b^2}\cdot\frac{b}{a}\,\text{arctg}\,\frac{t}{\frac{a}{b}} + c = \frac{1}{ab}\,\text{arctg}\left(\frac{b}{a}t\right) + c = \frac{1}{ab}\,\text{arctg}\left(\frac{b}{a}\,\text{tg}\,\frac{u}{2}\right) + c$$

$$= \frac{1}{ab}\,\text{arctg}\left(\frac{b}{a}\,\text{tg}\,x\right) + c,\ x\in R-\left\{\frac{\pi}{2}+k\pi,\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x} \qquad a > 0,\ b > 0$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x} \qquad a > 0,\ b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}}$$

Riešené príklady – 036

$$\int \frac{\mathrm{d}x}{a^2 \cos^2 x - b^2 \sin^2 x} \qquad a > 0,\ b > 0$$

$$= \int \frac{\mathrm{d}x}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg} \frac{a}{b} \right]$$

Riešené príklady – 036

$$\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x} \qquad a > 0,\ b > 0$$

$$= \int \frac{dx}{a^2 \frac{1+\cos 2x}{2} - b^2 \frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg} \tfrac{a}{b} \right] = \int \frac{2\, dx}{(a^2 - b^2) + (a^2 + b^2) \cos 2x}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} - b^2\,\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b}\right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right]$$

Riešené príklady – 036

$$\int \frac{\mathrm{d}x}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{\mathrm{d}x}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg} \tfrac{a}{b} \right] = \int \frac{2\,\mathrm{d}x}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ \mathrm{d}u = 2\,\mathrm{d}x & u\in R \end{array} \right] = \int \frac{\mathrm{d}u}{(a^2-b^2)+(a^2+b^2)\cos u}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} - b^2\,\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg} \frac{a}{b} \right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{l|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right]$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} - b^2\,\frac{1-\cos 2x}{2}} = \left[\, x \neq \pm \operatorname{arctg} \tfrac{a}{b} \,\right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

Riešené príklady – 036

$$\int \frac{\mathrm{d}x}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{\mathrm{d}x}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm\operatorname{arctg}\frac{a}{b} \right] = \int \frac{2\,\mathrm{d}x}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ \mathrm{d}u=2\,\mathrm{d}x & u\in R \end{array} \right] = \int \frac{\mathrm{d}u}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ \mathrm{d}u=\frac{2\,\mathrm{d}t}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,\mathrm{d}t}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,\mathrm{d}t}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2 \sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b} \right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b} \right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b} \right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2} = \int \frac{dt}{-b^2t^2+a^2}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b} \right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array} \right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2} = \int \frac{dt}{-b^2t^2+a^2} = -\frac{1}{b^2}\int \frac{dt}{t^2-\frac{a^2}{b^2}}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b} \right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2} = \int \frac{dt}{-b^2t^2+a^2} = -\frac{1}{b^2}\int \frac{dt}{t^2-\frac{a^2}{b^2}} = -\frac{1}{b^2}\int \frac{dt}{t^2-(\frac{a}{b})^2}$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b}\right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2} = \int \frac{dt}{-b^2t^2+a^2} = -\frac{1}{b^2}\int \frac{dt}{t^2-\frac{a^2}{b^2}} = -\frac{1}{b^2}\int \frac{dt}{t^2-(\frac{a}{b})^2} = -\frac{1}{b^2}\cdot\frac{b}{2a}\ln\left|\frac{t-\frac{a}{b}}{t+\frac{a}{b}}\right| + c$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg}\frac{a}{b}\right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{l|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2} = \int \frac{dt}{-b^2t^2+a^2} = -\frac{1}{b^2}\int \frac{dt}{t^2-\frac{a^2}{b^2}} = -\frac{1}{b^2}\int \frac{dt}{t^2-(\frac{a}{b})^2} = -\frac{1}{b^2}\cdot\frac{b}{2a}\ln\left|\frac{t-\frac{a}{b}}{t+\frac{a}{b}}\right|+c$$

$$= -\frac{1}{2ab}\ln\left|\frac{bt-a}{bt+a}\right| + c$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2 \sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\,\frac{1+\cos 2x}{2} - b^2\,\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg} \frac{a}{b} \right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg}x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2} = \int \frac{dt}{-b^2t^2+a^2} = -\frac{1}{b^2}\int \frac{dt}{t^2-\frac{a^2}{b^2}} = -\frac{1}{b^2}\int \frac{dt}{t^2-(\frac{a}{b})^2} = -\frac{1}{b^2}\cdot\frac{b}{2a}\ln\left|\frac{t-\frac{a}{b}}{t+\frac{a}{b}}\right| + c$$

$$= -\frac{1}{2ab}\ln\left|\frac{bt-a}{bt+a}\right| + c = \frac{1}{2ab}\ln\left|\frac{bt+a}{bt-a}\right| + c$$

Riešené príklady – 036

$$\int \frac{\mathrm{d}x}{a^2\cos^2 x - b^2\sin^2 x} \qquad a>0,\ b>0$$

$$= \int \frac{\mathrm{d}x}{a^2\,\frac{1+\cos 2x}{2} - b^2\,\frac{1-\cos 2x}{2}} = \left[x \neq \pm \operatorname{arctg} \tfrac{a}{b} \right] = \int \frac{2\,\mathrm{d}x}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ \mathrm{d}u = 2\,\mathrm{d}x & u\in R \end{array} \right] = \int \frac{\mathrm{d}u}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ \mathrm{d}u=\frac{2\,\mathrm{d}t}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,\mathrm{d}t}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,\mathrm{d}t}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,\mathrm{d}t}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,\mathrm{d}t}{-2b^2t^2+2a^2} = \int \frac{\mathrm{d}t}{-b^2t^2+a^2} = -\frac{1}{b^2}\int \frac{\mathrm{d}t}{t^2-\frac{a^2}{b^2}} = -\frac{1}{b^2}\int \frac{\mathrm{d}t}{t^2-(\frac{a}{b})^2} = -\frac{1}{b^2}\cdot\frac{b}{2a}\ln\left|\frac{t-\frac{a}{b}}{t+\frac{a}{b}}\right| + c$$

$$= -\frac{1}{2ab}\ln\left|\frac{bt-a}{bt+a}\right| + c = \frac{1}{2ab}\ln\left|\frac{bt+a}{bt-a}\right| + c = \frac{1}{2ab}\ln\left|\frac{b\operatorname{tg}\frac{u}{2}+a}{b\operatorname{tg}\frac{u}{2}-a}\right| + c$$

Riešené príklady – 036

$$\int \frac{dx}{a^2\cos^2 x - b^2\sin^2 x} = \frac{1}{2ab}\ln\left|\frac{b\,\mathrm{tg}\,x+a}{b\,\mathrm{tg}\,x-a}\right| + c \qquad a>0,\ b>0$$

$$= \int \frac{dx}{a^2\frac{1+\cos 2x}{2} - b^2\frac{1-\cos 2x}{2}} = \left[x \neq \pm\,\mathrm{arctg}\,\tfrac{a}{b}\right] = \int \frac{2\,dx}{(a^2-b^2)+(a^2+b^2)\cos 2x}$$

$$= \left[\begin{array}{l|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right] = \int \frac{du}{(a^2-b^2)+(a^2+b^2)\cos u}$$

$$= \left[\begin{array}{l|l|l} \text{UGS:} & t=\mathrm{tg}\,\frac{u}{2}=\mathrm{tg}\,x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{(a^2-b^2)+(a^2+b^2)\frac{1-t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{(a^2-b^2)(t^2+1)+(a^2+b^2)(1-t^2)} = \int \frac{2\,dt}{(a^2t^2+a^2-b^2t^2-b^2)+(a^2-a^2t^2+b^2-b^2t^2)}$$

$$= \int \frac{2\,dt}{-2b^2t^2+2a^2} = \int \frac{dt}{-b^2t^2+a^2} = -\frac{1}{b^2}\int \frac{dt}{t^2-\frac{a^2}{b^2}} = -\frac{1}{b^2}\int \frac{dt}{t^2-(\frac{a}{b})^2} = -\frac{1}{b^2}\cdot\frac{b}{2a}\ln\left|\frac{t-\frac{a}{b}}{t+\frac{a}{b}}\right| + c$$

$$= -\frac{1}{2ab}\ln\left|\frac{bt-a}{bt+a}\right| + c = \frac{1}{2ab}\ln\left|\frac{bt+a}{bt-a}\right| + c = \frac{1}{2ab}\ln\left|\frac{b\,\mathrm{tg}\,\frac{u}{2}+a}{b\,\mathrm{tg}\,\frac{u}{2}-a}\right| + c$$

$$= \frac{1}{2ab}\ln\left|\frac{b\,\mathrm{tg}\,x+a}{b\,\mathrm{tg}\,x-a}\right| + c,\ x\in R-\left\{\pm\,\mathrm{arctg}\,\frac{a}{b},\ \frac{\pi}{2}+k\pi,\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}}$$

$$= \int \frac{dx}{4\cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x + 1}$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4 - 3\sin^2 x}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x + \sin^2 x \\ \quad = 3\cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right]$$

$$= \int \frac{dx}{4\cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x + 1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2} + \frac{2}{2}}$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4 - 3\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 3\,\frac{1-\cos 2x}{2}}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x + \sin^2 x \\ \quad = 3\cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array}\right] = \int \frac{2\,dx}{5+3\cos 2x}$$

$$= \int \frac{dx}{4\cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x + 1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x}$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4 - 3\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x+\sin^2 x}$$

$$=\int \frac{dx}{4\,\frac{1+\cos 2x}{2}+\frac{1-\cos 2x}{2}} = \begin{bmatrix} 4\cos^2 x+\sin^2 x \\ =3\cos^2 x+\sin^2 x+\sin^2 x\geq 3 \end{bmatrix} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{dx}{4\cos^2 x+(1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x+1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2}+\frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{dx}{4(1-\sin^2 x)+\sin^2 x} = \int \frac{dx}{4-3\sin^2 x} = \int \frac{dx}{\frac{8}{2}-3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x + \sin^2 x \\ \quad = 3\cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array}\right] = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x + 1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4 - 3\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right]$$

$$= \int \frac{du}{5+3\cos u}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x + \sin^2 x \\ \quad = 3\cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array}\right] = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x + 1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{5+3\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{u}{2} = \operatorname{tg} x & u \in (-\pi + 2k\pi;\, \pi + 2k\pi),\ t \in R \\ du = \frac{2\,dt}{t^2+1} & \cos u = \frac{1-t^2}{t^2+1} & x \in (-\frac{\pi}{2} + k\pi;\, \frac{\pi}{2} + k\pi),\ k \in Z \end{array}\right]$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x + \sin^2 x \\ \quad = 3\cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array}\right] = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x + 1} = \int \frac{dx}{3\frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4-3\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 3\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{5+3\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{u}{2} = \operatorname{tg} x & u\in(-\pi+2k\pi;\, \pi+2k\pi),\ t\in R \\ du = \frac{2\,dt}{t^2+1} & \cos u = \frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+3\frac{1-t^2}{t^2+1}}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x+\sin^2 x}$$

$$=\int \frac{dx}{4\,\frac{1+\cos 2x}{2}+\frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x+\sin^2 x \\ \quad =3\cos^2 x+\sin^2 x+\sin^2 x \geq 3 \end{array}\right] = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{dx}{4\cos^2 x+(1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x+1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2}+\frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{dx}{4(1-\sin^2 x)+\sin^2 x} = \int \frac{dx}{4-3\sin^2 x} = \int \frac{dx}{\frac{8}{2}-3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{du}{5+3\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\,\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\,\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+3\,\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x+\sin^2 x}$$

$$=\int \frac{dx}{4\,\frac{1+\cos 2x}{2}+\frac{1-\cos 2x}{2}}=\left[\begin{array}{l}4\cos^2x+\sin^2x\\ \quad =3\cos^2x+\sin^2x+\sin^2x\geq 3\end{array}\right]=\int \frac{2\,dx}{5+3\cos 2x}=\left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right]$$

$$=\int \frac{dx}{4\cos^2x+(1-\cos^2x)}=\int \frac{dx}{3\cos^2x+1}=\int \frac{dx}{3\,\frac{1+\cos 2x}{2}+\frac{2}{2}}=\int \frac{2\,dx}{5+3\cos 2x}=\left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right]$$

$$=\int \frac{dx}{4(1-\sin^2x)+\sin^2x}=\int \frac{dx}{4-3\sin^2x}=\int \frac{dx}{\frac{8}{2}-3\,\frac{1-\cos 2x}{2}}=\int \frac{2\,dx}{5+3\cos 2x}=\left[\begin{array}{r|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right]$$

$$=\int \frac{du}{5+3\cos u}=\left[\begin{array}{l|l|l}\text{UGS:} & t=\text{tg}\,\frac{u}{2}=\text{tg}\,x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right]=\int \frac{\frac{2\,dt}{t^2+1}}{5+3\,\frac{1-t^2}{t^2+1}}=\int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$=\int \frac{2\,dt}{2t^2+8}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x+\sin^2 x}$$

$$=\int \frac{dx}{4\,\frac{1+\cos 2x}{2}+\frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x+\sin^2 x \\ \quad =3\cos^2 x+\sin^2 x+\sin^2 x\geq 3 \end{array}\right] = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{dx}{4\cos^2 x+(1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x+1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2}+\frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{dx}{4(1-\sin^2 x)+\sin^2 x} = \int \frac{dx}{4-3\sin^2 x} = \int \frac{dx}{\frac{8}{2}-3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$=\int \frac{du}{5+3\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\,\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\,\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+3\,\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$=\int \frac{2\,dt}{2t^2+8} = \int \frac{dt}{t^2+4}$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x + \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{array}{l} 4\cos^2 x + \sin^2 x \\ \quad = 3\cos^2 x + \sin^2 x + \sin^2 x \geq 3 \end{array} \right] = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array} \right]$$

$$= \int \frac{dx}{4\cos^2 x + (1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x + 1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2} + \frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) + \sin^2 x} = \int \frac{dx}{4 - 3\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array} \right]$$

$$= \int \frac{du}{5+3\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{u}{2} = \operatorname{tg} x & u \in (-\pi+2k\pi;\, \pi+2k\pi),\ t\in R \\ du = \frac{2\,dt}{t^2+1} & \cos u = \frac{1-t^2}{t^2+1} & x \in (-\frac{\pi}{2}+k\pi;\, \frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+3\,\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{2t^2+8} = \int \frac{dt}{t^2+4} = \frac{1}{2}\operatorname{arctg}\frac{t}{2} + c$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x+\sin^2 x}$$

$$=\int \frac{dx}{4\,\frac{1+\cos 2x}{2}+\frac{1-\cos 2x}{2}}=\left[\begin{array}{l}4\cos^2 x+\sin^2 x\\ \quad =3\cos^2 x+\sin^2 x+\sin^2 x\geq 3\end{array}\right]=\int\frac{2\,dx}{5+3\cos 2x}=\left[\begin{array}{l|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right]$$

$$=\int \frac{dx}{4\cos^2 x+(1-\cos^2 x)}=\int\frac{dx}{3\cos^2 x+1}=\int\frac{dx}{3\,\frac{1+\cos 2x}{2}+\frac{2}{2}}=\int\frac{2\,dx}{5+3\cos 2x}=\left[\begin{array}{l|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right]$$

$$=\int \frac{dx}{4(1-\sin^2 x)+\sin^2 x}=\int\frac{dx}{4-3\sin^2 x}=\int\frac{dx}{\frac{8}{2}-3\,\frac{1-\cos 2x}{2}}=\int\frac{2\,dx}{5+3\cos 2x}=\left[\begin{array}{l|l}\text{Subst. } u=2x & x\in R\\ du=2\,dx & u\in R\end{array}\right]$$

$$=\int\frac{du}{5+3\cos u}=\left[\begin{array}{l|l|l}\text{UGS:} & t=\mathrm{tg}\,\frac{u}{2}=\mathrm{tg}\,x & u\in(-\pi+2k\pi;\,\pi+2k\pi),\ t\in R\\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\,\frac{\pi}{2}+k\pi),\ k\in Z\end{array}\right]=\int\frac{\frac{2\,dt}{t^2+1}}{5+3\,\frac{1-t^2}{t^2+1}}=\int\frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$=\int\frac{2\,dt}{2t^2+8}=\int\frac{dt}{t^2+4}=\frac{1}{2}\,\mathrm{arctg}\,\frac{t}{2}+c=\frac{1}{2}\,\mathrm{arctg}\,\frac{\mathrm{tg}\,\frac{u}{2}}{2}+c$$

Riešené príklady – 037

$$\int \frac{dx}{4\cos^2 x+\sin^2 x} = \frac{1}{2}\,\mathrm{arctg}\,\frac{\mathrm{tg}\,x}{2} + c$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}} = \left[\begin{matrix} 4\cos^2 x+\sin^2 x \\ =3\cos^2 x+\sin^2 x+\sin^2 x \geq 3 \end{matrix}\right] = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{matrix}\text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R\end{matrix}\right]$$

$$= \int \frac{dx}{4\cos^2 x+(1-\cos^2 x)} = \int \frac{dx}{3\cos^2 x+1} = \int \frac{dx}{3\,\frac{1+\cos 2x}{2}+\frac{2}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{matrix}\text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R\end{matrix}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x)+\sin^2 x} = \int \frac{dx}{4-3\sin^2 x} = \int \frac{dx}{\frac{8}{2}-3\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{5+3\cos 2x} = \left[\begin{matrix}\text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R\end{matrix}\right]$$

$$= \int \frac{du}{5+3\cos u} = \left[\begin{matrix}\text{UGS:} & t=\mathrm{tg}\,\frac{u}{2}=\mathrm{tg}\,x & u\in(-\pi+2k\pi;\,\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\,\frac{\pi}{2}+k\pi),\ k\in Z\end{matrix}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{5+3\,\frac{1-t^2}{t^2+1}} = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{5t^2+5+3-3t^2}{t^2+1}}$$

$$= \int \frac{2\,dt}{2t^2+8} = \int \frac{dt}{t^2+4} = \frac{1}{2}\,\mathrm{arctg}\,\frac{t}{2} + c = \frac{1}{2}\,\mathrm{arctg}\,\frac{\mathrm{tg}\,\frac{u}{2}}{2} + c = \frac{1}{2}\,\mathrm{arctg}\,\frac{\mathrm{tg}\,x}{2} + c,$$

$$x\in R-\left\{\frac{\pi}{2}+k\pi,\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}}$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1}$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5\sin^2 x}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}}$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x}$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x}$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x-(1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x)-\sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{matrix} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{matrix} \right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{matrix} \text{Subst. } u=2x \\ du = 2\,dx \end{matrix} \middle| \begin{matrix} x \in R \\ u \in R \end{matrix} \right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{matrix} \text{Subst. } u=2x \\ du = 2\,dx \end{matrix} \middle| \begin{matrix} x \in R \\ u \in R \end{matrix} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4 - 5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{matrix} \text{Subst. } u=2x \\ du = 2\,dx \end{matrix} \middle| \begin{matrix} x \in R \\ u \in R \end{matrix} \right]$$

$$= \int \frac{du}{3+5\cos u}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right]$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{3+5\,\frac{1-t^2}{t^2+1}}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{3+5\,\frac{1-t^2}{t^2+1}} = \int \frac{2\,dt}{8-2t^2}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{3+5\,\frac{1-t^2}{t^2+1}} = \int \frac{2\,dt}{8-2t^2}$$

$$= -\int \frac{dt}{t^2-4}$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{3+5\,\frac{1-t^2}{t^2+1}} = \int \frac{2\,dt}{8-2t^2}$$

$$= -\int \frac{dt}{t^2-4} = -\frac{1}{2\cdot 2}\ln\left|\frac{t-2}{t+2}\right| + c$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array} \right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right]$$

$$= \int \frac{dx}{4\cos^2 x-(1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x-1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right]$$

$$= \int \frac{dx}{4(1-\sin^2 x)-\sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du=2\,dx & u\in R \end{array} \right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t=\operatorname{tg}\frac{u}{2}=\operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du=\frac{2\,dt}{t^2+1} & \cos u=\frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{3+5\,\frac{1-t^2}{t^2+1}} = \int \frac{2\,dt}{8-2t^2}$$

$$= -\int \frac{dt}{t^2-4} = -\frac{1}{2\cdot 2} \ln\left|\frac{t-2}{t+2}\right| + c = \frac{1}{4} \ln\left|\frac{t+2}{t-2}\right| + c$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x}$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{c|c} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{c|c} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{c|c} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u\in R \end{array}\right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{u}{2} = \operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du = \frac{2\,dt}{t^2+1} & \cos u = \frac{1-t^2}{t^2+1} & x\in(-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{3+5\,\frac{1-t^2}{t^2+1}} = \int \frac{2\,dt}{8-2t^2}$$

$$= -\int \frac{dt}{t^2-4} = -\frac{1}{2\cdot 2}\ln\left|\frac{t-2}{t+2}\right| + c = \frac{1}{4}\ln\left|\frac{t+2}{t-2}\right| + c = \frac{1}{4}\ln\left|\frac{\operatorname{tg}\frac{u}{2}+2}{\operatorname{tg}\frac{u}{2}-2}\right| + c$$

Riešené príklady – 038

$$\int \frac{dx}{4\cos^2 x - \sin^2 x} = \frac{1}{4}\ln\left|\frac{\operatorname{tg} x + 2}{\operatorname{tg} x - 2}\right| + c$$

$$= \int \frac{dx}{4\,\frac{1+\cos 2x}{2} - \frac{1-\cos 2x}{2}} = \left[\begin{array}{c} 4 \neq \frac{\sin^2 x}{\cos^2 x} \\ x \neq \pm \operatorname{arctg} 2 \end{array}\right] = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right]$$

$$= \int \frac{dx}{4\cos^2 x - (1-\cos^2 x)} = \int \frac{dx}{5\cos^2 x - 1} = \int \frac{dx}{5\,\frac{1+\cos 2x}{2} - \frac{2}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right]$$

$$= \int \frac{dx}{4(1-\sin^2 x) - \sin^2 x} = \int \frac{dx}{4-5\sin^2 x} = \int \frac{dx}{\frac{8}{2} - 5\,\frac{1-\cos 2x}{2}} = \int \frac{2\,dx}{3+5\cos 2x} = \left[\begin{array}{r|l} \text{Subst. } u=2x & x\in R \\ du = 2\,dx & u \in R \end{array}\right]$$

$$= \int \frac{du}{3+5\cos u} = \left[\begin{array}{l|l|l} \text{UGS:} & t = \operatorname{tg}\frac{u}{2} = \operatorname{tg} x & u\in(-\pi+2k\pi;\pi+2k\pi),\ t\in R \\ du = \frac{2\,dt}{t^2+1} & \cos u = \frac{1-t^2}{t^2+1} & x \in (-\frac{\pi}{2}+k\pi;\frac{\pi}{2}+k\pi),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{3+5\,\frac{1-t^2}{t^2+1}} = \int \frac{2\,dt}{8-2t^2}$$

$$= -\int \frac{dt}{t^2-4} = -\frac{1}{2\cdot 2}\ln\left|\frac{t-2}{t+2}\right| + c = \frac{1}{4}\ln\left|\frac{t+2}{t-2}\right| + c = \frac{1}{4}\ln\left|\frac{\operatorname{tg}\frac{u}{2}+2}{\operatorname{tg}\frac{u}{2}-2}\right| + c$$

$$= \frac{1}{4}\ln\left|\frac{\operatorname{tg} x+2}{\operatorname{tg} x-2}\right| + c,\ x\in R - \left\{\pm\operatorname{arctg} 2,\ \frac{\pi}{2}+k\pi,\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$\int x^2 \sin x^3\,dx$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$= \left[\begin{array}{r|l} \text{Subst. } u = x^3 & x \neq \pm\sqrt[3]{k\pi},\ u \neq \pm k\pi \\ du = 3x^2\,dx & k = 0, 1, 2, 3, 4, \ldots \end{array} \right]$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = x^3 & x \neq \pm\sqrt[3]{k\pi},\ t \neq \pm k\pi \\ dt = 3x^2\,dx & k = 0, 1, 2, 3, 4, \ldots \end{array} \right]$$

$$\int x^2 \sin x^3\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } u = x^3 & x \in R \\ du = 3x^2\,dx & u \in R \end{array} \right]$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\neq \pm\sqrt[3]{k\pi},\ u\neq \pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\ \ldots \end{array} \right] = \frac{1}{3}\int \frac{du}{\sin u}$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=x^3 & x\neq \pm\sqrt[3]{k\pi},\ t\neq \pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\ \ldots \end{array} \right] = \frac{1}{3}\int \frac{dt}{\sin t}$$

$$\int x^2 \sin x^3\,dx$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{array} \right] = \frac{1}{3}\int \sin u\,du$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\neq\pm\sqrt[3]{k\pi},\ u\neq\pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\ \dots \end{array}\right] = \frac{1}{3}\int\frac{du}{\sin u} = \left[\begin{array}{c|c|c} \text{UGS:} & du=\frac{2\,dt}{t^2+1} & u\in(-\pi+2l\pi;2l\pi),\ t\in(-\infty;0) \\ t=\text{tg}\,\frac{u}{2} & \sin u=\frac{2}{t^2+1} & u\in(2l\pi;\pi+2l\pi),\ t\in(0;\infty),\ l\in Z \end{array}\right]$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=x^3 & x\neq\pm\sqrt[3]{k\pi},\ t\neq\pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\ \dots \end{array}\right] = \frac{1}{3}\int\frac{dt}{\sin t} = \frac{1}{3}\int\frac{\sin t\,dt}{\sin^2 t}$$

$$\int x^2\sin x^3\,dx$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{array}\right] = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + c$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u = x^3 & x \neq \pm\sqrt[3]{k\pi},\ u \neq \pm k\pi \\ du = 3x^2\,dx & k = 0, 1, 2, 3, 4, \ldots \end{array} \right] = \frac{1}{3}\int \frac{du}{\sin u} = \left[\begin{array}{c|c|c} \text{UGS:} & du = \frac{2\,dt}{t^2+1} & u \in (-\pi+2l\pi; 2l\pi),\ t \in (-\infty; 0) \\ t = \operatorname{tg}\frac{u}{2} & \sin u = \frac{2}{t^2+1} & u \in (2l\pi; \pi+2l\pi),\ t \in (0; \infty),\ l \in Z \end{array} \right]$$

$$= \left[\begin{array}{c|c} \text{Subst. } t = x^3 & x \neq \pm\sqrt[3]{k\pi},\ t \neq \pm k\pi \\ dt = 3x^2\,dx & k = 0, 1, 2, 3, 4, \ldots \end{array} \right] = \frac{1}{3}\int \frac{dt}{\sin t} = \frac{1}{3}\int \frac{\sin t\,dt}{\sin^2 t} = \frac{1}{3}\int \frac{\sin t\,dt}{1-\cos^2 t}$$

$$\int x^2 \sin x^3\,dx = -\frac{1}{3}\cos x^3 + c$$

$$= \left[\begin{array}{c|c} \text{Subst. } u = x^3 & x \in R \\ du = 3x^2\,dx & u \in R \end{array} \right] = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos x^3 + c,\ x \in R,\ c \in R.$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\neq\pm\sqrt[3]{k\pi},\ u\neq\pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array}\right] = \frac{1}{3}\int\frac{du}{\sin u} = \left[\begin{array}{c|c|c} \text{UGS:} & du=\frac{2\,dt}{t^2+1} & u\in(-\pi+2l\pi;2l\pi),\ t\in(-\infty;0) \\ t=\operatorname{tg}\frac{u}{2} & \sin u=\frac{2}{t^2+1} & u\in(2l\pi;\pi+2l\pi),\ t\in(0;\infty),\ l\in Z \end{array}\right]$$

$$= \frac{1}{3}\int\frac{1+t^2}{2t}\,\frac{2\,du}{1+t^2}$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=x^3 & x\neq\pm\sqrt[3]{k\pi},\ t\neq\pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array}\right] = \frac{1}{3}\int\frac{dt}{\sin t} = \frac{1}{3}\int\frac{\sin t\,dt}{\sin^2 t} = \frac{1}{3}\int\frac{\sin t\,dt}{1-\cos^2 t} = \frac{1}{3}\int\frac{-\sin t\,dt}{\cos^2 t-1}$$

$$\int x^2\sin x^3\,dx = -\frac{1}{3}\cos x^3 + c$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{array}\right] = \frac{1}{3}\int\sin u\,du = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos x^3 + c,\ x\in R,\ c\in R.$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x \neq \pm\sqrt[3]{k\pi},\ u\neq \pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array} \right] = \frac{1}{3}\int \frac{du}{\sin u} = \left[\begin{array}{c|c|c} \text{UGS:} & du=\frac{2\,dt}{t^2+1} & u\in(-\pi+2l\pi;2l\pi),\ t\in(-\infty;0) \\ t=\operatorname{tg}\frac{u}{2} & \sin u=\frac{2}{t^2+1} & u\in(2l\pi;\pi+2l\pi),\ t\in(0;\infty),\ l\in Z \end{array} \right]$$

$$= \frac{1}{3}\int \frac{1+t^2}{2t}\,\frac{2\,du}{1+t^2} = \frac{1}{3}\int \frac{dt}{t}$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=x^3 & x \neq \pm\sqrt[3]{k\pi},\ t\neq \pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array} \right] = \frac{1}{3}\int \frac{dt}{\sin t} = \frac{1}{3}\int \frac{\sin t\,dt}{\sin^2 t} = \frac{1}{3}\int \frac{\sin t\,dt}{1-\cos^2 t} = \frac{1}{3}\int \frac{-\sin t\,dt}{\cos^2 t - 1}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=\cos t & t\neq \pm k\pi,\ u\neq \pm 1 \\ du=-\sin t\,dt & k=0,1,2,3,\ldots \end{array} \right]$$

$$\int x^2 \sin x^3\,dx = -\frac{1}{3}\cos x^3 + c$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{array} \right] = \frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos x^3 + c,\ x\in R,\ c\in R.$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3}$$

$$= \left[\begin{array}{l|l} \text{Subst. } u=x^3 & x\neq\pm\sqrt[3]{k\pi},\ u\neq\pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array}\right] = \frac{1}{3}\int\frac{du}{\sin u} = \left[\begin{array}{l|l|l} \text{UGS:} & du=\frac{2\,dt}{t^2+1} & u\in(-\pi+2l\pi;2l\pi),\ t\in(-\infty;0) \\ t=\operatorname{tg}\frac{u}{2} & \sin u=\frac{2}{t^2+1} & u\in(2l\pi;\pi+2l\pi),\ t\in(0;\infty),\ l\in Z \end{array}\right]$$

$$= \frac{1}{3}\int\frac{1+t^2}{2t}\,\frac{2\,du}{1+t^2} = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln|t| + c_1$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=x^3 & x\neq\pm\sqrt[3]{k\pi},\ t\neq\pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array}\right] = \frac{1}{3}\int\frac{dt}{\sin t} = \frac{1}{3}\int\frac{\sin t\,dt}{\sin^2 t} = \frac{1}{3}\int\frac{\sin t\,dt}{1-\cos^2 t} = \frac{1}{3}\int\frac{-\sin t\,dt}{\cos^2 t-1}$$

$$= \left[\begin{array}{l|l} \text{Subst. } u=\cos t & t\neq\pm k\pi,\ u\neq\pm 1 \\ du=-\sin t\,dt & k=0,1,2,3,\ldots \end{array}\right] = \frac{1}{3}\int\frac{du}{u^2-1}$$

$$\int x^2\sin x^3\,dx = -\frac{1}{3}\cos x^3 + c$$

$$= \left[\begin{array}{l|l} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{array}\right] = \frac{1}{3}\int\sin u\,du = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos x^3 + c,\ x\in R,\ c\in R.$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3} = \frac{1}{3}\ln\left|\operatorname{tg}\frac{x^3}{2}\right| + c_1$$

$$= \begin{bmatrix} \text{Subst. } u=x^3 & x\neq\pm\sqrt[3]{k\pi},\ u\neq\pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\ldots \end{bmatrix} = \frac{1}{3}\int\frac{du}{\sin u} = \begin{bmatrix} \text{UGS:} & du=\frac{2\,dt}{t^2+1} & u\in(-\pi+2l\pi;2l\pi),\ t\in(-\infty;0) \\ t=\operatorname{tg}\frac{u}{2} & \sin u=\frac{2}{t^2+1} & u\in(2l\pi;\pi+2l\pi),\ t\in(0;\infty),\ l\in Z \end{bmatrix}$$

$$= \frac{1}{3}\int\frac{1+t^2}{2t}\,\frac{2\,du}{1+t^2} = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln|t| + c_1 = \frac{1}{3}\ln\left|\operatorname{tg}\frac{x^3}{2}\right| + c_1,$$

$$x\in R-\{\pm\sqrt[3]{k\pi},\ k=0,1,2,3,4,\ldots\},\ c_1\in R.$$

$$= \begin{bmatrix} \text{Subst. } t=x^3 & x\neq\pm\sqrt[3]{k\pi},\ t\neq\pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\ldots \end{bmatrix} = \frac{1}{3}\int\frac{dt}{\sin t} = \frac{1}{3}\int\frac{\sin t\,dt}{\sin^2 t} = \frac{1}{3}\int\frac{\sin t\,dt}{1-\cos^2 t} = \frac{1}{3}\int\frac{-\sin t\,dt}{\cos^2 t-1}$$

$$= \begin{bmatrix} \text{Subst. } u=\cos t & t\neq\pm k\pi,\ u\neq\pm 1 \\ du=-\sin t\,dt & k=0,1,2,3,\ldots \end{bmatrix} = \frac{1}{3}\int\frac{du}{u^2-1} = \frac{1}{3}\cdot\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + c_2$$

$$\int x^2\sin x^3\,dx = -\frac{1}{3}\cos x^3 + c$$

$$= \begin{bmatrix} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{bmatrix} = \frac{1}{3}\int\sin u\,du = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos x^3 + c,\ x\in R,\ c\in R.$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3} = \frac{1}{3}\ln\left|\operatorname{tg}\frac{x^3}{2}\right| + c_1$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\neq\pm\sqrt[3]{k\pi},\ u\neq\pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array}\right] = \frac{1}{3}\int\frac{du}{\sin u} = \left[\begin{array}{l|l|l} \text{UGS:} & du=\frac{2\,dt}{t^2+1} & u\in(-\pi+2l\pi;2l\pi),\ t\in(-\infty;0) \\ t=\operatorname{tg}\frac{u}{2} & \sin u=\frac{2}{t^2+1} & u\in(2l\pi;\pi+2l\pi),\ t\in(0;\infty),\ l\in Z \end{array}\right]$$

$$= \frac{1}{3}\int\frac{1+t^2}{2t}\,\frac{2\,du}{1+t^2} = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln|t| + c_1 = \frac{1}{3}\ln\left|\operatorname{tg}\frac{x^3}{2}\right| + c_1,$$

$$x\in R-\{\pm\sqrt[3]{k\pi},\ k=0,1,2,3,4,\ldots\},\ c_1\in R.$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=x^3 & x\neq\pm\sqrt[3]{k\pi},\ t\neq\pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\ldots \end{array}\right] = \frac{1}{3}\int\frac{dt}{\sin t} = \frac{1}{3}\int\frac{\sin t\,dt}{\sin^2 t} = \frac{1}{3}\int\frac{\sin t\,dt}{1-\cos^2 t} = \frac{1}{3}\int\frac{-\sin t\,dt}{\cos^2 t-1}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=\cos t & t\neq\pm k\pi,\ u\neq\pm 1 \\ du=-\sin t\,dt & k=0,1,2,3,\ldots \end{array}\right] = \frac{1}{3}\int\frac{du}{u^2-1} = \frac{1}{3}\cdot\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + c_2 = \frac{1}{6}\ln\left|\frac{\cos t-1}{\cos t+1}\right| + c_2$$

$$\int x^2\sin x^3\,dx = -\frac{1}{3}\cos x^3 + c$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{array}\right] = \frac{1}{3}\int\sin u\,du = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos x^3 + c,\ x\in R,\ c\in R.$$

Riešené príklady – 039, 040

$$\int \frac{x^2\,dx}{\sin x^3} = \frac{1}{3}\ln\left|\operatorname{tg}\frac{x^3}{2}\right| + c_1 = \frac{1}{6}\ln\left|\frac{\cos x^3-1}{\cos x^3+1}\right| + c_2$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\neq\pm\sqrt[3]{k\pi},\ u\neq\pm k\pi \\ du=3x^2\,dx & k=0,1,2,3,4,\dots \end{array}\right] = \frac{1}{3}\int\frac{du}{\sin u} = \left[\begin{array}{c|c|c} \text{UGS:} & du=\frac{2\,dt}{t^2+1} & u\in(-\pi+2l\pi;2l\pi),\ t\in(-\infty;0) \\ t=\operatorname{tg}\frac{u}{2} & \sin u=\frac{2}{t^2+1} & u\in(2l\pi;\pi+2l\pi),\ t\in(0;\infty),\ l\in Z \end{array}\right]$$

$$= \frac{1}{3}\int\frac{1+t^2}{2t}\,\frac{2\,du}{1+t^2} = \frac{1}{3}\int\frac{dt}{t} = \frac{1}{3}\ln|t| + c_1 = \frac{1}{3}\ln\left|\operatorname{tg}\frac{x^3}{2}\right| + c_1,$$

$$x\in R-\{\pm\sqrt[3]{k\pi},\ k=0,1,2,3,4,\dots\},\ c_1\in R.$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=x^3 & x\neq\pm\sqrt[3]{k\pi},\ t\neq\pm k\pi \\ dt=3x^2\,dx & k=0,1,2,3,4,\dots \end{array}\right] = \frac{1}{3}\int\frac{dt}{\sin t} = \frac{1}{3}\int\frac{\sin t\,dt}{\sin^2 t} = \frac{1}{3}\int\frac{\sin t\,dt}{1-\cos^2 t} = \frac{1}{3}\int\frac{-\sin t\,dt}{\cos^2 t-1}$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=\cos t & t\neq\pm k\pi,\ u\neq\pm 1 \\ du=-\sin t\,dt & k=0,1,2,3,\dots \end{array}\right] = \frac{1}{3}\int\frac{du}{u^2-1} = \frac{1}{3}\cdot\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + c_2 = \frac{1}{6}\ln\left|\frac{\cos t-1}{\cos t+1}\right| + c_2$$

$$= \frac{1}{6}\ln\left|\frac{\cos x^3-1}{\cos x^3+1}\right| + c_2,\ x\in R-\{\pm\sqrt[3]{k\pi},\ k=0,1,2,3,4,\dots\},\ c_2\in R.$$

$$\int x^2\sin x^3\,dx = -\frac{1}{3}\cos x^3 + c$$

$$= \left[\begin{array}{c|c} \text{Subst. } u=x^3 & x\in R \\ du=3x^2\,dx & u\in R \end{array}\right] = \frac{1}{3}\int\sin u\,du = -\frac{1}{3}\cos u + c = -\frac{1}{3}\cos x^3 + c,\ x\in R,\ c\in R.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x \, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x \, dx$$

$$\int \frac{\cos x \, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x \, dx$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x} \, dx$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x\,dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \cos x & x \in \left(-\frac{\pi}{2} + 2k\pi; 0 + 2k\pi\right\rangle,\ t \in (0;1\rangle \\ dt = -\sin x\,dx & x \in \left\langle 0 + 2k\pi; \frac{\pi}{2} + 2k\pi\right),\ t \in (0;1\rangle,\ k \in Z \end{array}\right]$$

$$\int \frac{\cos x\,dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \sin x & x \in (2k\pi; \pi + 2k\pi\rangle,\ t \in (0;1\rangle \\ dt = \cos x\,dx & x \in \langle \pi + 2k\pi; 2\pi + 2k\pi),\ t \in (0;1\rangle \end{array}\right]$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$$

$$= \int \frac{(\sin x + \cos x)'}{\sin x + \cos x}\,dx$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x\,dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=\cos x & x\in\left(-\frac{\pi}{2}+2k\pi;0+2k\pi\right\rangle,\ t\in(0;1\rangle \\ dt=-\sin x\,dx & x\in\left\langle 0+2k\pi;\frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle,\ k\in Z \end{array}\right] = -\int t^{-\frac{5}{2}}\,dt$$

$$\int \frac{\cos x\,dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=\sin x & x\in(2k\pi;\pi+2k\pi\rangle,\ t\in(0;1\rangle \\ dt=\cos x\,dx & x\in\langle\pi+2k\pi;2\pi+2k\pi),\ t\in(0;1\rangle \end{array}\right] = \int t^{-\frac{2}{3}}\,dt$$

$$\int \frac{\cos x-\sin x}{\cos x+\sin x}\,dx = \ln|\sin x+\cos x|+c$$

$$= \int \frac{(\sin x+\cos x)'}{\sin x+\cos x}\,dx = \ln|\sin x+\cos x|+c,\ x\in R-\left\{\frac{3\pi}{4}+k\pi,\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x\,dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=\cos x & x\in\left(-\frac{\pi}{2}+2k\pi; 0+2k\pi\right\rangle,\ t\in(0;1\rangle \\ dt=-\sin x\,dx & x\in\left\langle 0+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle,\ k\in Z \end{array}\right] = -\int t^{-\frac{5}{2}}\,dt = -\frac{t^{-\frac{3}{2}}}{-\frac{3}{2}}+c$$

$$\int \frac{\cos x\,dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=\sin x & x\in(2k\pi; \pi+2k\pi\rangle,\ t\in(0;1\rangle \\ dt=\cos x\,dx & x\in\langle \pi+2k\pi; 2\pi+2k\pi),\ t\in(0;1\rangle \end{array}\right] = \int t^{-\frac{2}{3}}\,dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}}+c$$

$$\int \frac{\cos x-\sin x}{\cos x+\sin x}\,dx = \ln|\sin x+\cos x|+c$$

$$= \int \frac{(\sin x+\cos x)'}{\sin x+\cos x}\,dx = \ln|\sin x+\cos x|+c,\ x\in R-\left\{\frac{3\pi}{4}+k\pi,\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x\,dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=\cos x & x\in\left(-\frac{\pi}{2}+2k\pi; 0+2k\pi\right\rangle,\ t\in(0;1\rangle \\ dt=-\sin x\,dx & x\in\left\langle 0+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t\in(0;1\rangle,\ k\in Z \end{array} \right] = -\int t^{-\frac{5}{2}}\,dt = -\frac{t^{-\frac{3}{2}}}{-\frac{3}{2}}+c = \frac{2}{3}\,\frac{1}{\sqrt{t^3}}+c$$

$$\int \frac{\cos x\,dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x\,dx = 3\sqrt[3]{\sin x}+c$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=\sin x & x\in(2k\pi; \pi+2k\pi\rangle,\ t\in(0;1\rangle \\ dt=\cos x\,dx & x\in\langle \pi+2k\pi; 2\pi+2k\pi),\ t\in(0;1\rangle \end{array} \right] = \int t^{-\frac{2}{3}}\,dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}}+c = 3\sqrt[3]{t}+c$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \ln|\sin x+\cos x|+c$$

$$= \int \frac{(\sin x+\cos x)'}{\sin x+\cos x}\,dx = \ln|\sin x+\cos x|+c,\ x\in R-\left\{\frac{3\pi}{4}+k\pi,\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 041, 042, 043

$$\int \frac{\sin x\, dx}{\sqrt{\cos^5 x}} = \int \cos^{-\frac{5}{2}} x \cdot \sin x\, dx = \frac{2}{3\sqrt{\cos^3 x}} + c$$

$$= \left[\begin{array}{l|l} \text{Subst. } t = \cos x & x \in \left(-\frac{\pi}{2}+2k\pi; 0+2k\pi\right\rangle,\ t \in (0;1\rangle \\ dt = -\sin x\, dx & x \in \left\langle 0+2k\pi; \frac{\pi}{2}+2k\pi\right),\ t \in (0;1\rangle,\ k \in Z \end{array} \right] = -\int t^{-\frac{5}{2}}\, dt = -\frac{t^{-\frac{3}{2}}}{-\frac{3}{2}} + c = \frac{2}{3}\frac{1}{\sqrt{t^3}} + c$$

$$= \frac{2}{3\sqrt{\cos^3 x}} + c,\ x \in \left(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi\right),\ k \in Z,\ c \in R.$$

$$\int \frac{\cos x\, dx}{\sqrt[3]{\sin^2 x}} = \int \sin^{-\frac{2}{3}} x \cdot \cos x\, dx = 3\sqrt[3]{\sin x} + c$$

$$= \left[\begin{array}{l|l} \text{Subst. } t = \sin x & x \in (2k\pi; \pi+2k\pi\rangle,\ t \in (0;1\rangle \\ dt = \cos x\, dx & x \in \langle \pi+2k\pi; 2\pi+2k\pi),\ t \in (0;1\rangle \end{array} \right] = \int t^{-\frac{2}{3}}\, dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + c = 3\sqrt[3]{t} + c = 3\sqrt[3]{\sin x} + c,$$

$$x \in R - \{2k\pi,\ k \in Z\},\ c \in R.$$

$$\int \frac{\cos x - \sin x}{\cos x + \sin x}\, dx = \ln|\sin x + \cos x| + c$$

$$= \int \frac{(\sin x + \cos x)'}{\sin x + \cos x}\, dx = \ln|\sin x + \cos x| + c,\ x \in R - \left\{\frac{3\pi}{4} + k\pi,\ k \in Z\right\},\ c \in R.$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x}\,\mathrm{d}x$$

$$\int \frac{\mathrm{d}x}{\sin x \cos x}$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x}\, dx$$

$$= \begin{bmatrix} \cos x > 0,\ \sin x \neq 0, \quad k \in Z \\ x \in \left(-\frac{\pi}{2} + 2k\pi; \frac{\pi}{2} + 2k\pi\right),\ x \neq k\pi \end{bmatrix}$$

$$\int \frac{dx}{\sin x \cos x}$$

$$= \int \frac{(\cos^2 x + \sin^2 x)\, dx}{\sin x \cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}}$$

Riešené príklady – 044, 045

$\int \frac{\ln\cos x}{\sin^2 x}\,dx$

$$= \begin{bmatrix} \cos x > 0,\ \sin x \neq 0, \quad k \in Z \\ x \in \left(-\frac{\pi}{2}+2k\pi; \frac{\pi}{2}+2k\pi\right),\ x \neq k\pi \end{bmatrix} = \left[\begin{array}{l|l} u = \ln\cos x & u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} & v = -\operatorname{cotg} x = -\frac{\cos x}{\sin x} \end{array}\right]$$

$\int \frac{dx}{\sin x \cos x}$

$$= \int \frac{(\cos^2 x + \sin^2 x)\,dx}{\sin x \cos x} = \int \frac{\cos^2 x\,dx}{\sin x \cos x} + \int \frac{\sin^2 x\,dx}{\sin x \cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}}$$

Riešené príklady – 044, 045

$$\int \frac{\ln\cos x}{\sin^2 x}\,dx$$

$$= \left[\begin{matrix} \cos x>0,\ \sin x\neq 0, \quad k\in Z \\ x\in\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right),\ x\neq k\pi \end{matrix}\right] = \left[\begin{array}{l|l} u=\ln\cos x & u'=\frac{-\sin x}{\cos x} \\ v'=\frac{1}{\sin^2 x} & v=-\operatorname{cotg} x=-\frac{\cos x}{\sin x} \end{array}\right] = -\operatorname{cotg} x\cdot\ln\cos x - \int dx$$

$$\int \frac{dx}{\sin x\cos x}$$

$$= \int \frac{(\cos^2 x+\sin^2 x)\,dx}{\sin x\cos x} = \int \frac{\cos^2 x\,dx}{\sin x\cos x} + \int \frac{\sin^2 x\,dx}{\sin x\cos x} = \int \frac{\cos x\,dx}{\sin x} - \int \frac{-\sin x\,dx}{\cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x\cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)'\,dx}{\operatorname{tg} x}$$

Riešené príklady – 044, 045

$$\int \frac{\ln\cos x}{\sin^2 x}\,dx = -\operatorname{cotg} x\cdot\ln\cos x - x + c$$

$$= \left[\begin{matrix}\cos x>0,\ \sin x\neq 0, & k\in Z\\ x\in\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right),\ x\neq k\pi\end{matrix}\right] = \left[\begin{array}{l|l} u=\ln\cos x & u'=\frac{-\sin x}{\cos x}\\ v'=\frac{1}{\sin^2 x} & v=-\operatorname{cotg} x=-\frac{\cos x}{\sin x}\end{array}\right] = -\operatorname{cotg} x\cdot\ln\cos x - \int dx$$

$$= -\operatorname{cotg} x\cdot\ln\cos x - x + c,\ x\in\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right)-\{2k\pi\},\ k\in Z,\ c\in R.$$

$$\int \frac{dx}{\sin x\cos x} = \ln|\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x+\sin^2 x)\,dx}{\sin x\cos x} = \int \frac{\cos^2 x\,dx}{\sin x\cos x} + \int \frac{\sin^2 x\,dx}{\sin x\cos x} = \int \frac{\cos x\,dx}{\sin x} - \int \frac{-\sin x\,dx}{\cos x}$$

$$= \int \frac{(\sin x)'\,dx}{\sin x} - \int \frac{(\cos x)'\,dx}{\cos x}$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x\cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)'\,dx}{\operatorname{tg} x} = \ln|\operatorname{tg} x| + c,\ x\in R-\left\{\frac{k\pi}{2},\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 044, 045

$$\int \frac{\ln\cos x}{\sin^2 x}\,dx = -\operatorname{cotg} x\cdot\ln\cos x - x + c$$

$$= \begin{bmatrix} \cos x>0,\ \sin x\neq 0, \quad k\in Z \\ x\in\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right),\ x\neq k\pi \end{bmatrix} = \left[\begin{array}{l|l} u=\ln\cos x & u'=\frac{-\sin x}{\cos x} \\ v'=\frac{1}{\sin^2 x} & v=-\operatorname{cotg} x=-\frac{\cos x}{\sin x} \end{array}\right] = -\operatorname{cotg} x\cdot\ln\cos x - \int dx$$

$$= -\operatorname{cotg} x\cdot\ln\cos x - x + c,\ x\in\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right)-\{2k\pi\},\ k\in Z,\ c\in R.$$

$$\int \frac{dx}{\sin x\cos x} = \ln|\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x+\sin^2 x)\,dx}{\sin x\cos x} = \int \frac{\cos^2 x\,dx}{\sin x\cos x} + \int \frac{\sin^2 x\,dx}{\sin x\cos x} = \int \frac{\cos x\,dx}{\sin x} - \int \frac{-\sin x\,dx}{\cos x}$$

$$= \int \frac{(\sin x)'\,dx}{\sin x} - \int \frac{(\cos x)'\,dx}{\cos x} = \ln|\sin x| - \ln|\cos x| + c$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x\cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)'\,dx}{\operatorname{tg} x} = \ln|\operatorname{tg} x| + c,\ x\in R-\left\{\frac{k\pi}{2},\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 044, 045

$$\int \frac{\ln \cos x}{\sin^2 x}\,dx = -\operatorname{cotg} x \cdot \ln \cos x - x + c$$

$$= \left[\begin{matrix} \cos x > 0,\ \sin x \neq 0, \quad k \in Z \\ x \in \left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right),\ x \neq k\pi \end{matrix}\right] = \left[\begin{array}{l|l} u = \ln \cos x & u' = \frac{-\sin x}{\cos x} \\ v' = \frac{1}{\sin^2 x} & v = -\operatorname{cotg} x = -\frac{\cos x}{\sin x} \end{array}\right] = -\operatorname{cotg} x \cdot \ln \cos x - \int dx$$

$$= -\operatorname{cotg} x \cdot \ln \cos x - x + c,\ x \in \left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right) - \{2k\pi\},\ k \in Z,\ c \in R.$$

$$\int \frac{dx}{\sin x \cos x} = \ln|\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x + \sin^2 x)\,dx}{\sin x \cos x} = \int \frac{\cos^2 x\,dx}{\sin x \cos x} + \int \frac{\sin^2 x\,dx}{\sin x \cos x} = \int \frac{\cos x\,dx}{\sin x} - \int \frac{-\sin x\,dx}{\cos x}$$

$$= \int \frac{(\sin x)'\,dx}{\sin x} - \int \frac{(\cos x)'\,dx}{\cos x} = \ln|\sin x| - \ln|\cos x| + c = \ln\left|\frac{\sin x}{\cos x}\right| + c$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)'\,dx}{\operatorname{tg} x} = \ln|\operatorname{tg} x| + c,\ x \in R - \left\{\frac{k\pi}{2},\ k \in Z\right\},\ c \in R.$$

Riešené príklady – 044, 045

$$\int \frac{\ln\cos x}{\sin^2 x}\,dx = -\operatorname{cotg} x\cdot\ln\cos x - x + c$$

$$= \left[\begin{matrix} \cos x>0,\ \sin x\neq 0,\quad k\in Z \\ x\in\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right),\ x\neq k\pi\end{matrix}\right] = \left[\begin{array}{l|l} u=\ln\cos x & u'=\frac{-\sin x}{\cos x} \\ v'=\frac{1}{\sin^2 x} & v=-\operatorname{cotg} x=-\frac{\cos x}{\sin x}\end{array}\right] = -\operatorname{cotg} x\cdot\ln\cos x - \int dx$$

$$= -\operatorname{cotg} x\cdot\ln\cos x - x + c,\ x\in\left(-\frac{\pi}{2}+2k\pi;\ \frac{\pi}{2}+2k\pi\right)-\{2k\pi\},\ k\in Z,\ c\in R.$$

$$\int \frac{dx}{\sin x\cos x} = \ln|\operatorname{tg} x| + c$$

$$= \int \frac{(\cos^2 x+\sin^2 x)\,dx}{\sin x\cos x} = \int \frac{\cos^2 x\,dx}{\sin x\cos x} + \int \frac{\sin^2 x\,dx}{\sin x\cos x} = \int \frac{\cos x\,dx}{\sin x} - \int \frac{-\sin x\,dx}{\cos x}$$

$$= \int \frac{(\sin x)'\,dx}{\sin x} - \int \frac{(\cos x)'\,dx}{\cos x} = \ln|\sin x| - \ln|\cos x| + c = \ln\left|\frac{\sin x}{\cos x}\right| + c$$

$$= \ln|\operatorname{tg} x| + c,\ x\in R-\left\{\frac{k\pi}{2},\ k\in Z\right\},\ c\in R.$$

$$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x\cos x}{\cos^2 x}} = \int \frac{\frac{dx}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \int \frac{(\operatorname{tg} x)'\,dx}{\operatorname{tg} x} = \ln|\operatorname{tg} x| + c,\ x\in R-\left\{\frac{k\pi}{2},\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

Riešené príklady – 046

$$\int \frac{\mathrm{d}x}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x)\,\mathrm{d}x}{\cos^2 x - \sin^2 x}$$

$$= \left[\begin{array}{r|l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & \sin x \neq -\cos x & x \in \left(-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi\right),\, t \in \left(-\infty; 1-\sqrt{2}\right) \\ \mathrm{d}x = \frac{2\,\mathrm{d}t}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \neq \frac{3\pi}{4} + k\pi & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\, t \in \left(1-\sqrt{2}; 1+\sqrt{2}\right) \\ & & & x \in \left(\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi\right),\, t \in \left(1+\sqrt{2}; \infty\right),\, k \in Z \end{array} \right]$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x)\,dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x\,dx}{(1-\sin^2 x) - \sin^2 x} - \int \frac{\sin x\,dx}{\cos^2 x - (1-\cos^2 x)}$$

$$= \left[\begin{array}{r|l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & \sin x \neq -\cos x & x \in \left(-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi\right),\ t \in \left(-\infty; 1-\sqrt{2}\right) \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \neq \frac{3\pi}{4} + k\pi & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in \left(1-\sqrt{2}; 1+\sqrt{2}\right) \\ & & & x \in \left(\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi\right),\ t \in \left(1+\sqrt{2}; \infty\right),\ k \in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}}$$

Riešené príklady – 046

$\int \frac{dx}{\cos x + \sin x}$

$$= \int \frac{(\cos x - \sin x)\,dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x\,dx}{(1-\sin^2 x) - \sin^2 x} - \int \frac{\sin x\,dx}{\cos^2 x - (1-\cos^2 x)} = \int \frac{\cos x\,dx}{1 - 2\sin^2 x} - \int \frac{\sin x\,dx}{2\cos^2 x - 1}$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & \sin x \neq -\cos x & x \in \left(-\pi + 2k\pi; -\frac{\pi}{4} + 2k\pi\right),\, t \in \left(-\infty; 1-\sqrt{2}\right) \\ \quad dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & x \neq \frac{3\pi}{4} + k\pi & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\, t \in \left(1-\sqrt{2}; 1+\sqrt{2}\right) \\ & & & x \in \left(\frac{3\pi}{4} + 2k\pi; \pi + 2k\pi\right),\, t \in \left(1+\sqrt{2}; \infty\right),\, k \in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1} + \frac{2t}{t^2+1}}$$

$$= \int \frac{2\,dt}{2t + 1 - t^2}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x)\,dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x\,dx}{(1-\sin^2 x) - \sin^2 x} - \int \frac{\sin x\,dx}{\cos^2 x - (1-\cos^2 x)} = \int \frac{\cos x\,dx}{1-2\sin^2 x} - \int \frac{\sin x\,dx}{2\cos^2 x - 1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=\sin x & du=\cos x\,dx & \sin x \neq \pm\cos x \\ v=\cos x & dv=-\sin x\,dx & x\neq \frac{3\pi}{4}+k\pi,\ k\in Z \end{array} \right]$$

$$= \left[\begin{array}{r|l|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1} & \sin x \neq -\cos x & x\in\left(-\pi+2k\pi; -\frac{\pi}{4}+2k\pi\right), t\in\left(-\infty; 1-\sqrt{2}\right) \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\neq\frac{3\pi}{4}+k\pi & x\in\left(-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi\right), t\in\left(1-\sqrt{2}; 1+\sqrt{2}\right) \\ & & & x\in\left(\frac{3\pi}{4}+2k\pi; \pi+2k\pi\right), t\in\left(1+\sqrt{2}; \infty\right), k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}+\frac{2t}{t^2+1}}$$

$$= \int \frac{2\,dt}{2t+1-t^2} = \int \frac{-2\,dt}{t^2-2t+1-2}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x+\sin x}$$

$$= \int \frac{(\cos x-\sin x)\,dx}{\cos^2 x-\sin^2 x} = \int \frac{\cos x\,dx}{(1-\sin^2 x)-\sin^2 x} - \int \frac{\sin x\,dx}{\cos^2 x-(1-\cos^2 x)} = \int \frac{\cos x\,dx}{1-2\sin^2 x} - \int \frac{\sin x\,dx}{2\cos^2 x-1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=\sin x & du=\cos x\,dx & \sin x \neq \pm\cos x \\ v=\cos x & dv=-\sin x\,dx & x\neq \frac{3\pi}{4}+k\pi,\ k\in Z \end{array} \right] = \int \frac{du}{1-2u^2} - \int \frac{-\,dv}{2v^2-1}$$

$$= \left[\begin{array}{r|l|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1} & \sin x\neq-\cos x & x\in\left(-\pi+2k\pi;-\frac{\pi}{4}+2k\pi\right),\ t\in\left(-\infty;1-\sqrt{2}\right) \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\neq\frac{3\pi}{4}+k\pi & x\in\left(-\frac{\pi}{4}+2k\pi;\frac{3\pi}{4}+2k\pi\right),\ t\in\left(1-\sqrt{2};1+\sqrt{2}\right) \\ & & & x\in\left(\frac{3\pi}{4}+2k\pi;\pi+2k\pi\right),\ t\in\left(1+\sqrt{2};\infty\right),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}+\frac{2t}{t^2+1}}$$

$$= \int \frac{2\,dt}{2t+1-t^2} = \int \frac{-2\,dt}{t^2-2t+1-2} = \int \frac{-2\,dt}{(t-1)^2-(\sqrt{2})^2}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x+\sin x}$$

$$=\int\frac{(\cos x-\sin x)\,dx}{\cos^2 x-\sin^2 x}=\int\frac{\cos x\,dx}{(1-\sin^2 x)-\sin^2 x}-\int\frac{\sin x\,dx}{\cos^2 x-(1-\cos^2 x)}=\int\frac{\cos x\,dx}{1-2\sin^2 x}-\int\frac{\sin x\,dx}{2\cos^2 x-1}$$

$$=\left[\begin{array}{r|l|l}\text{Subst. } u=\sin x & du=\cos x\,dx & \sin x\neq\pm\cos x\\ v=\cos x & dv=-\sin x\,dx & x\neq\frac{3\pi}{4}+k\pi,\ k\in Z\end{array}\right]=\int\frac{du}{1-2u^2}-\int\frac{-\,dv}{2v^2-1}=\int\frac{\frac12\,dv}{v^2-\frac24}-\int\frac{\frac12\,du}{u^2-\frac24}$$

$$=\left[\begin{array}{r|l|l|l}\text{UGS: } t=\mathrm{tg}\,\frac{x}{2} & \sin x=\frac{2t}{t^2+1} & \sin x\neq-\cos x & x\in\left(-\pi+2k\pi;-\frac{\pi}{4}+2k\pi\right),\ t\in\left(-\infty;1-\sqrt2\right)\\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\neq\frac{3\pi}{4}+k\pi & x\in\left(-\frac{\pi}{4}+2k\pi;\frac{3\pi}{4}+2k\pi\right),\ t\in\left(1-\sqrt2;1+\sqrt2\right)\\ & & & x\in\left(\frac{3\pi}{4}+2k\pi;\pi+2k\pi\right),\ t\in\left(1+\sqrt2;\infty\right),\ k\in Z\end{array}\right]=\int\frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}+\frac{2t}{t^2+1}}$$

$$=\int\frac{2\,dt}{2t+1-t^2}=\int\frac{-2\,dt}{t^2-2t+1-2}=\int\frac{-2\,dt}{(t-1)^2-(\sqrt2)^2}=\left[\begin{array}{r|l}\text{Subst. } u=t-1 & x\neq1\pm\sqrt2\\ du=dt & t\neq\pm\sqrt2\end{array}\right]$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x + \sin x}$$

$$= \int \frac{(\cos x - \sin x)\,dx}{\cos^2 x - \sin^2 x} = \int \frac{\cos x\,dx}{(1-\sin^2 x) - \sin^2 x} - \int \frac{\sin x\,dx}{\cos^2 x - (1-\cos^2 x)} = \int \frac{\cos x\,dx}{1-2\sin^2 x} - \int \frac{\sin x\,dx}{2\cos^2 x - 1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=\sin x & du=\cos x\,dx & \sin x \neq \pm\cos x \\ v=\cos x & dv=-\sin x\,dx & x\neq \frac{3\pi}{4}+k\pi,\ k\in Z \end{array} \right] = \int \frac{du}{1-2u^2} - \int \frac{-\,dv}{2v^2-1} = \int \frac{\frac{1}{2}\,dv}{v^2-\frac{2}{4}} - \int \frac{\frac{1}{2}\,du}{u^2-\frac{2}{4}}$$

$$= \frac{\frac{1}{2}}{2\frac{\sqrt{2}}{2}} \ln \left| \frac{v-\frac{\sqrt{2}}{2}}{v+\frac{\sqrt{2}}{2}} \right| - \frac{\frac{1}{2}}{2\frac{\sqrt{2}}{2}} \ln \left| \frac{u-\frac{\sqrt{2}}{2}}{u+\frac{\sqrt{2}}{2}} \right| + c_1$$

$$= \left[\begin{array}{r|l|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1} & \sin x\neq -\cos x & x\in(-\pi+2k\pi; -\frac{\pi}{4}+2k\pi),\ t\in(-\infty; 1-\sqrt{2}) \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\neq \frac{3\pi}{4}+k\pi & x\in(-\frac{\pi}{4}+2k\pi; \frac{3\pi}{4}+2k\pi),\ t\in(1-\sqrt{2}; 1+\sqrt{2}) \\ & & & x\in(\frac{3\pi}{4}+2k\pi; \pi+2k\pi),\ t\in(1+\sqrt{2}; \infty),\ k\in Z \end{array} \right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}+\frac{2t}{t^2+1}}$$

$$= \int \frac{2\,dt}{2t+1-t^2} = \int \frac{-2\,dt}{t^2-2t+1-2} = \int \frac{-2\,dt}{(t-1)^2-(\sqrt{2})^2} = \left[\begin{array}{r|l} \text{Subst. } u=t-1 & x\neq 1\pm\sqrt{2} \\ du=dt & t\neq \pm\sqrt{2} \end{array} \right] = \int \frac{-2\,du}{u^2-(\sqrt{2})^2}$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x+\sin x}$$

$$=\int\frac{(\cos x-\sin x)\,dx}{\cos^2 x-\sin^2 x}=\int\frac{\cos x\,dx}{(1-\sin^2 x)-\sin^2 x}-\int\frac{\sin x\,dx}{\cos^2 x-(1-\cos^2 x)}=\int\frac{\cos x\,dx}{1-2\sin^2 x}-\int\frac{\sin x\,dx}{2\cos^2 x-1}$$

$$=\left[\begin{array}{r|l|l}\text{Subst. } u=\sin x & du=\cos x\,dx & \sin x\neq\pm\cos x\\ v=\cos x & dv=-\sin x\,dx & x\neq\frac{3\pi}{4}+k\pi,\ k\in Z\end{array}\right]=\int\frac{du}{1-2u^2}-\int\frac{-\,dv}{2v^2-1}=\int\frac{\frac{1}{2}\,dv}{v^2-\frac{2}{4}}-\int\frac{\frac{1}{2}\,du}{u^2-\frac{2}{4}}$$

$$=\frac{\frac{1}{2}}{2\frac{\sqrt2}{2}}\ln\left|\frac{v-\frac{\sqrt2}{2}}{v+\frac{\sqrt2}{2}}\right|-\frac{\frac{1}{2}}{2\frac{\sqrt2}{2}}\ln\left|\frac{u-\frac{\sqrt2}{2}}{u+\frac{\sqrt2}{2}}\right|+c_1=\frac{1}{2\sqrt2}\ln\left|\frac{2v-\sqrt2}{2v+\sqrt2}\right|-\frac{1}{2\sqrt2}\ln\left|\frac{2u-\sqrt2}{2u+\sqrt2}\right|+c_1$$

$$=\left[\begin{array}{r|l|l|l}\text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1} & \sin x\neq-\cos x & x\in\left(-\pi+2k\pi;-\frac{\pi}{4}+2k\pi\right),\ t\in\left(-\infty;1-\sqrt2\right)\\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\neq\frac{3\pi}{4}+k\pi & x\in\left(-\frac{\pi}{4}+2k\pi;\frac{3\pi}{4}+2k\pi\right),\ t\in\left(1-\sqrt2;1+\sqrt2\right)\\ & & & x\in\left(\frac{3\pi}{4}+2k\pi;\pi+2k\pi\right),\ t\in\left(1+\sqrt2;\infty\right),\ k\in Z\end{array}\right]=\int\frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}+\frac{2t}{t^2+1}}$$

$$=\int\frac{2\,dt}{2t+1-t^2}=\int\frac{-2\,dt}{t^2-2t+1-2}=\int\frac{-2\,dt}{(t-1)^2-(\sqrt2)^2}=\left[\begin{array}{r|l}\text{Subst. } u=t-1 & x\neq1\pm\sqrt2\\ du=dt & t\neq\pm\sqrt2\end{array}\right]=\int\frac{-2\,du}{u^2-(\sqrt2)^2}$$

$$=\frac{-2}{2\sqrt2}\ln\left|\frac{u-\sqrt2}{u+\sqrt2}\right|+c_2$$

Riešené príklady – 046

$$\int \frac{dx}{\cos x+\sin x} = \frac{\sqrt{2}}{4}\ln\left|\frac{2\cos x-\sqrt{2}}{2\cos x+\sqrt{2}}\right| - \frac{\sqrt{2}}{4}\ln\left|\frac{2\sin x-\sqrt{2}}{2\sin x+\sqrt{2}}\right| + c_1$$

$$= \int \frac{(\cos x-\sin x)\,dx}{\cos^2 x-\sin^2 x} = \int \frac{\cos x\,dx}{(1-\sin^2 x)-\sin^2 x} - \int \frac{\sin x\,dx}{\cos^2 x-(1-\cos^2 x)} = \int \frac{\cos x\,dx}{1-2\sin^2 x} - \int \frac{\sin x\,dx}{2\cos^2 x-1}$$

$$= \left[\begin{array}{r|l|l} \text{Subst. } u=\sin x & du=\cos x\,dx & \sin x \neq \pm\cos x \\ v=\cos x & dv=-\sin x\,dx & x\neq \frac{3\pi}{4}+k\pi,\ k\in Z \end{array}\right] = \int \frac{du}{1-2u^2} - \int \frac{-\,dv}{2v^2-1} = \int \frac{\frac{1}{2}\,dv}{v^2-\frac{2}{4}} - \int \frac{\frac{1}{2}\,du}{u^2-\frac{2}{4}}$$

$$= \frac{\frac{1}{2}}{2\frac{\sqrt{2}}{2}}\ln\left|\frac{v-\frac{\sqrt{2}}{2}}{v+\frac{\sqrt{2}}{2}}\right| - \frac{\frac{1}{2}}{2\frac{\sqrt{2}}{2}}\ln\left|\frac{u-\frac{\sqrt{2}}{2}}{u+\frac{\sqrt{2}}{2}}\right| + c_1 = \frac{1}{2\sqrt{2}}\ln\left|\frac{2v-\sqrt{2}}{2v+\sqrt{2}}\right| - \frac{1}{2\sqrt{2}}\ln\left|\frac{2u-\sqrt{2}}{2u+\sqrt{2}}\right| + c_1$$

$$= \frac{\sqrt{2}}{4}\ln\left|\frac{2\cos x-\sqrt{2}}{2\cos x+\sqrt{2}}\right| - \frac{\sqrt{2}}{4}\ln\left|\frac{2\sin x-\sqrt{2}}{2\sin x+\sqrt{2}}\right| + c_1,\ x\in R-\left\{\frac{3\pi}{4}+k\pi,\ k\in Z\right\},\ c_1\in R.$$

$$= \left[\begin{array}{l|l|l|l} \text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1} & \sin x\neq -\cos x & x\in(-\pi+2k\pi;-\frac{\pi}{4}+2k\pi),\ t\in(-\infty;1-\sqrt{2}) \\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\neq \frac{3\pi}{4}+k\pi & x\in(-\frac{\pi}{4}+2k\pi;\frac{3\pi}{4}+2k\pi),\ t\in(1-\sqrt{2};1+\sqrt{2}) \\ & & & x\in(\frac{3\pi}{4}+2k\pi;\pi+2k\pi),\ t\in(1+\sqrt{2};\infty),\ k\in Z \end{array}\right] = \int \frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}+\frac{2t}{t^2+1}}$$

$$= \int \frac{2\,dt}{2t+1-t^2} = \int \frac{-2\,dt}{t^2-2t+1-2} = \int \frac{-2\,dt}{(t-1)^2-(\sqrt{2})^2} = \left[\begin{array}{r|l} \text{Subst. } u=t-1 & x\neq 1\pm\sqrt{2} \\ du=dt & t\neq\pm\sqrt{2} \end{array}\right] = \int \frac{-2\,du}{u^2-(\sqrt{2})^2}$$

$$= \frac{-2}{2\sqrt{2}}\ln\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right| + c_2 = \frac{1}{\sqrt{2}}\ln\left|\frac{(t-1)+\sqrt{2}}{(t-1)-\sqrt{2}}\right| + c_2$$

Riešené príklady – 046

$$\int\frac{dx}{\cos x+\sin x}=\frac{\sqrt2}{4}\ln\left|\frac{2\cos x-\sqrt2}{2\cos x+\sqrt2}\right|-\frac{\sqrt2}{4}\ln\left|\frac{2\sin x-\sqrt2}{2\sin x+\sqrt2}\right|+c_1=\frac{\sqrt2}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-1+\sqrt2}{\operatorname{tg}\frac{x}{2}-1-\sqrt2}\right|+c_2$$

$$=\int\frac{(\cos x-\sin x)\,dx}{\cos^2x-\sin^2x}=\int\frac{\cos x\,dx}{(1-\sin^2x)-\sin^2x}-\int\frac{\sin x\,dx}{\cos^2x-(1-\cos^2x)}=\int\frac{\cos x\,dx}{1-2\sin^2x}-\int\frac{\sin x\,dx}{2\cos^2x-1}$$

$$=\left[\begin{array}{r|l|l}\text{Subst. } u=\sin x & du=\cos x\,dx & \sin x\neq\pm\cos x\\ v=\cos x & dv=-\sin x\,dx & x\neq\frac{3\pi}{4}+k\pi,\ k\in Z\end{array}\right]=\int\frac{du}{1-2u^2}-\int\frac{-\,dv}{2v^2-1}=\int\frac{\frac12\,dv}{v^2-\frac24}-\int\frac{\frac12\,du}{u^2-\frac24}$$

$$=\frac{\frac12}{2\frac{\sqrt2}{2}}\ln\left|\frac{v-\frac{\sqrt2}{2}}{v+\frac{\sqrt2}{2}}\right|-\frac{\frac12}{2\frac{\sqrt2}{2}}\ln\left|\frac{u-\frac{\sqrt2}{2}}{u+\frac{\sqrt2}{2}}\right|+c_1=\frac{1}{2\sqrt2}\ln\left|\frac{2v-\sqrt2}{2v+\sqrt2}\right|-\frac{1}{2\sqrt2}\ln\left|\frac{2u-\sqrt2}{2u+\sqrt2}\right|+c_1$$

$$=\frac{\sqrt2}{4}\ln\left|\frac{2\cos x-\sqrt2}{2\cos x+\sqrt2}\right|-\frac{\sqrt2}{4}\ln\left|\frac{2\sin x-\sqrt2}{2\sin x+\sqrt2}\right|+c_1,\ x\in R-\left\{\frac{3\pi}{4}+k\pi,\ k\in Z\right\},\ c_1\in R.$$

$$=\left[\begin{array}{l|l|l|l}\text{UGS: } t=\operatorname{tg}\frac{x}{2} & \sin x=\frac{2t}{t^2+1} & \sin x\neq-\cos x & x\in(-\pi+2k\pi;-\frac{\pi}{4}+2k\pi),\ t\in(-\infty;1-\sqrt2)\\ dx=\frac{2\,dt}{t^2+1} & \cos x=\frac{1-t^2}{t^2+1} & x\neq\frac{3\pi}{4}+k\pi & x\in(-\frac{\pi}{4}+2k\pi;\frac{3\pi}{4}+2k\pi),\ t\in(1-\sqrt2;1+\sqrt2)\\ & & & x\in(\frac{3\pi}{4}+2k\pi;\pi+2k\pi),\ t\in(1+\sqrt2;\infty),\ k\in Z\end{array}\right]=\int\frac{\frac{2\,dt}{t^2+1}}{\frac{1-t^2}{t^2+1}+\frac{2t}{t^2+1}}$$

$$=\int\frac{2\,dt}{2t+1-t^2}=\int\frac{-2\,dt}{t^2-2t+1-2}=\int\frac{-2\,dt}{(t-1)^2-(\sqrt2)^2}=\left[\begin{array}{r|l}\text{Subst. } u=t-1 & x\neq1\pm\sqrt2\\ du=dt & t\neq\pm\sqrt2\end{array}\right]=\int\frac{-2\,du}{u^2-(\sqrt2)^2}$$

$$=\frac{-2}{2\sqrt2}\ln\left|\frac{u-\sqrt2}{u+\sqrt2}\right|+c_2=\frac{1}{\sqrt2}\ln\left|\frac{(t-1)+\sqrt2}{(t-1)-\sqrt2}\right|+c_2=\frac{\sqrt2}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}-1+\sqrt2}{\operatorname{tg}\frac{x}{2}-1-\sqrt2}\right|+c_2,$$

$$x\in R-\left\{\frac{3\pi}{4}+k\pi,\ k\in Z\right\},\ c_2\in R.$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x)\, dx}{\sqrt[4]{\sin x + \cos x}}$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x}\, dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x)\,dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\begin{array}{r|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & x \in (-\pi + 2k\pi; \pi + 2k\pi), \quad \sin x \neq -\cos x,\ x \neq \frac{3\pi}{4} + k\pi, \quad k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & \sin x + \cos x > 0,\ x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in \left(1-\sqrt{2}; 1+\sqrt{2}\right) \end{array} \right]$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \sin x + \cos x & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{\pi}{4} + 2k\pi\right\rangle, t \in (0; \sqrt{2}\rangle \\ dt = (\cos x - \sin x)\,dx & x \in \left\langle\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right), t \in (0; \sqrt{2}\rangle, k \in Z \end{array} \right]$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x}\,dx$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}\,dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x)\,dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\begin{array}{l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & x \in (-\pi + 2k\pi; \pi + 2k\pi), \quad \sin x \neq -\cos x,\ x \neq \frac{3\pi}{4} + k\pi, \quad k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & \sin x + \cos x > 0,\ x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in (1-\sqrt{2}; 1+\sqrt{2}) \end{array} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}\right)\frac{2\,dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}}$$

$$= \left[\begin{array}{l|l} \text{Subst. } t = \sin x + \cos x & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{\pi}{4} + 2k\pi\right\rangle, t \in (0; \sqrt{2}\rangle \\ dt = (\cos x - \sin x)\,dx & x \in \left\langle \frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right), t \in (0; \sqrt{2}\rangle, k \in Z \end{array} \right] = -\int \frac{dt}{\sqrt[4]{t}}$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x}\,dx$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}\,dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x)\,dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\begin{array}{l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & x \in (-\pi + 2k\pi; \pi + 2k\pi), \quad \sin x \neq -\cos x,\ x \neq \frac{3\pi}{4} + k\pi, \quad k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & \sin x + \cos x > 0,\ x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in (1-\sqrt{2}; 1+\sqrt{2}) \end{array} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}\right)\frac{2\,dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1)\,dt}{\sqrt[4]{\frac{1+2t-t^2}{1+t^2}}\,(1+t^2)^2}$$

$$= \left[\begin{array}{l|l} \text{Subst. } t = \sin x + \cos x & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{\pi}{4} + 2k\pi\right\rangle,\ t \in (0; \sqrt{2}\rangle \\ dt = (\cos x - \sin x)\,dx & x \in \left\langle \frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in (0; \sqrt{2}\rangle,\ k \in Z \end{array} \right] = -\int \frac{dt}{\sqrt[4]{t}} = -\int t^{-\frac{1}{4}}\,dt$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x}\,dx$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}\,dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x}\,dx$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x)\,dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\begin{array}{l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & x \in (-\pi + 2k\pi; \pi + 2k\pi), \quad \sin x \neq -\cos x,\ x \neq \frac{3\pi}{4} + k\pi, \quad k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & \sin x + \cos x > 0,\ x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in (1-\sqrt{2}; 1+\sqrt{2}) \end{array} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}\right)\frac{2\,dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1)\,dt}{\sqrt[4]{\frac{1+2t-t^2}{1+t^2}}\,(1+t^2)^2} = \cdots$$

[Príliš veľa práce pre normálneho smrteľníka.]

$$= \left[\begin{array}{l|l} \text{Subst. } t = \sin x + \cos x & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{\pi}{4} + 2k\pi\right\rangle, t \in (0; \sqrt{2}\rangle \\ dt = (\cos x - \sin x)\,dx & x \in \left\langle \frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right), t \in (0; \sqrt{2}\rangle, k \in Z \end{array} \right] = -\int \frac{dt}{\sqrt[4]{t}} = -\int t^{-\frac{1}{4}}\,dt = -\frac{t^{\frac{3}{4}}}{\frac{3}{4}} + c$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x}\,dx = \ln|\sin x + \cos x| + c$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}\,dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x}\,dx = \ln|\sin x + \cos x| + c,$$

$$x \in R - \left\{\frac{\pi}{2} + k\pi,\ -\frac{\pi}{4} + k\pi,\ k \in Z\right\},\ c \in R.$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x)\,dx}{\sqrt[4]{\sin x + \cos x}}$$

$$= \left[\begin{array}{l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & x \in (-\pi + 2k\pi; \pi + 2k\pi), \quad \sin x \neq -\cos x,\ x \neq \frac{3\pi}{4} + k\pi, \quad k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & \sin x + \cos x > 0,\ x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in (1-\sqrt{2}; 1+\sqrt{2}) \end{array} \right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}\right)\frac{2\,dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1)\,dt}{\sqrt[4]{\frac{1+2t-t^2}{1+t^2}}\,(1+t^2)^2} = \cdots$$

[Príliš veľa práce pre normálneho smrteľníka.]

$$= \left[\begin{array}{l|l} \text{Subst. } t = \sin x + \cos x & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{\pi}{4} + 2k\pi\right\rangle,\ t \in (0; \sqrt{2}\rangle \\ dt = (\cos x - \sin x)\,dx & x \in \left\langle\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in (0; \sqrt{2}\rangle,\ k \in Z \end{array} \right] = -\int \frac{dt}{\sqrt[4]{t}} = -\int t^{-\frac{1}{4}}\,dt = -\frac{t^{\frac{3}{4}}}{\frac{3}{4}} + c$$

$$= c - \frac{4}{3}\sqrt[4]{t^3}$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x}\,dx = \ln|\sin x + \cos x| + c$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}\,dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x}\,dx = \ln|\sin x + \cos x| + c,$$

$$x \in R - \left\{\frac{\pi}{2} + k\pi,\ -\frac{\pi}{4} + k\pi,\ k \in Z\right\},\ c \in R.$$

Riešené príklady – 047, 048

$$\int \frac{(\sin x - \cos x)\,dx}{\sqrt[4]{\sin x + \cos x}} = c - \frac{4}{3}\sqrt[4]{(\sin x + \cos x)^3}$$

$$= \left[\begin{array}{l|l|l} \text{UGS: } t = \operatorname{tg}\frac{x}{2} & \sin x = \frac{2t}{t^2+1} & x \in (-\pi + 2k\pi; \pi + 2k\pi), \quad \sin x \neq -\cos x,\ x \neq \frac{3\pi}{4} + k\pi, \quad k \in Z \\ dx = \frac{2\,dt}{t^2+1} & \cos x = \frac{1-t^2}{t^2+1} & \sin x + \cos x > 0,\ x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ t \in (1-\sqrt{2}; 1+\sqrt{2}) \end{array}\right]$$

$$= \int \frac{\left(\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}\right)\frac{2\,dt}{1+t^2}}{\sqrt[4]{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}} = \int \frac{(t^2+2t-1)\,dt}{\sqrt[4]{\frac{1+2t-t^2}{1+t^2}}\,(1+t^2)^2} = \cdots$$

[Príliš veľa práce pre normálneho smrteľníka.]

$$= \left[\begin{array}{l|l} \text{Subst. } t = \sin x + \cos x & x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{\pi}{4} + 2k\pi\right\rangle, t \in (0; \sqrt{2}\rangle \\ dt = (\cos x - \sin x)\,dx & x \in \left\langle\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right), t \in (0; \sqrt{2}\rangle, k \in Z \end{array}\right] = -\int \frac{dt}{\sqrt[4]{t}} = -\int t^{-\frac{1}{4}}\,dt = -\frac{t^{\frac{3}{4}}}{\frac{3}{4}} + c$$

$$= c - \frac{4}{3}\sqrt[4]{t^3} = c - \frac{4}{3}\sqrt[4]{(\sin x + \cos x)^3},\ x \in \left(-\frac{\pi}{4} + 2k\pi; \frac{3\pi}{4} + 2k\pi\right),\ k \in Z,\ c \in R.$$

$$\int \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x}\,dx = \ln|\sin x + \cos x| + c$$

$$= \int \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}\,dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \int \frac{(\sin x + \cos x)'}{\sin x + \cos x}\,dx = \ln|\sin x + \cos x| + c,$$

$$x \in R - \left\{\frac{\pi}{2} + k\pi,\ -\frac{\pi}{4} + k\pi,\ k \in Z\right\},\ c \in R.$$

Riešené príklady – 049, 050

$\int \cos^n ax \cdot \sin ax \, dx$ $\qquad a \in R - \{0\}, \; n \in N$

$\int \frac{\sin ax \, dx}{\cos^n ax}$ $\qquad a \in R - \{0\}, \; n \in N$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \cos ax & ax \in \langle 0 + k\pi; \pi + k\pi \rangle \\ & dt = -a \sin ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right]$$

$$\int \frac{\sin ax \, dx}{\cos^n ax} \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t = \cos ax & ax \in \langle 0 + k\pi; \frac{\pi}{2} + k\pi) \cup (\frac{\pi}{2} + k\pi; \pi + k\pi \rangle \\ & dt = -a \sin ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right]$$

Riešené príklady – 049, 050

$\int \cos^n ax \cdot \sin ax \, dx$ $\quad a \in R - \{0\},\ n \in N$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cos ax & ax \in \langle 0 + k\pi ; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{t^n \, dt}{-a}$$

$\int \frac{\sin ax \, dx}{\cos^n ax}$ $\quad a \in R - \{0\},\ n \in N$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \cos ax & ax \in \langle 0 + k\pi ; \frac{\pi}{2} + k\pi) \cup (\frac{\pi}{2} + k\pi ; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{dt}{-at^n}$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t = \cos ax & ax \in \langle 0 + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{t^n \, dt}{-a} = -\frac{1}{a} \int t^n \, dt$$

$$\int \frac{\sin ax \, dx}{\cos^n ax} \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t = \cos ax & ax \in \langle 0 + k\pi; \frac{\pi}{2} + k\pi) \cup (\frac{\pi}{2} + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{dt}{-at^n} = -\frac{1}{a} \int \frac{dt}{t^n}$$

$n = 1$: $$= -\frac{1}{a} \int \frac{dt}{t}$$

$n > 1$: $$= -\frac{1}{a} \int t^{-n} \, dt$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t = \cos ax & ax \in \langle 0 + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{t^n \, dt}{-a} = -\frac{1}{a} \int t^n \, dt = -\frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + c$$

$$\int \frac{\sin ax \, dx}{\cos^n ax} \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t = \cos ax & ax \in \langle 0 + k\pi; \frac{\pi}{2} + k\pi) \cup (\frac{\pi}{2} + k\pi; \pi + k\pi \rangle \\ dt = -a \sin ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{dt}{-at^n} = -\frac{1}{a} \int \frac{dt}{t^n}$$

$n = 1$: $$= -\frac{1}{a} \int \frac{dt}{t} = -\frac{1}{a} \ln |t| + c$$

$n > 1$: $$= -\frac{1}{a} \int t^{-n} \, dt = -\frac{1}{a} \cdot \frac{t^{-n+1}}{-n+1} + c$$

Riešené príklady – 049, 050

$$\int \cos^n ax \cdot \sin ax \, dx = -\frac{\cos^{n+1} ax}{a(n+1)} + c \qquad a\in R-\{0\},\ n\in N$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=\cos ax & ax\in\langle 0+k\pi;\pi+k\pi\rangle \\ dt=-a\sin ax\,dx & t\in\langle -1;1\rangle,\quad k\in Z \end{array}\right] = \int \frac{t^n\,dt}{-a} = -\frac{1}{a}\int t^n\,dt = -\frac{1}{a}\cdot\frac{t^{n+1}}{n+1} + c$$

$$= -\frac{\cos^{n+1} ax}{a(n+1)} + c,\ x\in R,\ c\in R.$$

$$\int \frac{\sin ax\,dx}{\cos^n ax} = -\frac{1}{a}\ln|\cos ax| + c \text{ resp. } \frac{1}{a(n-1)\cos^{n-1} ax} + c \qquad a\in R-\{0\},\ n\in N$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=\cos ax & ax\in\langle 0+k\pi;\frac{\pi}{2}+k\pi)\cup(\frac{\pi}{2}+k\pi;\pi+k\pi\rangle \\ dt=-a\sin ax\,dx & t\in\langle -1;0)\cup(0;1\rangle,\quad k\in Z \end{array}\right] = \int \frac{dt}{-at^n} = -\frac{1}{a}\int\frac{dt}{t^n}$$

$n=1$: $$= -\frac{1}{a}\int\frac{dt}{t} = -\frac{1}{a}\ln|t| + c = -\frac{1}{a}\ln|\cos ax| + c,\ n=1,$$

$n>1$: $$= -\frac{1}{a}\int t^{-n}\,dt = -\frac{1}{a}\cdot\frac{t^{-n+1}}{-n+1} + c = \frac{1}{a(n-1)\cos^{n-1} ax} + c,\ n=2,3,4,\ldots,$$

$$x\in R-\left\{\frac{\pi+2k\pi}{2a},\ k\in Z\right\},\ c\in R.$$

Riešené príklady – 051, 052

$\int \sin^n ax \cdot \cos ax \, dx$ $\quad a \in R - \{0\},\ n \in N$

$I = \int \frac{\cos ax \, dx}{\sin^n ax}$ $\quad a \in R - \{0\},\ n \in N$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right]$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax} \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi) \cup (0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right]$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{t^n \, dt}{a}$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax} \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi) \cup (0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{dt}{at^n}$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{t^n \, dt}{a} = \frac{1}{a} \int t^n \, dt$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax} \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{r|l} \text{Subst.} \quad t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi) \cup (0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{dt}{at^n} = \frac{1}{a} \int \frac{dt}{t^n}$$

$n = 1$: $$= \frac{1}{a} \int \frac{dt}{t}$$

$n > 1$: $$= \frac{1}{a} \int t^{-n} \, dt$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{c|l} \text{Subst. } \ t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{t^n \, dt}{a} = \frac{1}{a} \int t^n \, dt = \frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + c$$

$$I = \int \frac{\cos ax \, dx}{\sin^n ax} \qquad a \in R - \{0\},\ n \in N$$

$$= \left[\begin{array}{c|l} \text{Subst. } \ t = \sin ax & ax \in \langle -\frac{\pi}{2} + k\pi; 0 + k\pi) \cup (0 + k\pi; \frac{\pi}{2} + k\pi \rangle \\ dt = a \cos ax \, dx & t \in \langle -1; 0) \cup (0; 1 \rangle, \quad k \in Z \end{array} \right] = \int \frac{dt}{at^n} = \frac{1}{a} \int \frac{dt}{t^n}$$

$n=1$ $$= \frac{1}{a} \int \frac{dt}{t} = \frac{1}{a} \ln |t| + c$$

$n>1$ $$= \frac{1}{a} \int t^{-n} \, dt = \frac{1}{a} \cdot \frac{t^{-n+1}}{-n+1} + c$$

Riešené príklady – 051, 052

$$\int \sin^n ax \cdot \cos ax \, dx = \frac{\sin^{n+1} ax}{a(n+1)} + c \qquad a\in R-\{0\},\ n\in N$$

$$= \left[\begin{array}{c|l} \text{Subst.}\quad t=\sin ax & ax\in\langle -\frac{\pi}{2}+k\pi; \frac{\pi}{2}+k\pi\rangle \\ dt = a\cos ax\, dx & t\in\langle -1;1\rangle,\quad k\in Z \end{array} \right] = \int \frac{t^n\, dt}{a} = \frac{1}{a}\int t^n\, dt = \frac{1}{a}\cdot\frac{t^{n+1}}{n+1} + c$$

$$= \frac{\sin^{n+1} ax}{a(n+1)} + c,\ x\in R,\ c\in R.$$

$$I = \int \frac{\cos ax\, dx}{\sin^n ax} = \frac{1}{a}\ln|\sin ax| + c \ \text{ resp. }\ I = -\frac{1}{a(n-1)\sin^{n-1} ax} + c \qquad a\in R-\{0\},\ n\in N$$

$$= \left[\begin{array}{c|l} \text{Subst.}\quad t=\sin ax & ax\in\langle -\frac{\pi}{2}+k\pi; 0+k\pi) \cup (0+k\pi; \frac{\pi}{2}+k\pi\rangle \\ dt = a\cos ax\, dx & t\in\langle -1;0)\cup(0;1\rangle,\quad k\in Z \end{array} \right] = \int \frac{dt}{at^n} = \frac{1}{a}\int\frac{dt}{t^n}$$

$n=1$: $$= \frac{1}{a}\int\frac{dt}{t} = \frac{1}{a}\ln|t| + c = \frac{1}{a}\ln|\sin ax| + c,\ n=1,$$

$n>1$: $$= \frac{1}{a}\int t^{-n}\, dt = \frac{1}{a}\cdot\frac{t^{-n+1}}{-n+1} + c = -\frac{1}{a(n-1)\sin^{n-1} ax} + c,\ n=2,3,4,\ldots,$$

$$x\in R-\left\{\frac{k\pi}{a}, k\in Z\right\},\ c\in R.$$

Riešené príklady – 053

$I = \int \sin ax \cdot \cos bx \, dx$

$a, b \in R - \{0\}, \; a \neq \pm b$

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$$I = \int \sin ax \cdot \cos bx \, \mathrm{d}x$$

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$$= \left[\begin{array}{l} \sin\alpha \cdot \cos\beta \\ \quad = \frac{\sin(\alpha-\beta)+\sin(\alpha+\beta)}{2} \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \sin ax & u' = a\cos ax \\ v' = \cos bx & v = \frac{1}{b}\sin bx \end{array} \right]$$

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$$I=\int \sin ax\cdot\cos bx\,dx = c_1-\frac{\cos(a-b)x}{2(a-b)}-\frac{\cos(a+b)x}{2(a+b)}$$

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Riešené príklady – 053

$$I=\int \sin ax\cdot\cos bx\,dx = c_1-\frac{\cos(a-b)x}{2(a-b)}-\frac{\cos(a+b)x}{2(a+b)} = \frac{b\sin ax\cdot\sin bx+a\cos ax\cdot\cos bx}{b^2-a^2}+c_2$$

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Riešené príklady – 054

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$$I=\int \cos ax\cdot\cos bx\,\mathrm{d}x=\frac{\sin(a-b)x}{2(a-b)}+\frac{\sin(a+b)x}{2(a+b)}+c_1=\frac{b\cos ax\cdot\sin bx-a\sin ax\cdot\cos bx}{b^2-a^2}+c_2$$

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$$=\left[\begin{array}{l}\sin\alpha\cdot\sin\beta\\ \quad=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}\end{array}\right]=\int\frac{\cos(ax-bx)-\cos(ax+bx)}{2}\,dx=\int\frac{\cos(a-b)x\,dx}{2}-\int\frac{\cos(a+b)x\,dx}{2}$$

$$=\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)}+c_1,\ x\in R,\ c_1\in R.$$

$$=\left[\begin{array}{l|l}u=\sin ax & u'=\ \ a\cos ax\\ v'=\sin bx & v=-\frac{1}{b}\cos bx\end{array}\right]=-\frac{\sin ax\cdot\cos bx}{b}+\frac{a}{b}\int\cos ax\cdot\cos bx\,dx$$

$$=\left[\begin{array}{l|l}u=\cos ax & u'=-a\sin ax\\ v'=\cos bx & v=\ \ \frac{1}{b}\sin bx\end{array}\right]=-\frac{\sin ax\cdot\cos bx}{b}+\frac{a}{b}\left[\frac{\cos ax\cdot\sin bx}{b}+\frac{a}{b}\int\sin ax\cdot\sin bx\,dx\right]$$

$$=-\frac{b\sin ax\cdot\cos bx}{b^2}+\frac{a\cos ax\cdot\sin bx}{b^2}+\frac{a^2}{b^2}\int\sin ax\cdot\sin bx\,dx$$

Riešené príklady – 055

$$I=\int \sin ax\cdot\sin bx\,dx=\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)}+c_1$$

$a,b\in R-\{0\},\ a\neq\pm b$

$$=\left[\begin{array}{l}\sin\alpha\cdot\sin\beta\\ \quad=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}\end{array}\right]=\int\frac{\cos(ax-bx)-\cos(ax+bx)}{2}\,dx=\int\frac{\cos(a-b)x\,dx}{2}-\int\frac{\cos(a+b)x\,dx}{2}$$

$$=\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)}+c_1,\ x\in R,\ c_1\in R.$$

$$=\left[\begin{array}{l|l}u=\sin ax & u'=\ a\cos ax\\ v'=\sin bx & v=-\frac{1}{b}\cos bx\end{array}\right]=-\frac{\sin ax\cdot\cos bx}{b}+\frac{a}{b}\int\cos ax\cdot\cos bx\,dx$$

$$=\left[\begin{array}{l|l}u=\cos ax & u'=-a\sin ax\\ v'=\cos bx & v=\ \frac{1}{b}\sin bx\end{array}\right]=-\frac{\sin ax\cdot\cos bx}{b}+\frac{a}{b}\left[\frac{\cos ax\cdot\sin bx}{b}+\frac{a}{b}\int\sin ax\cdot\sin bx\,dx\right]$$

$$=-\frac{b\sin ax\cdot\cos bx}{b^2}+\frac{a\cos ax\cdot\sin bx}{b^2}+\frac{a^2}{b^2}\int\sin ax\cdot\sin bx\,dx$$

$$=\left[\text{Rovnica}\quad I=-\frac{b\sin ax\cdot\cos bx}{b^2}+\frac{a\cos ax\cdot\sin bx}{b^2}+\frac{a^2}{b^2}I\ \Rightarrow\ \frac{b^2-a^2}{b^2}I=\left(1-\frac{a^2}{b^2}\right)I=-\frac{b\sin ax\cdot\cos bx}{b^2}+\frac{a\cos ax\cdot\sin bx}{b^2}\right]$$

Riešené príklady – 055

$$I=\int \sin ax\cdot\sin bx\,dx=\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)}+c_1=\frac{-b\sin ax\cdot\cos bx+a\cos ax\cdot\sin bx}{b^2-a^2}+c_2$$

$a,b\in R-\{0\},\ a\neq\pm b$

$$=\left[\begin{array}{l}\sin\alpha\cdot\sin\beta\\ \quad=\frac{\cos(\alpha-\beta)-\cos(\alpha+\beta)}{2}\end{array}\right]=\int\frac{\cos(ax-bx)-\cos(ax+bx)}{2}\,dx=\int\frac{\cos(a-b)x\,dx}{2}-\int\frac{\cos(a+b)x\,dx}{2}$$

$$=\frac{\sin(a-b)x}{2(a-b)}-\frac{\sin(a+b)x}{2(a+b)}+c_1,\ x\in R,\ c_1\in R.$$

$$=\left[\begin{array}{l|l}u=\sin ax & u'=\ a\cos ax\\ v'=\sin bx & v=-\frac{1}{b}\cos bx\end{array}\right]=-\frac{\sin ax\cdot\cos bx}{b}+\frac{a}{b}\int\cos ax\cdot\cos bx\,dx$$

$$=\left[\begin{array}{l|l}u=\cos ax & u'=-a\sin ax\\ v'=\cos bx & v=\ \frac{1}{b}\sin bx\end{array}\right]=-\frac{\sin ax\cdot\cos bx}{b}+\frac{a}{b}\left[\frac{\cos ax\cdot\sin bx}{b}+\frac{a}{b}\int\sin ax\cdot\sin bx\,dx\right]$$

$$=-\frac{b\sin ax\cdot\cos bx}{b^2}+\frac{a\cos ax\cdot\sin bx}{b^2}+\frac{a^2}{b^2}\int\sin ax\cdot\sin bx\,dx$$

$$=\left[\text{Rovnica}\quad I=-\frac{b\sin ax\cdot\cos bx}{b^2}+\frac{a\cos ax\cdot\sin bx}{b^2}+\frac{a^2}{b^2}I\ \Rightarrow\ \frac{b^2-a^2}{b^2}I=(1-\frac{a^2}{b^2})I=-\frac{b\sin ax\cdot\cos bx}{b^2}+\frac{a\cos ax\cdot\sin bx}{b^2}\right]$$

$$=-\frac{b}{b^2-a^2}\sin ax\cdot\cos bx+\frac{a}{b^2-a^2}\cos ax\cdot\sin bx+c_2,\ x\in R,\ c_2\in R.$$

Riešené príklady – 056, 057, 058

$\int \sin ax \cdot \cos ax \, dx$ $\qquad a \in R, \; a \neq 0$

$\int \cos^2 ax \, dx$ $\qquad a \in R, \; a \neq 0$

$\int \sin^2 ax \, dx$ $\qquad a \in R, \; a \neq 0$

Riešené príklady – 056, 057, 058

$\int \sin ax \cdot \cos ax \, dx$ $\quad a \in R, \; a \neq 0$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2}$$

$\int \cos^2 ax \, dx$ $\quad a \in R, \; a \neq 0$

$$= \int \frac{1 + \cos 2ax}{2} \, dx$$

$\int \sin^2 ax \, dx$ $\quad a \in R, \; a \neq 0$

$$= \int \frac{1 - \cos 2ax}{2} \, dx$$

Riešené príklady – 056, 057, 058

$$\int \sin ax \cdot \cos ax \, dx \qquad a \in R, \ a \neq 0$$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2} = \int \frac{\sin 2ax \, dx}{2}$$

$$\int \cos^2 ax \, dx \qquad a \in R, \ a \neq 0$$

$$= \int \frac{1+\cos 2ax}{2} \, dx = \int \frac{dx}{2} + \int \frac{\cos 2ax \, dx}{2}$$

$$\int \sin^2 ax \, dx \qquad a \in R, \ a \neq 0$$

$$= \int \frac{1-\cos 2ax}{2} \, dx = \int \frac{dx}{2} - \int \frac{\cos 2ax \, dx}{2}$$

Riešené príklady – 056, 057, 058

$\int \sin ax \cdot \cos ax \, dx \qquad a \in R, \ a \neq 0$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2} = \int \frac{\sin 2ax \, dx}{2} = \frac{-\cos 2ax}{2 \cdot 2a} + c$$

$\int \cos^2 ax \, dx \qquad a \in R, \ a \neq 0$

$$= \int \frac{1+\cos 2ax}{2} \, dx = \int \frac{dx}{2} + \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} + \frac{\sin 2ax}{2 \cdot 2a} + c$$

$\int \sin^2 ax \, dx \qquad a \in R, \ a \neq 0$

$$= \int \frac{1-\cos 2ax}{2} \, dx = \int \frac{dx}{2} - \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} - \frac{\sin 2ax}{2 \cdot 2a} + c$$

Riešené príklady – 056, 057, 058

$$\int \sin ax \cdot \cos ax \, dx = -\frac{\cos 2ax}{4a} + c \qquad a \in R,\ a \neq 0$$

$$= \int \frac{2 \sin ax \cdot \cos ax \, dx}{2} = \int \frac{\sin 2ax \, dx}{2} = \frac{-\cos 2ax}{2 \cdot 2a} + c = -\frac{\cos 2ax}{4a} + c,\ x \in R,\ c \in R.$$

$$\int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + c \qquad a \in R,\ a \neq 0$$

$$= \int \frac{1 + \cos 2ax}{2} \, dx = \int \frac{dx}{2} + \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} + \frac{\sin 2ax}{2 \cdot 2a} + c = \frac{x}{2} + \frac{\sin 2ax}{4a} + c,$$
$$x \in R,\ c \in R.$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + c \qquad a \in R,\ a \neq 0$$

$$= \int \frac{1 - \cos 2ax}{2} \, dx = \int \frac{dx}{2} - \int \frac{\cos 2ax \, dx}{2} = \frac{x}{2} - \frac{\sin 2ax}{2 \cdot 2a} + c = \frac{x}{2} - \frac{\sin 2ax}{4a} + c,$$
$$x \in R,\ c \in R.$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$\int x \operatorname{cotg}^2 x \, dx$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x}$$

$$\int x \operatorname{cotg}^2 x \, dx$$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x}$$

Riešené príklady – 059, 060

$\int x \operatorname{tg}^2 x \, dx$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x} = \int \frac{x(1-\cos^2 x) \, dx}{\cos^2 x}$$

$\int x \operatorname{cotg}^2 x \, dx$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1-\sin^2 x) \, dx}{\sin^2 x}$$

Riešené príklady – 059, 060

$$\int x \,\mathrm{tg}^2 x \,\mathrm{d}x$$

$$= \int \frac{x \sin^2 x \,\mathrm{d}x}{\cos^2 x} = \int \frac{x(1-\cos^2 x)\,\mathrm{d}x}{\cos^2 x} = \int \frac{x \,\mathrm{d}x}{\cos^2 x} - \int x \,\mathrm{d}x$$

$$\int x \,\mathrm{cotg}^2 x \,\mathrm{d}x$$

$$= \int \frac{x \cos^2 x \,\mathrm{d}x}{\sin^2 x} = \int \frac{x(1-\sin^2 x)\,\mathrm{d}x}{\sin^2 x} = \int \frac{x \,\mathrm{d}x}{\sin^2 x} - \int x \,\mathrm{d}x$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x} = \int \frac{x(1-\cos^2 x)\, dx}{\cos^2 x} = \int \frac{x \, dx}{\cos^2 x} - \int x \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\cos^2 x} & v = \operatorname{tg} x = \frac{\sin x}{\cos x} \end{array} \right]$$

$$\int x \operatorname{cotg}^2 x \, dx$$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1-\sin^2 x)\, dx}{\sin^2 x} = \int \frac{x \, dx}{\sin^2 x} - \int x \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\sin^2 x} & v = -\operatorname{cotg} x = -\frac{\cos x}{\sin x} \end{array} \right]$$

Riešené príklady – 059, 060

$$\int x \operatorname{tg}^2 x \, dx$$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x} = \int \frac{x(1-\cos^2 x)\, dx}{\cos^2 x} = \int \frac{x\, dx}{\cos^2 x} - \int x \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\cos^2 x} & v = \operatorname{tg} x = \frac{\sin x}{\cos x} \end{array} \right]$$

$$= x \operatorname{tg} x - \int \frac{\sin x \, dx}{\cos x} - \int x \, dx$$

$$\int x \operatorname{cotg}^2 x \, dx$$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1-\sin^2 x)\, dx}{\sin^2 x} = \int \frac{x\, dx}{\sin^2 x} - \int x \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\sin^2 x} & v = -\operatorname{cotg} x = -\frac{\cos x}{\sin x} \end{array} \right]$$

$$= -x \operatorname{cotg} x + \int \frac{\cos x \, dx}{\sin x} - \int x \, dx$$

Riešené príklady – 059, 060

$\int x \operatorname{tg}^2 x \, dx$

$$= \int \frac{x \sin^2 x \, dx}{\cos^2 x} = \int \frac{x(1-\cos^2 x)\, dx}{\cos^2 x} = \int \frac{x\, dx}{\cos^2 x} - \int x\, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\cos^2 x} & v = \operatorname{tg} x = \frac{\sin x}{\cos x} \end{array} \right]$$

$$= x \operatorname{tg} x - \int \frac{\sin x\, dx}{\cos x} - \int x\, dx = x \operatorname{tg} x + \int \frac{-\sin x\, dx}{\cos x} - \frac{x^2}{2}$$

$\int x \operatorname{cotg}^2 x \, dx = -x \operatorname{cotg} x + \ln|\sin x| - \frac{x^2}{2} + c$

$$= \int \frac{x \cos^2 x \, dx}{\sin^2 x} = \int \frac{x(1-\sin^2 x)\, dx}{\sin^2 x} = \int \frac{x\, dx}{\sin^2 x} - \int x\, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\sin^2 x} & v = -\operatorname{cotg} x = -\frac{\cos x}{\sin x} \end{array} \right]$$

$$= -x \operatorname{cotg} x + \int \frac{\cos x\, dx}{\sin x} - \int x\, dx = -x \operatorname{cotg} x + \ln|\sin x| - \frac{x^2}{2} + c,$$

$$x \in R - \{k\pi, k \in Z\}.$$

Riešené príklady – 059, 060

$$\int x\,\mathrm{tg}^2 x\,dx = x\,\mathrm{tg}\,x + \ln|\cos x| - \frac{x^2}{2} + c$$

$$= \int \frac{x\sin^2 x\,dx}{\cos^2 x} = \int \frac{x(1-\cos^2 x)\,dx}{\cos^2 x} = \int \frac{x\,dx}{\cos^2 x} - \int x\,dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\cos^2 x} & v = \mathrm{tg}\,x = \frac{\sin x}{\cos x} \end{array}\right]$$

$$= x\,\mathrm{tg}\,x - \int \frac{\sin x\,dx}{\cos x} - \int x\,dx = x\,\mathrm{tg}\,x + \int \frac{-\sin x\,dx}{\cos x} - \frac{x^2}{2}$$

$$= x\,\mathrm{tg}\,x + \ln|\cos x| - \frac{x^2}{2} + c,\ x \in R - \{\tfrac{\pi}{2} + k\pi,\ k \in Z\}.$$

$$\int x\,\mathrm{cotg}^2 x\,dx = -x\,\mathrm{cotg}\,x + \ln|\sin x| - \frac{x^2}{2} + c$$

$$= \int \frac{x\cos^2 x\,dx}{\sin^2 x} = \int \frac{x(1-\sin^2 x)\,dx}{\sin^2 x} = \int \frac{x\,dx}{\sin^2 x} - \int x\,dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \frac{1}{\sin^2 x} & v = -\,\mathrm{cotg}\,x = -\frac{\cos x}{\sin x} \end{array}\right]$$

$$= -x\,\mathrm{cotg}\,x + \int \frac{\cos x\,dx}{\sin x} - \int x\,dx = -\,x\,\mathrm{cotg}\,x + \ln|\sin x| - \frac{x^2}{2} + c,$$

$$x \in R - \{k\pi,\ k \in Z\}.$$

Riešené príklady – 061

$S_1 = \int x \sin ax \, dx$ $\quad a \in R, \ a \neq 0$

Riešené príklady – 061

$S_1 = \int x \sin ax \, dx$ $\qquad a \in R, \; a \neq 0$

$$= \left[\begin{array}{l|l} u = x & u' = \;\; 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right]$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

Riešené príklady – 061

$S_1 = \int x \sin ax \, dx$ $\qquad a \in R, \ a \neq 0$

$$= \left[\begin{array}{l|l} u = x & u' = \ 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \sin ax = \left[\beta \sin ax + (\alpha + \beta x) a \cos ax \right] + \left[\delta \cos ax - (\gamma + \delta x) a \sin ax \right].$$

Riešené príklady – 061

$S_1 = \int x \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{1}{a}\cos ax \end{array} \right] = -\frac{x\cos ax}{a} + \int \frac{\cos ax}{a}\, dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x)\sin ax + (\gamma + \delta x)\cos ax + c$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha+\beta x)\sin ax + (\gamma+\delta x)\cos ax + c$]

$$x \sin ax = \left[\beta \sin ax + (\alpha+\beta x) a \cos ax\right] + \left[\delta \cos ax - (\gamma+\delta x) a \sin ax\right].$$

$$\left[0+1\cdot x\right]\sin ax + \left[0+0\cdot x\right]\cos ax = \left[\beta - \gamma a - \delta a x\right]\sin ax + \left[\alpha a + \delta + \beta a x\right]\cos ax.$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{matrix} u = x \\ v' = \sin ax \end{matrix} \middle| \begin{matrix} u' = 1 \\ v = -\frac{1}{a}\cos ax \end{matrix} \right] = -\frac{x\cos ax}{a} + \int \frac{\cos ax}{a}\, dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a\cdot a} + c$$

$$= \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} + c,\ x \in R,\ c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x)\sin ax + (\gamma + \delta x)\cos ax + c$$

[Derivácia rovnosti $\int x \sin ax\, dx = (\alpha+\beta x)\sin ax + (\gamma+\delta x)\cos ax + c$]

$$x \sin ax = \left[\beta \sin ax + (\alpha+\beta x) a \cos ax\right] + \left[\delta \cos ax - (\gamma+\delta x) a \sin ax\right].$$

$$\left[0 + 1\cdot x\right]\sin ax + \left[0 + 0\cdot x\right]\cos ax = \left[\beta - \gamma a - \delta a x\right]\sin ax + \left[\alpha a + \delta + \beta a x\right]\cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

$$= \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c, \ x \in R, \ c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \sin ax = \left[\beta \sin ax + (\alpha + \beta x) a \cos ax\right] + \left[\delta \cos ax - (\gamma + \delta x) a \sin ax\right].$$

$$\left[0 + 1 \cdot x\right] \sin ax + \left[0 + 0 \cdot x\right] \cos ax = \left[\beta - \gamma a - \delta a x\right] \sin ax + \left[\alpha a + \delta + \beta a x\right] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a, \ 1 = -\delta a$ pre $\sin ax$,

Riešené príklady – 061

$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{matrix} u = x \\ v' = \sin ax \end{matrix} \middle| \begin{matrix} u' = 1 \\ v = -\frac{1}{a} \cos ax \end{matrix} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

$$= \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c, \ x \in R, \ c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \sin ax = \big[\beta \sin ax + (\alpha + \beta x) a \cos ax\big] + \big[\delta \cos ax - (\gamma + \delta x) a \sin ax\big].$$

$$\big[0 + 1 \cdot x\big] \sin ax + \big[0 + 0 \cdot x\big] \cos ax = \big[\beta - \gamma a - \delta ax\big] \sin ax + \big[\alpha a + \delta + \beta ax\big] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a, \ 1 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a, \ 0 = \beta a$ pre $\cos ax$.

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{matrix} u = x \\ v' = \sin ax \end{matrix} \middle| \begin{matrix} u' = 1 \\ v = -\frac{1}{a}\cos ax \end{matrix} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

$$= \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c,\ x \in R,\ c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha+\beta x)\sin ax + (\gamma+\delta x)\cos ax + c$]

$$x \sin ax = \big[\beta \sin ax + (\alpha + \beta x) a \cos ax\big] + \big[\delta \cos ax - (\gamma + \delta x) a \sin ax\big].$$

$$\big[0 + 1 \cdot x\big] \sin ax + \big[0 + 0 \cdot x\big] \cos ax = \big[\beta - \gamma a - \delta a x\big] \sin ax + \big[\alpha a + \delta + \beta a x\big] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a,\ 1 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a,\ 0 = \beta a$ pre $\cos ax$.

$$\Rightarrow \quad \beta = \frac{0}{a},$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

$$= \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c,\ x \in R,\ c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

[Derivácia rovnosti $\int x \sin ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \sin ax = \big[\beta \sin ax + (\alpha + \beta x) a \cos ax\big] + \big[\delta \cos ax - (\gamma + \delta x) a \sin ax\big].$$

$$\big[0 + 1 \cdot x\big] \sin ax + \big[0 + 0 \cdot x\big] \cos ax = \big[\beta - \gamma a - \delta a x\big] \sin ax + \big[\alpha a + \delta + \beta a x\big] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a$, $1 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a$, $0 = \beta a$ pre $\cos ax$.

$$\Rightarrow \quad \beta = \frac{0}{a},\ \gamma = \frac{\beta}{a},$$

Riešené príklady – 061

$$S_1 = \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{1}{a} \cos ax \end{array} \right] = -\frac{x \cos ax}{a} + \int \frac{\cos ax}{a} \, dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a} + c$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a$, $1 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a$, $0 = \beta a$ pre $\cos ax$.

$$\Rightarrow \quad \beta = \frac{0}{a},\ \gamma = \frac{\beta}{a},\ \delta = \frac{1}{-a},$$

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Riešené príklady – 061

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Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$

$0 = \beta - \gamma a,\ 1 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a,\ 0 = \beta a$ pre $\cos ax$.

$$\Rightarrow\ \beta = \frac{0}{a},\ \gamma = \frac{\beta}{a},\ \delta = \frac{1}{-a},\ \alpha = \frac{\delta}{-a} \quad \Rightarrow\ \beta = 0,\ \gamma = 0,$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$$0 = \beta - \gamma a,\ 1 = -\delta a \text{ pre } \sin ax,\ 0 = \delta + \alpha a,\ 0 = \beta a \text{ pre } \cos ax.$$

$$\Rightarrow\ \beta = \frac{0}{a},\ \gamma = \frac{\beta}{a},\ \delta = \frac{1}{-a},\ \alpha = \frac{\delta}{-a} \quad \Rightarrow\ \beta = 0,\ \gamma = 0,\ \delta = -\frac{1}{a},\ \alpha = \frac{1}{a^2}.$$

Riešené príklady – 061

$$S_1=\int x\sin ax\,\mathrm{d}x=\frac{\sin ax}{a^2}-\frac{x\cos ax}{a}+c \qquad a\in R,\ a\neq 0$$

$$=\left[\begin{array}{l|l} u=x & u'=\ \ 1 \\ v'=\sin ax & v=-\frac{1}{a}\cos ax\end{array}\right]=-\frac{x\cos ax}{a}+\int\frac{\cos ax}{a}\,\mathrm{d}x=-\frac{x\cos ax}{a}+\frac{\sin ax}{a\cdot a}+c$$

$$=\frac{\sin ax}{a^2}-\frac{x\cos ax}{a}+c,\ x\in R,\ c\in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_1=(\alpha+\beta x)\sin ax+(\gamma+\delta x)\cos ax+c=\frac{\sin ax}{a^2}-\frac{x\cos ax}{a}+c,\ x\in R,\ c\in R.$$

[Derivácia rovnosti $\int x\sin ax\,\mathrm{d}x=(\alpha+\beta x)\sin ax+(\gamma+\delta x)\cos ax+c$]

$$x\sin ax=\left[\beta\sin ax+(\alpha+\beta x)a\cos ax\right]+\left[\delta\cos ax-(\gamma+\delta x)a\sin ax\right].$$

$$\left[0+1\cdot x\right]\sin ax+\left[0+0\cdot x\right]\cos ax=\left[\beta-\gamma a-\delta ax\right]\sin ax+\left[\alpha a+\delta+\beta ax\right]\cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha,\beta,\gamma,\delta$]

$$0=\beta-\gamma a,\ 1=-\delta a \ \text{ pre } \sin ax,\ 0=\delta+\alpha a,\ 0=\beta a \ \text{ pre } \cos ax.$$

$$\Rightarrow\ \beta=\frac{0}{a},\ \gamma=\frac{\beta}{a},\ \delta=\frac{1}{-a},\ \alpha=\frac{\delta}{-a}\quad\Rightarrow\ \beta=0,\ \gamma=0,\ \delta=-\frac{1}{a},\ \alpha=\frac{1}{a^2}.$$

Riešené príklady – 062

$C_1 = \int x \cos ax \, \mathrm{d}x$ $\qquad a \in R,\ a \neq 0$

Riešené príklady – 062

$C_1 = \int x \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{1}{a} \sin ax \end{array} \right]$$

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$$C_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$$

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$C_1 = \int x \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

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Riešené príklady – 062

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$$= \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + c, \ x \in R, \ c \in R.$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

Riešené príklady – 062

$$C_1 = \int x\cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} + c \qquad a\in R,\ a\neq 0$$

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$0=\beta-\gamma a,\ 0=-\delta a$ pre $\sin ax$,

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$0 = \beta - \gamma a$, $0 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a$, $1 = \beta a$ pre $\cos ax$.

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$$C_1=\int x\cos ax\,dx=\frac{\cos ax}{a^2}+\frac{x\sin ax}{a}+c \qquad a\in R,\ a\neq 0$$

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$$x \cos ax = \big[\beta \sin ax + (\alpha + \beta x) a \cos ax\big] + \big[\delta \cos ax - (\gamma + \delta x) a \sin ax\big].$$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a,\ 0 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a,\ 1 = \beta a$ pre $\cos ax$.

$$\Rightarrow\quad \beta = \frac{1}{a},\ \gamma = \frac{\beta}{a},\ \delta = \frac{0}{-a},\ \alpha = \frac{-\delta}{a}$$

Riešené príklady – 062

$C_1 = \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + c$ $\quad a \in R,\ a \neq 0$

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a, \ 0 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a, \ 1 = \beta a$ pre $\cos ax$.

$$\Rightarrow \ \beta = \frac{1}{a}, \ \gamma = \frac{\beta}{a}, \ \delta = \frac{0}{-a}, \ \alpha = \frac{-\delta}{a} \quad \Rightarrow \ \beta = \frac{1}{a}, \ \gamma = \frac{1}{a^2},$$

Riešené príklady – 062

$C_1 = \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + c$ $\quad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{1}{a}\sin ax \end{array}\right] = \frac{x \sin ax}{a} - \int \frac{\sin ax}{a} \, dx = \frac{x \sin ax}{a} - \frac{-\cos ax}{a \cdot a} + c$$

$$= \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + c,\ x \in R,\ c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_1 = (\alpha + \beta x)\sin ax + (\gamma + \delta x)\cos ax + c$$

[Derivácia rovnosti $\int x \cos ax \, dx = (\alpha+\beta x)\sin ax + (\gamma+\delta x)\cos ax + c$]

$$x \cos ax = \left[\beta \sin ax + (\alpha + \beta x) a \cos ax\right] + \left[\delta \cos ax - (\gamma + \delta x) a \sin ax\right].$$

$$\left[0 + 0 \cdot x\right] \sin ax + \left[0 + 1 \cdot x\right] \cos ax = \left[\beta - \gamma a - \delta a x\right] \sin ax + \left[\alpha a + \delta + \beta a x\right] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a,\ 0 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a,\ 1 = \beta a$ pre $\cos ax$.

$$\Rightarrow \quad \beta = \frac{1}{a},\ \gamma = \frac{\beta}{a},\ \delta = \frac{0}{-a},\ \alpha = \frac{-\delta}{a} \quad \Rightarrow \quad \beta = \frac{1}{a},\ \gamma = \frac{1}{a^2},\ \delta = 0,\ \alpha = 0.$$

Riešené príklady – 062

$$C_1 = \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{1}{a} \sin ax \end{array}\right] = \frac{x \sin ax}{a} - \int \frac{\sin ax}{a} \, dx = \frac{x \sin ax}{a} - \frac{-\cos ax}{a \cdot a} + c$$

$$= \frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + c,\ x \in R,\ c \in R.$$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_1 = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c = \frac{x \sin ax}{a} + \frac{\cos ax}{a^2} + c,\ x \in R,\ c \in R.$$

[Derivácia rovnosti $\int x \cos ax \, dx = (\alpha + \beta x) \sin ax + (\gamma + \delta x) \cos ax + c$]

$$x \cos ax = \left[\beta \sin ax + (\alpha + \beta x) a \cos ax\right] + \left[\delta \cos ax - (\gamma + \delta x) a \sin ax\right].$$

$$\left[0 + 0 \cdot x\right] \sin ax + \left[0 + 1 \cdot x\right] \cos ax = \left[\beta - \gamma a - \delta a x\right] \sin ax + \left[\alpha a + \delta + \beta a x\right] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \beta - \gamma a$, $0 = -\delta a$ pre $\sin ax$, $0 = \delta + \alpha a$, $1 = \beta a$ pre $\cos ax$.

$$\Rightarrow\ \beta = \frac{1}{a},\ \gamma = \frac{\beta}{a},\ \delta = \frac{0}{-a},\ \alpha = \frac{-\delta}{a} \quad \Rightarrow\ \beta = \frac{1}{a},\ \gamma = \frac{1}{a^2},\ \delta = 0,\ \alpha = 0.$$

Riešené príklady – 063, 064

$\int x^2 \sinh ax \, dx$ $\qquad a \in R,\ a \neq 0$

$\int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

Riešené príklady – 063, 064

$\int x^2 \sinh ax \, dx$ $a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right]$$

$\int x^2 \sin ax \, dx$ $a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

Riešené príklady – 063, 064

$\int x^2 \sinh ax \, dx$ $a \in R, \ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx$$

$\int x^2 \sin ax \, dx$ $a \in R, \ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx$$

Riešené príklady – 063, 064

$\int x^2 \sinh ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right]$$

$\int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

Riešené príklady – 063, 064

$\int x^2 \sinh ax \, dx$ $\quad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right]$$

$$= \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{1}{a} \int \sinh ax \, dx \right]$$

$\int x^2 \sin ax \, dx$ $\quad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

$$= -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{1}{a} \int \sin ax \, dx \right]$$

Riešené príklady – 063, 064

$\int x^2 \sinh ax \, dx$ $\quad a \in R, \ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \int x \cosh ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right]$$

$$= \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{1}{a} \int \sinh ax \, dx \right] = \frac{x^2 \cosh ax}{a} - \frac{2}{a} \left[\frac{x \sinh ax}{a} - \frac{\cosh ax}{a \cdot a} \right]$$

$\int x^2 \sin ax \, dx$ $\quad a \in R, \ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \int x \cos ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

$$= -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{1}{a} \int \sin ax \, dx \right] = -\frac{x^2 \cos ax}{a} + \frac{2}{a} \left[\frac{x \sin ax}{a} - \frac{-\cos ax}{a \cdot a} \right]$$

Riešené príklady – 063, 064

$$\int x^2 \sinh ax \, dx = \frac{2\cosh ax}{a^3} - \frac{2x\sinh ax}{a^2} + \frac{x^2\cosh ax}{a} + c \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u=x^2 & u'=2x \\ v'=\sinh ax & v=\frac{\cosh ax}{a}\end{array}\right] = \frac{x^2\cosh ax}{a} - \frac{2}{a}\int x\cosh ax\,dx = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\cosh ax & v=\frac{\sinh ax}{a}\end{array}\right]$$

$$= \frac{x^2\cosh ax}{a} - \frac{2}{a}\left[\frac{x\sinh ax}{a} - \frac{1}{a}\int \sinh ax\,dx\right] = \frac{x^2\cosh ax}{a} - \frac{2}{a}\left[\frac{x\sinh ax}{a} - \frac{\cosh ax}{a\cdot a}\right]$$

$$= \frac{2\cosh ax}{a^3} - \frac{2x\sinh ax}{a^2} + \frac{x^2\cosh ax}{a} + c,\ x\in R,\ c\in R.$$

$$\int x^2 \sin ax \, dx = \frac{2\cos ax}{a^3} + \frac{2x\sin ax}{a^2} - \frac{x^2\cos ax}{a} + c \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u=x^2 & u'=2x \\ v'=\sin ax & v=-\frac{\cos ax}{a}\end{array}\right] = -\frac{x^2\cos ax}{a} + \frac{2}{a}\int x\cos ax\,dx = \left[\begin{array}{l|l} u=x & u'=1 \\ v'=\cos ax & v=\frac{\sin ax}{a}\end{array}\right]$$

$$= -\frac{x^2\cos ax}{a} + \frac{2}{a}\left[\frac{x\sin ax}{a} - \frac{1}{a}\int \sin ax\,dx\right] = -\frac{x^2\cos ax}{a} + \frac{2}{a}\left[\frac{x\sin ax}{a} - \frac{-\cos ax}{a\cdot a}\right]$$

$$= \frac{2\cos ax}{a^3} + \frac{2x\sin ax}{a^2} - \frac{x^2\cos ax}{a} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R, \; a \neq 0$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\quad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 1 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 0 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2\big] \sin ax + \big[\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2\big] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 1 = -\varphi a$ pre $\sin ax$,

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 0 = \gamma a$ pre $\cos ax$.

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 0 = \gamma a$ pre $\cos ax$.

$\Rightarrow \quad \gamma = \frac{0}{a},$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 1 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 0 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\big] \sin ax + \big[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\big] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $0 = \gamma a$ pre $\cos ax$.

$\Rightarrow \quad \gamma = \frac{0}{a}$, $\psi = \frac{2\gamma}{a}$,

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\quad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \sin ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0\cdot x + 1\cdot x^2\big] \sin ax + \big[0 + 0\cdot x + 0\cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\big] \sin ax + \big[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\big] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 0 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \quad \gamma = \frac{0}{a},\ \psi = \frac{2\gamma}{a},\ \alpha = \frac{\psi}{-a},$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $0 = \gamma a$ pre $\cos ax$.

$\Rightarrow \quad \gamma = \frac{0}{a}$, $\psi = \frac{2\gamma}{a}$, $\alpha = \frac{\psi}{-a}$, $\varphi = \frac{1}{-a}$,

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\quad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \sin ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 1 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 0 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\big] \sin ax + \big[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\big] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 0 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \quad \gamma = \frac{0}{a},\ \psi = \frac{2\gamma}{a},\ \alpha = \frac{\psi}{-a},\ \varphi = \frac{1}{-a},\ \beta = \frac{2\varphi}{-a},$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R, \ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2\right] \sin ax + \left[\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2\right] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $0 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \quad \gamma = \frac{0}{a}, \ \psi = \frac{2\gamma}{a}, \ \alpha = \frac{\psi}{-a}, \ \varphi = \frac{1}{-a}, \ \beta = \frac{2\varphi}{-a}, \ \delta = \frac{\beta}{a}$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $0 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \gamma = \frac{0}{a},\ \psi = \frac{2\gamma}{a},\ \alpha = \frac{\psi}{-a},\ \varphi = \frac{1}{-a},\ \beta = \frac{2\varphi}{-a},\ \delta = \frac{\beta}{a}$$

$$\Rightarrow \gamma = 0,\ \psi = 0,\ \alpha = 0,$$

Riešené príklady – 064

$S_2 = \int x^2 \sin ax \, dx$ $\qquad a \in R, \; a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \sin ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \sin ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 0 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $1 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $0 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \; \gamma = \frac{0}{a}, \; \psi = \frac{2\gamma}{a}, \; \alpha = \frac{\psi}{-a}, \; \varphi = \frac{1}{-a}, \; \beta = \frac{2\varphi}{-a}, \; \delta = \frac{\beta}{a}$$

$$\Rightarrow \; \gamma = 0, \; \psi = 0, \; \alpha = 0, \; \varphi = -\frac{1}{a}, \; \beta = \frac{2}{a^2}, \; \delta = \frac{2}{a^3}.$$

Riešené príklady – 064

$$S_2 = \int x^2 \sin ax\, dx = \frac{2\cos ax}{a^3} + \frac{2x\sin ax}{a^2} - \frac{x^2\cos ax}{a} + c \qquad a\in R,\ a\neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$S_2 = (\alpha+\beta x+\gamma x^2)\sin ax + (\delta+\psi x+\varphi x^2)\cos ax + c$$
$$= \frac{2x\sin ax}{a^2} + \frac{2\cos ax}{a^3} - \frac{x^2\cos ax}{a} + c,\ x\in R,\ c\in R.$$

[Derivácia rovnosti $\int x^2 \sin ax\, dx = (\alpha+\beta x+\gamma x^2)\sin ax + (\delta+\psi x+\varphi x^2)\cos ax + c$]

$$x^2\sin ax = \big[(\beta+2\gamma x)\sin ax + (\alpha+\beta x+\gamma x^2)a\cos ax\big] + \big[(\psi+2\varphi x)\cos ax - (\delta+\psi x+\varphi x^2)a\sin ax\big].$$

$$\big[0+0\cdot x+1\cdot x^2\big]\sin ax + \big[0+0\cdot x+0\cdot x^2\big]\cos ax$$
$$= \big[\beta-\delta a+2\gamma x-\psi ax-\varphi ax^2\big]\sin ax + \big[\alpha a+\psi+\beta ax+2\varphi x+\gamma ax^2\big]\cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha,\beta,\gamma,\delta,\psi,\varphi$]

$0=\beta-\delta a,\ 0=2\gamma-\psi a,\ 1=-\varphi a$ pre $\sin ax$, $0=\alpha a+\psi,\ 0=\beta a+2\varphi,\ 0=\gamma a$ pre $\cos ax$.

$$\Rightarrow\ \gamma=\frac{0}{a},\ \psi=\frac{2\gamma}{a},\ \alpha=\frac{\psi}{-a},\ \varphi=\frac{1}{-a},\ \beta=\frac{2\varphi}{-a},\ \delta=\frac{\beta}{a}$$
$$\Rightarrow\ \gamma=0,\ \psi=0,\ \alpha=0,\ \varphi=-\frac{1}{a},\ \beta=\frac{2}{a^2},\ \delta=\frac{2}{a^3}.$$

Riešené príklady – 065, 066

$\int x^2 \cosh ax \, dx$ $\quad a \in R, \ a \neq 0$

$\int x^2 \cos ax \, dx$ $\quad a \in R, \ a \neq 0$

Riešené príklady – 065, 066

$\int x^2 \cosh ax \, dx$ $a \in R, \ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right]$$

$\int x^2 \cos ax \, dx$ $a \in R, \ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

Riešené príklady – 065, 066

$\int x^2 \cosh ax \, dx$ $\quad a \in R, \; a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \int x \sinh ax \, dx$$

$\int x^2 \cos ax \, dx$ $\quad a \in R, \; a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax \, dx \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \int x \sinh ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right]$$

$$\int x^2 \cos ax \, dx \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

Riešené príklady – 065, 066

$\int x^2 \cosh ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array} \right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a} \int x \sinh ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array} \right]$$

$$= \frac{x^2 \sinh ax}{a} - \frac{2}{a} \left[\frac{x \cosh ax}{a} - \frac{1}{a} \int \cosh ax \, dx \right]$$

$\int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

$$= \frac{x^2 \sin ax}{a} - \frac{2}{a} \left[-\frac{x \cos ax}{a} + \frac{1}{a} \int \cos ax \, dx \right]$$

Riešené príklady – 065, 066

$\int x^2 \cosh ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array}\right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a}\int x \sinh ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array}\right]$$

$$= \frac{x^2 \sinh ax}{a} - \frac{2}{a}\left[\frac{x \cosh ax}{a} - \frac{1}{a}\int \cosh ax \, dx\right] = \frac{x^2 \sinh ax}{a} - \frac{2}{a}\left[\frac{x \cosh ax}{a} - \frac{\sinh ax}{a \cdot a}\right]$$

$\int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array}\right] = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax \, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array}\right]$$

$$= \frac{x^2 \sin ax}{a} - \frac{2}{a}\left[-\frac{x \cos ax}{a} + \frac{1}{a}\int \cos ax \, dx\right] = \frac{x^2 \sin ax}{a} - \frac{2}{a}\left[-\frac{x \cos ax}{a} + \frac{\sin ax}{a \cdot a}\right]$$

Riešené príklady – 065, 066

$$\int x^2 \cosh ax\, dx = \frac{2\sinh ax}{a^3} - \frac{2x\cosh ax}{a^2} + \frac{x^2\sinh ax}{a} + c \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cosh ax & v = \frac{\sinh ax}{a} \end{array}\right] = \frac{x^2\sinh ax}{a} - \frac{2}{a}\int x\sinh ax\, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sinh ax & v = \frac{\cosh ax}{a} \end{array}\right]$$

$$= \frac{x^2\sinh ax}{a} - \frac{2}{a}\left[\frac{x\cosh ax}{a} - \frac{1}{a}\int \cosh ax\, dx\right] = \frac{x^2\sinh ax}{a} - \frac{2}{a}\left[\frac{x\cosh ax}{a} - \frac{\sinh ax}{a\cdot a}\right]$$

$$= \frac{2\sinh ax}{a^3} - \frac{2x\cosh ax}{a^2} + \frac{x^2\sinh ax}{a} + c,\ x\in R,\ c\in R.$$

$$\int x^2 \cos ax\, dx = -\frac{2\sin ax}{a^3} + \frac{2x\cos ax}{a^2} + \frac{x^2\sin ax}{a} + c \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array}\right] = \frac{x^2\sin ax}{a} - \frac{2}{a}\int x\sin ax\, dx = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array}\right]$$

$$= \frac{x^2\sin ax}{a} - \frac{2}{a}\left[-\frac{x\cos ax}{a} + \frac{1}{a}\int \cos ax\, dx\right] = \frac{x^2\sin ax}{a} - \frac{2}{a}\left[-\frac{x\cos ax}{a} + \frac{\sin ax}{a\cdot a}\right]$$

$$= -\frac{2\sin ax}{a^3} + \frac{2x\cos ax}{a^2} + \frac{x^2\sin ax}{a} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R, \; a \neq 0$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx \qquad a \in R, \ a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha+\beta x+\gamma x^2) \sin ax + (\delta+\psi x+\varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 0 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 1 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2\right] \sin ax + \left[\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2\right] \cos ax.$$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 0 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 1 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2\big] \sin ax + \big[\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2\big] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $a \in R, \ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 0 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 1 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2\right] \sin ax + \left[\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2\right] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a, \ 0 = 2\gamma - \psi a, \ 0 = -\varphi a$ pre $\sin ax$,

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 0 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 1 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 1 = \gamma a$ pre $\cos ax$.

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 0 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 1 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\big] \sin ax + \big[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\big] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 1 = \gamma a$ pre $\cos ax$.

$\Rightarrow \quad \varphi = \frac{0}{-a},$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 0 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 1 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\big] \sin ax + \big[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\big] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $1 = \gamma a$ pre $\cos ax$.

$\Rightarrow \quad \varphi = \frac{0}{-a}$, $\beta = \frac{2\varphi}{-a}$,

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\quad a \in R,\ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \cos ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 0 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 1 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 1 = \gamma a$ pre $\cos ax$.

$\Rightarrow \quad \varphi = \frac{0}{-a},\ \beta = \frac{2\varphi}{-a},\ \delta = \frac{\beta}{a},$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 0 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 1 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\big] \sin ax + \big[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\big] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 1 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \quad \varphi = \frac{0}{-a},\ \beta = \frac{2\varphi}{-a},\ \delta = \frac{\beta}{a},\ \gamma = \frac{1}{a},$$

Riešené príklady – 066

$C_2=\int x^2 \cos ax \,dx$ $\qquad a\in R,\ a\neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$C_2=(\alpha+\beta x+\gamma x^2)\sin ax+(\delta+\psi x+\varphi x^2)\cos ax+c$$

Derivácia rovnosti $\int x^2\cos ax\,dx=(\alpha+\beta x+\gamma x^2)\sin ax+(\delta+\psi x+\varphi x^2)\cos ax+c$

$$x^2\cos ax=\big[(\beta+2\gamma x)\sin ax+(\alpha+\beta x+\gamma x^2)a\cos ax\big] + \big[(\psi+2\varphi x)\cos ax-(\delta+\psi x+\varphi x^2)a\sin ax\big].$$

$$\big[0+0\cdot x+0\cdot x^2\big]\sin ax+\big[0+0\cdot x+1\cdot x^2\big]\cos ax =\big[\beta-\delta a+2\gamma x-\psi ax-\varphi ax^2\big]\sin ax+\big[\alpha a+\psi+\beta ax+2\varphi x+\gamma ax^2\big]\cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha,\beta,\gamma,\delta,\psi,\varphi$

$0=\beta-\delta a,\ 0=2\gamma-\psi a,\ 0=-\varphi a$ pre $\sin ax$, $0=\alpha a+\psi,\ 0=\beta a+2\varphi,\ 1=\gamma a$ pre $\cos ax$.

$$\Rightarrow\quad \varphi=\frac{0}{-a},\ \beta=\frac{2\varphi}{-a},\ \delta=\frac{\beta}{a},\ \gamma=\frac{1}{a},\ \psi=\frac{2\gamma}{a},$$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \big[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\big] + \big[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\big].$$

$$\big[0 + 0 \cdot x + 0 \cdot x^2\big] \sin ax + \big[0 + 0 \cdot x + 1 \cdot x^2\big] \cos ax = \big[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\big] \sin ax + \big[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\big] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $1 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \quad \varphi = \frac{0}{-a},\ \beta = \frac{2\varphi}{-a},\ \delta = \frac{\beta}{a},\ \gamma = \frac{1}{a},\ \psi = \frac{2\gamma}{a},\ \alpha = \frac{\psi}{-a}$$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\quad a \in R, \ a \neq 0$

Odhad riešenia metódou neurčitých koeficientov

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$

$$x^2 \cos ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 0 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 1 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi ax - \varphi ax^2\right] \sin ax + \left[\alpha a + \psi + \beta ax + 2\varphi x + \gamma ax^2\right] \cos ax.$$

Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$

$0 = \beta - \delta a$, $0 = 2\gamma - \psi a$, $0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi$, $0 = \beta a + 2\varphi$, $1 = \gamma a$ pre $\cos ax$.

$$\Rightarrow \ \varphi = \frac{0}{-a}, \ \beta = \frac{2\varphi}{-a}, \ \delta = \frac{\beta}{a}, \ \gamma = \frac{1}{a}, \ \psi = \frac{2\gamma}{a}, \ \alpha = \frac{\psi}{-a}$$

$$\Rightarrow \ \varphi = 0, \ \beta = 0, \ \delta = 0,$$

Riešené príklady – 066

$C_2 = \int x^2 \cos ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$$

[Derivácia rovnosti $\int x^2 \cos ax \, dx = (\alpha + \beta x + \gamma x^2) \sin ax + (\delta + \psi x + \varphi x^2) \cos ax + c$]

$$x^2 \cos ax = \left[(\beta + 2\gamma x) \sin ax + (\alpha + \beta x + \gamma x^2) a \cos ax\right] + \left[(\psi + 2\varphi x) \cos ax - (\delta + \psi x + \varphi x^2) a \sin ax\right].$$

$$\left[0 + 0 \cdot x + 0 \cdot x^2\right] \sin ax + \left[0 + 0 \cdot x + 1 \cdot x^2\right] \cos ax = \left[\beta - \delta a + 2\gamma x - \psi a x - \varphi a x^2\right] \sin ax + \left[\alpha a + \psi + \beta a x + 2\varphi x + \gamma a x^2\right] \cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha, \beta, \gamma, \delta, \psi, \varphi$]

$0 = \beta - \delta a,\ 0 = 2\gamma - \psi a,\ 0 = -\varphi a$ pre $\sin ax$, $0 = \alpha a + \psi,\ 0 = \beta a + 2\varphi,\ 1 = \gamma a$ pre $\cos ax$.

$$\Rightarrow\ \varphi = \frac{0}{-a},\ \beta = \frac{2\varphi}{-a},\ \delta = \frac{\beta}{a},\ \gamma = \frac{1}{a},\ \psi = \frac{2\gamma}{a},\ \alpha = \frac{\psi}{-a}$$

$$\Rightarrow\ \varphi = 0,\ \beta = 0,\ \delta = 0,\ \gamma = \frac{1}{a},\ \psi = \frac{2}{a^2},\ \alpha = -\frac{2}{a^3}.$$

Riešené príklady – 066

$$C_2 = \int x^2 \cos ax \, dx = \frac{2x\cos ax}{a^2} - \frac{2\sin ax}{a^3} + \frac{x^2 \sin ax}{a} + c \qquad a\in R,\ a\neq 0$$

[Odhad riešenia metódou neurčitých koeficientov]

$$C_2 = (\alpha+\beta x+\gamma x^2)\sin ax + (\delta+\psi x+\varphi x^2)\cos ax + c$$
$$= \frac{2x\cos ax}{a^2} - \frac{2\sin ax}{a^3} + \frac{x^2\sin ax}{a} + c,\ x\in R,\ c\in R.$$

[Derivácia rovnosti $\int x^2\cos ax\,dx = (\alpha+\beta x+\gamma x^2)\sin ax + (\delta+\psi x+\varphi x^2)\cos ax + c$]

$$x^2\cos ax = \big[(\beta+2\gamma x)\sin ax + (\alpha+\beta x+\gamma x^2)a\cos ax\big] + \big[(\psi+2\varphi x)\cos ax - (\delta+\psi x+\varphi x^2)a\sin ax\big].$$

$$\big[0+0\cdot x+0\cdot x^2\big]\sin ax + \big[0+0\cdot x+1\cdot x^2\big]\cos ax = \big[\beta-\delta a+2\gamma x-\psi ax-\varphi ax^2\big]\sin ax + \big[\alpha a+\psi+\beta ax+2\varphi x+\gamma ax^2\big]\cos ax.$$

[Riešenie 6 rovníc so šiestimi neznámymi $\alpha,\beta,\gamma,\delta,\psi,\varphi$]

$$0=\beta-\delta a,\ 0=2\gamma-\psi a,\ 0=-\varphi a \text{ pre } \sin ax,\ 0=\alpha a+\psi,\ 0=\beta a+2\varphi,\ 1=\gamma a \text{ pre } \cos ax.$$
$$\Rightarrow\ \varphi=\frac{0}{-a},\ \beta=\frac{2\varphi}{-a},\ \delta=\frac{\beta}{a},\ \gamma=\frac{1}{a},\ \psi=\frac{2\gamma}{a},\ \alpha=\frac{\psi}{-a}$$
$$\Rightarrow\ \varphi=0,\ \beta=0,\ \delta=0,\ \gamma=\frac{1}{a},\ \psi=\frac{2}{a^2},\ \alpha=-\frac{2}{a^3}.$$

Riešené príklady – 067, 068

$S_n = \int x^n \sin ax \, dx$ $\quad n \in N, \ a \in R, \ a \neq 0$

$C_n = \int x^n \cos ax \, dx$ $\quad n \in N, \ a \in R, \ a \neq 0$

Riešené príklady – 067, 068

$$S_n = \int x^n \sin ax \, dx \qquad n \in N,\ a \in R,\ a \neq 0$$

$$S_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

$$S_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right]$$

$$C_n = \int x^n \cos ax \, dx \qquad n \in N,\ a \in R,\ a \neq 0$$

$$C_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

$$C_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right]$$

Riešené príklady – 067, 068

$$S_n = \int x^n \sin ax \, dx \qquad n \in N,\ a \in R,\ a \neq 0$$

$$S_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = \frac{-x^n \cos ax}{a} + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

$$S_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = \frac{-x \cos ax}{a} + \int \frac{\cos ax \, dx}{a}$$

$$C_n = \int x^n \cos ax \, dx \qquad n \in N,\ a \in R,\ a \neq 0$$

$$C_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^n \sin ax}{a} - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

$$C_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x \sin ax}{a} - \int \frac{\sin ax \, dx}{a}$$

Riešené príklady – 067, 068

$$S_n = \int x^n \sin ax \, dx = -\frac{x^n \cos ax}{a} + \frac{n}{a} C_{n-1} \qquad n \in N,\ a \in R,\ a \neq 0$$

$$S_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = \frac{-x^n \cos ax}{a} + \frac{n}{a} \int x^{n-1} \cos ax \, dx = -\frac{x^n \cos ax}{a} + \frac{n}{a} C_{n-1},$$

$$S_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \sin ax & v = -\frac{\cos ax}{a} \end{array} \right] = \frac{-x \cos ax}{a} + \int \frac{\cos ax \, dx}{a} = -\frac{x \cos ax}{a} + \frac{C_0}{a} = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} + c,$$

$$x \in R,\ c \in R.$$

$$C_n = \int x^n \cos ax \, dx = \frac{x^n \sin ax}{a} - \frac{n}{a} S_{n-1} \qquad n \in N,\ a \in R,\ a \neq 0$$

$$C_n = \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x^n \sin ax}{a} - \frac{n}{a} \int x^{n-1} \sin ax \, dx = \frac{x^n \sin ax}{a} - \frac{n}{a} S_{n-1},$$

$$C_1 = \left[\begin{array}{l|l} u = x & u' = 1 \\ v' = \cos ax & v = \frac{\sin ax}{a} \end{array} \right] = \frac{x \sin ax}{a} - \int \frac{\sin ax \, dx}{a} = \frac{x \sin ax}{a} - \frac{S_0}{a} = \frac{x \sin ax}{a} + \frac{\cos ax}{a^2} + c,$$

$$x \in R,\ c \in R.$$

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $a \in R, \ a \neq 0$

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $\quad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a}\int x^2 \cos ax \, dx$ | $\int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax \, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $\qquad a \in R, \; a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a} \int x^2 \cos ax \, dx$ | $\int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a} \int x \sin ax \, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax\right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax\right].$$

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax\, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a}\int x^2 \cos ax\, dx$ | $\int x^2 \cos ax\, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax\, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax\, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax\right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax\right].$$

$$\left[0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3\right] \sin ax + \left[0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3\right] \cos ax = \left[(2\beta - \gamma a)x - \delta a x^3\right] \sin ax + \left[(\alpha a + \gamma) + (\beta a + 3\delta)x^2\right] \cos ax.$$

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $\qquad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax\, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a}\int x^2 \cos ax \, dx$ | $\int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax\, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax\, dx = (\alpha+\beta x^2)\sin ax + (\gamma x+\delta x^3)\cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha+\beta x^2) a \cos ax\right] + \left[(\gamma + 3\delta x^2)\cos ax - (\gamma x + \delta x^3) a \sin ax\right].$$

$$\left[0+0\cdot x+0\cdot x^2+1\cdot x^3\right]\sin ax + \left[0+0\cdot x+0\cdot x^2+0\cdot x^3\right]\cos ax = \left[(2\beta-\gamma a)x - \delta a x^3\right]\sin ax + \left[(\alpha a+\gamma)+(\beta a+3\delta)x^2\right]\cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $\quad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 | Pr. 68 | $\int x^3 \sin ax\,dx = -\frac{x^3\cos ax}{a} + \frac{3}{a}\int x^2 \cos ax\,dx$ | $\int x^2 \cos ax\,dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax\,dx$]

$$S_3 = (\alpha + \beta x^2)\sin ax + (\gamma x + \delta x^3)\cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax\,dx = (\alpha+\beta x^2)\sin ax + (\gamma x + \delta x^3)\cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha+\beta x^2)a\cos ax\right] + \left[(\gamma+3\delta x^2)\cos ax - (\gamma x+\delta x^3)a \sin ax\right].$$

$$\left[0+0\cdot x+0\cdot x^2+1\cdot x^3\right]\sin ax + \left[0+0\cdot x+0\cdot x^2+0\cdot x^3\right]\cos ax = \left[(2\beta-\gamma a)x - \delta a x^3\right]\sin ax + \left[(\alpha a+\gamma)+(\beta a+3\delta)x^2\right]\cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = 2\beta - \gamma a,\ 1 = -\delta a$ pre $\sin ax$,

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax\, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a}\int x^2 \cos ax\, dx$ | $\int x^2 \cos ax\, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax\, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax\, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax\right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax\right].$$

$$\left[0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3\right] \sin ax + \left[0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3\right] \cos ax = \left[(2\beta - \gamma a)x - \delta a x^3\right] \sin ax + \left[(\alpha a + \gamma) + (\beta a + 3\delta)x^2\right] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = 2\beta - \gamma a,\ 1 = -\delta a$ pre $\sin ax$, $0 = \alpha a + \gamma,\ 0 = \beta a + 3\delta$ pre $\cos ax$.

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $\quad a \in R,\ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a}\int x^2 \cos ax \, dx$ | $\int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax \, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax \, dx = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax\right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax\right].$$

$$\left[0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3\right] \sin ax + \left[0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3\right] \cos ax = \left[(2\beta - \gamma a)x - \delta a x^3\right] \sin ax + \left[(\alpha a + \gamma) + (\beta a + 3\delta)x^2\right] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = 2\beta - \gamma a$, $1 = -\delta a$ pre $\sin ax$, $0 = \alpha a + \gamma$, $0 = \beta a + 3\delta$ pre $\cos ax$.

$\Rightarrow \quad \delta = \frac{1}{-a}$,

Riešené príklady – 069

$S_3 = \int x^3 \sin ax \, dx$ $\qquad a \in R, \ a \neq 0$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax \, dx = -\frac{x^3 \cos ax}{a} + \frac{3}{a}\int x^2 \cos ax \, dx$ | $\int x^2 \cos ax \, dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax \, dx$]

$$S_3 = (\alpha + \beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$$

[Derivácia rovnosti $\int x^3 \sin ax \, dx = (\alpha+\beta x^2) \sin ax + (\gamma x + \delta x^3) \cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha+\beta x^2) a \cos ax\right] + \left[(\gamma + 3\delta x^2) \cos ax - (\gamma x + \delta x^3) a \sin ax\right].$$

$$\left[0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3\right] \sin ax + \left[0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3\right] \cos ax = \left[(2\beta - \gamma a)x - \delta a x^3\right] \sin ax + \left[(\alpha a + \gamma) + (\beta a + 3\delta)x^2\right] \cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = 2\beta - \gamma a$, $1 = -\delta a$ pre $\sin ax$, $0 = \alpha a + \gamma$, $0 = \beta a + 3\delta$ pre $\cos ax$.

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Riešené príklady – 069

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Riešené príklady – 069

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Riešené príklady – 069

$$S_3 = \int x^3 \sin ax \, dx = -\frac{6\sin ax}{a^4} + \frac{3x^2 \sin ax}{a^2} + \frac{6x\cos ax}{a^3} - \frac{x^3 \cos ax}{a} + c \qquad a \in R,\ a \neq 0$$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3 \sin ax\,dx = -\frac{x^3\cos ax}{a} + \frac{3}{a}\int x^2 \cos ax\,dx$ | $\int x^2 \cos ax\,dx = \frac{x^2 \sin ax}{a} - \frac{2}{a}\int x \sin ax\,dx$]

$$S_3 = (\alpha + \beta x^2)\sin ax + (\gamma x + \delta x^3)\cos ax + c$$
$$= -\frac{6\sin ax}{a^4} + \frac{3x^2\sin ax}{a^2} + \frac{6x\cos ax}{a^3} - \frac{x^3\cos ax}{a} + c,\ x \in R,\ c \in R.$$

[Derivácia rovnosti $\int x^3 \sin ax\,dx = (\alpha+\beta x^2)\sin ax + (\gamma x + \delta x^3)\cos ax + c$]

$$x^3 \sin ax = \left[2\beta x \sin ax + (\alpha + \beta x^2) a \cos ax\right] + \left[(\gamma + 3\delta x^2)\cos ax - (\gamma x + \delta x^3) a \sin ax\right].$$

$$\left[0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3\right]\sin ax + \left[0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3\right]\cos ax$$
$$= \left[(2\beta - \gamma a)x - \delta a x^3\right]\sin ax + \left[(\alpha a + \gamma) + (\beta a + 3\delta)x^2\right]\cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = 2\beta - \gamma a,\ 1 = -\delta a$ pre $\sin ax$, $0 = \alpha a + \gamma,\ 0 = \beta a + 3\delta$ pre $\cos ax$.

$$\Rightarrow\ \delta = \frac{1}{-a},\ \beta = \frac{3\delta}{-a},\ \gamma = \frac{2\beta}{a},\ \alpha = \frac{\gamma}{-a} \quad \Rightarrow\ \delta = -\frac{1}{a},\ \beta = \frac{3}{a^2},\ \gamma = \frac{6}{a^3},\ \alpha = -\frac{6}{a^4}.$$

Riešené príklady – 070

$C_3 = \int x^3 \cos ax \, dx$ $a \in R,\ a \neq 0$

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$$C_3 = (\alpha x + \beta x^3) \sin ax + (\gamma + \delta x^2) \cos ax + c$$

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$$x^3 \cos ax = \left[(\alpha + 3\beta x^2) \sin ax + (\alpha x + \beta x^3) a \cos ax\right] + \left[2\delta x \cos ax - (\gamma + \delta x^2) a \sin ax\right].$$

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$$\big[0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3\big]\sin ax + \big[0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3\big]\cos ax = \big[(\alpha - \gamma a) + (3\beta - \delta a)x^2\big]\sin ax + \big[(\alpha a + 2\delta)x + \beta a x^3\big]\cos ax.$$

Riešené príklady – 070

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

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$0 = \alpha - \gamma a,\ 0 = 3\beta - \delta a$ pre $\sin ax$, $0 = \alpha a + 2\delta,\ 1 = \beta a$ pre $\cos ax$.

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[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

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$0 = \alpha - \gamma a,\ 0 = 3\beta - \delta a$ pre $\sin ax$, $0 = \alpha a + 2\delta,\ 1 = \beta a$ pre $\cos ax$.

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Riešené príklady – 070

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$0 = \alpha - \gamma a$, $0 = 3\beta - \delta a$ pre $\sin ax$, $0 = \alpha a + 2\delta$, $1 = \beta a$ pre $\cos ax$.

$$\Rightarrow \quad \beta = \frac{1}{a},\ \delta = \frac{3\beta}{a},\ \alpha = \frac{2\delta}{-a},\ \gamma = \frac{\alpha}{a}$$

Riešené príklady – 070

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$$\big[0 + 0\cdot x + 0\cdot x^2 + 0\cdot x^3\big]\sin ax + \big[0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3\big]\cos ax = \big[(\alpha - \gamma a) + (3\beta - \delta a)x^2\big]\sin ax + \big[(\alpha a + 2\delta)x + \beta a x^3\big]\cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha, \beta, \gamma, \delta$]

$0 = \alpha - \gamma a,\ 0 = 3\beta - \delta a$ pre $\sin ax$, $0 = \alpha a + 2\delta,\ 1 = \beta a$ pre $\cos ax$.

$$\Rightarrow\ \beta = \frac{1}{a},\ \delta = \frac{3\beta}{a},\ \alpha = \frac{2\delta}{-a},\ \gamma = \frac{\alpha}{a} \quad \Rightarrow\ \beta = \frac{1}{a},\ \delta = \frac{3}{a^2},\ \alpha = -\frac{6}{a^3},\ \gamma = -\frac{6}{a^4}.$$

Riešené príklady – 070

$$C_3=\int x^3\cos ax\,dx=-\frac{6x\sin ax}{a^3}+\frac{x^3\sin ax}{a}-\frac{6\cos ax}{a^4}+\frac{3x^2\cos ax}{a^2}+c \qquad a\in R,\ a\neq 0$$

[Odhad riešenia metódou neurčitých koeficientov | Pr. 67 Pr. 68 | $\int x^3\cos ax\,dx=\frac{x^3\sin ax}{a}-\frac{3}{a}\int x^2\sin ax\,dx$ | $\int x^2\sin ax\,dx=-\frac{x^2\cos ax}{a}+\frac{2}{a}\int x\cos ax\,dx$]

$$C_3=(\alpha x+\beta x^3)\sin ax+(\gamma+\delta x^2)\cos ax+c$$
$$=-\frac{6x\sin ax}{a^3}+\frac{x^3\sin ax}{a}-\frac{6\cos ax}{a^4}+\frac{3x^2\cos ax}{a^2}+c,\ x\in R,\ c\in R.$$

[Derivácia rovnosti $\int x^3\cos ax\,dx=(\alpha x+\beta x^3)\sin ax+(\gamma+\delta x^2)\cos ax+c$]

$$x^3\cos ax=\left[(\alpha+3\beta x^2)\sin ax+(\alpha x+\beta x^3)a\cos ax\right]+\left[2\delta x\cos ax-(\gamma+\delta x^2)a\sin ax\right].$$

$$\left[0+0\cdot x+0\cdot x^2+0\cdot x^3\right]\sin ax+\left[0+0\cdot x+0\cdot x^2+1\cdot x^3\right]\cos ax$$
$$=\left[(\alpha-\gamma a)+(3\beta-\delta a)x^2\right]\sin ax+\left[(\alpha a+2\delta)x+\beta a x^3\right]\cos ax.$$

[Riešenie 4 rovníc so štyrmi neznámymi $\alpha,\beta,\gamma,\delta$]

$0=\alpha-\gamma a,\ 0=3\beta-\delta a$ pre $\sin ax$, $0=\alpha a+2\delta,\ 1=\beta a$ pre $\cos ax$.

$$\Rightarrow\ \beta=\frac{1}{a},\ \delta=\frac{3\beta}{a},\ \alpha=\frac{2\delta}{-a},\ \gamma=\frac{\alpha}{a}\quad\Rightarrow\ \beta=\frac{1}{a},\ \delta=\frac{3}{a^2},\ \alpha=-\frac{6}{a^3},\ \gamma=-\frac{6}{a^4}.$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,dx$$

Riešené príklady – 071

$$\int\sqrt{1+\frac{1}{\sin x}}\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=1+\frac{1}{\sin x},\ \sin x\in(0;1\rangle & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 \\ \cos x\,dx=-(t-1)^{-2}\,dt=\frac{-dt}{(t-1)^2}\ \Rightarrow\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{1}{\cos x},\ \cos x\neq 0 & \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & \end{array}\right|\begin{array}{l} x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array}\right]$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,\mathrm{d}x$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\quad t=1+\frac{1}{\sin x},\ \sin x\in(0;1\rangle & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 & \mathrm{d}x=\frac{-\mathrm{d}t}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-\mathrm{d}t}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ \cos x\,\mathrm{d}x=-(t-1)^{-2}\,\mathrm{d}t=\frac{-\mathrm{d}t}{(t-1)^2}\ \Rightarrow\ \mathrm{d}x=\frac{-\mathrm{d}t}{(t-1)^2}\cdot\frac{1}{\cos x},\ \cos x\neq 0 & & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & & \mathrm{d}x=\frac{-\mathrm{d}t}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{\mathrm{d}t}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array}\right]$$

$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z$

$$=\int\frac{-\sqrt{t}\,\mathrm{d}t}{(t-1)\sqrt{t}\sqrt{t-2}}$$

$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z$

$$=\int\frac{\sqrt{t}\,\mathrm{d}t}{(t-1)\sqrt{t}\sqrt{t-2}}$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\quad t=1+\frac{1}{\sin x},\ \sin x\in(0;1\rangle & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 & dx=\frac{-dt}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ \cos x\,dx=-(t-1)^{-2}\,dt=\frac{-dt}{(t-1)^2}\ \Rightarrow\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{1}{\cos x},\quad \cos x\neq 0 & & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & & dx=\frac{-dt}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array}\right]$$

$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z$

$$=\int\frac{-\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{-\,dt}{(t-1)\sqrt{t-2}}$$

$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z$

$$=\int\frac{\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{dt}{(t-1)\sqrt{t-2}}$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\quad t=1+\frac{1}{\sin x},\ \sin x\in(0;1\rangle & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 & dx=\frac{-dt}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ \cos x\,dx=-(t-1)^{-2}\,dt=\frac{-dt}{(t-1)^2}\ \Rightarrow\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{1}{\cos x},\ \cos x\neq 0 & & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & & dx=\frac{-dt}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array} \right]$$

$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z$

$$=\int\frac{-\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{-\,dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty)\\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z$

$$=\int\frac{\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty)\\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } t=1+\frac{1}{\sin x},\ \sin x\in(0;1\rangle & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 & dx=\frac{-dt}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ \cos x\,dx=-(t-1)^{-2}\,dt=\frac{-dt}{(t-1)^2}\ \Rightarrow\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{1}{\cos x},\ \cos x\neq 0 & & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & & dx=\frac{-dt}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array}\right]$$

$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z$

$$=\int\frac{-\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{-\,dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst. } u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty) \\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

$$=\int\frac{-2u\,du}{(u^2+1)u}$$

$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z$

$$=\int\frac{\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst. } u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty) \\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

$$=\int\frac{2u\,du}{(u^2+1)u}$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\quad t=1+\frac{1}{\sin x},\ \sin x\in(0;1\rangle & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 & dx=\frac{-dt}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ \cos x\,dx=-(t-1)^{-2}\,dt=\frac{-dt}{(t-1)^2}\ \Rightarrow\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{1}{\cos x},\ \cos x\neq 0 & & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & & dx=\frac{-dt}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array}\right]$$

$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z$

$$=\int\frac{-\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{-dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{l|l} \text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty) \\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty) \end{array}\right]$$

$$=\int\frac{-2u\,du}{(u^2+1)u}=-2\int\frac{du}{u^2+1}$$

$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z$

$$=\int\frac{\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{l|l} \text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty) \\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty) \end{array}\right]$$

$$=\int\frac{2u\,du}{(u^2+1)u}=2\int\frac{du}{u^2+1}$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,dx$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=1+\frac{1}{\sin x},\ \sin x\in(0;1\rangle & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 \\ \cos x\,dx=-(t-1)^{-2}\,dt=\frac{-dt}{(t-1)^2}\ \Rightarrow\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{1}{\cos x},\ \cos x\neq 0 & \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & \end{array}\right|\begin{array}{l} x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array}\right]$$

$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z$

$$=\int\frac{-\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{-dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty)\\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

$$=\int\frac{-2u\,du}{(u^2+1)u}=-2\int\frac{du}{u^2+1}=-2\operatorname{arctg}u+c$$

$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z$

$$=\int\frac{\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty)\\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

$$=\int\frac{2u\,du}{(u^2+1)u}=2\int\frac{du}{u^2+1}=2\operatorname{arctg}u+c$$

Riešené príklady – 071

$$\int \sqrt{1+\frac{1}{\sin x}}\,dx = \mp 2\,\mathrm{arctg}\,\sqrt{\frac{1}{\sin x}-1}+c$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\quad t=1+\frac{1}{\sin x},\ \sin x\in(0;1) & x\in(0+2k\pi;\pi+2k\pi),\ k\in Z & x\in(0+2k\pi;\frac{\pi}{2}+2k\pi):\ 0<\cos x=\frac{\sqrt{t(t-2)}}{t-1} \\ \sin x=\frac{1}{t-1}=(t-1)^{-1},\ t\in\langle 2;\infty) & \sin x\in(-1;0)\Rightarrow 1+\frac{1}{\sin x}<0 & dx=\frac{-dt}{(t-1)^2}\cdot\frac{t-1}{\sqrt{t(t-2)}}=\frac{-dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \\ \cos x\,dx=-(t-1)^{-2}\,dt=\frac{-dt}{(t-1)^2}\ \Rightarrow\ dx=\frac{-dt}{(t-1)^2}\cdot\frac{1}{\cos x},\ \cos x\neq 0 & & x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi):\ 0>\cos x=-\frac{\sqrt{t(t-2)}}{t-1} \\ \cos^2 x=1-\sin^2 x=1-\frac{1}{(t-1)^2}=\frac{(t-1)^2-1}{(t-1)^2}=\frac{t^2-2t+1-1}{(t-1)^2}=\frac{t^2-2t}{(t-1)^2}=\frac{t(t-2)}{(t-1)^2} & & dx=\frac{-dt}{(t-1)^2}\cdot\frac{-(t-1)}{\sqrt{t(t-2)}}=\frac{dt}{(t-1)\sqrt{t}\sqrt{t-2}},\ t\in(2;\infty) \end{array}\right]$$

$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z$

$$=\int\frac{-\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{-dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty)\\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

$$=\int\frac{-2u\,du}{(u^2+1)u}=-2\int\frac{du}{u^2+1}=-2\,\mathrm{arctg}\,u+c=-2\,\mathrm{arctg}\,\sqrt{\frac{1}{\sin x}-1}+c,$$

$$x\in(0+2k\pi;\frac{\pi}{2}+2k\pi),\ k\in Z,\ c\in R.$$

$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z$

$$=\int\frac{\sqrt{t}\,dt}{(t-1)\sqrt{t}\sqrt{t-2}}=\int\frac{dt}{(t-1)\sqrt{t-2}}=\left[\begin{array}{r|l}\text{Subst.}\ u=\sqrt{t-2}=\sqrt{\frac{1}{\sin x}-1} & t\in(2;\infty)\\ u^2=t-2,\quad 2u\,du=dt & u\in(0;\infty)\end{array}\right]$$

$$=\int\frac{2u\,du}{(u^2+1)u}=2\int\frac{du}{u^2+1}=2\,\mathrm{arctg}\,u+c=+2\,\mathrm{arctg}\,\sqrt{\frac{1}{\sin x}-1}+c,$$

$$x\in(\frac{\pi}{2}+2k\pi;\pi+2k\pi),\ k\in Z,\ c\in R.$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx$$

$$\int x \operatorname{arctg} x \, dx$$

Riešené príklady – 072, 073

$\int \operatorname{arctg} x \, dx$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right]$$

$\int x \operatorname{arctg} x \, dx$

$$= \left[\begin{array}{l|l} u' = x & u = \frac{x^2}{2} \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right]$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2}$$

$$\int x \operatorname{arctg} x \, dx$$

$$= \left[\begin{array}{l|l} u' = x & u = \frac{x^2}{2} \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 \, dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(1+x^2-1) \, dx}{1+x^2}$$

Riešené príklady – 072, 073

$\int \operatorname{arctg} x \, dx$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2} = x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$\int x \operatorname{arctg} x \, dx$

$$= \left[\begin{array}{l|l} u' = x & u = \frac{x^2}{2} \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 \, dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(1+x^2-1) \, dx}{1+x^2}$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{1+x^2}$$

Riešené príklady – 072, 073

$\int \operatorname{arctg} x \, dx$

$$= \left[\begin{array}{l|l} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x \, dx}{1+x^2} = x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$= x \operatorname{arctg} x - \frac{1}{2} \ln \left| 1+x^2 \right| + c$$

$\int x \operatorname{arctg} x \, dx$

$$= \left[\begin{array}{l|l} u' = x & u = \frac{x^2}{2} \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2 \, dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{(1+x^2-1) \, dx}{1+x^2}$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{1}{2} \operatorname{arctg} x + c$$

Riešené príklady – 072, 073

$$\int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \ln \sqrt{1+x^2} + c$$

$$= \left[\begin{array}{l|l} u'=1 & u=x \\ v=\operatorname{arctg} x & v'=\frac{1}{1+x^2} \end{array} \right] = x \operatorname{arctg} x - \int \frac{x\,dx}{1+x^2} = x \operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$= x \operatorname{arctg} x - \tfrac{1}{2} \ln\left|1+x^2\right| + c = x \operatorname{arctg} x - \tfrac{1}{2} \ln\left(1+x^2\right) + c$$

$$= x \operatorname{arctg} x - \ln\sqrt{1+x^2} + c, \ x \in R, \ c \in R.$$

$$\int x \operatorname{arctg} x \, dx = \frac{1+x^2}{2} \operatorname{arctg} x - \frac{x}{2} + c$$

$$= \left[\begin{array}{l|l} u'=x & u=\frac{x^2}{2} \\ v=\operatorname{arctg} x & v'=\frac{1}{1+x^2} \end{array} \right] = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2}\int \frac{x^2\,dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2}\int \frac{(1+x^2-1)\,dx}{1+x^2}$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{1}{2} \operatorname{arctg} x + c$$

$$= \frac{1+x^2}{2} \operatorname{arctg} x - \frac{x}{2} + c, \ x \in R, \ c \in R.$$

Riešené príklady – 074, 075

$$\int \ln x \, dx$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx$$

Riešené príklady – 074, 075

$$\int \ln x \, \mathrm{d}x$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{array} \right]$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, \mathrm{d}x$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \operatorname{arctg} x & x \in (0; \infty) \\ \mathrm{d}t = \frac{\mathrm{d}x}{x^2+1} & t \in (0; \frac{\pi}{2}) \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \ln \operatorname{arctg} x & u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} & v = \operatorname{arctg} x \end{array} \right]$$

Riešené príklady – 074, 075

$$\int \ln x \,\mathrm{d}x$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{array}\right] = x \ln x - \int \frac{x\,\mathrm{d}x}{x}$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \operatorname{arctg} x & x \in (0;\infty) \\ \mathrm{d}t = \frac{\mathrm{d}x}{x^2+1} & t \in (0; \frac{\pi}{2}) \end{array}\right] = \int \ln t \,\mathrm{d}t$$

$$= \left[\begin{array}{l|l} u = \ln \operatorname{arctg} x & u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} & v = \operatorname{arctg} x \end{array}\right] = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{\mathrm{d}x}{x^2+1}$$

Riešené príklady – 074, 075

$$\int \ln x \, dx$$

$$= \begin{bmatrix} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{bmatrix} = x \ln x - \int \frac{x \, dx}{x} = x \ln x - \int dx$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \begin{bmatrix} \text{Subst. } t = \operatorname{arctg} x & x \in (0;\infty) \\ dt = \frac{dx}{x^2+1} & t \in (0; \frac{\pi}{2}) \end{bmatrix} = \int \ln t \, dt = \begin{bmatrix} u = \ln t & u' = \frac{1}{t} \\ v' = 1 & v = t \end{bmatrix}$$

$$= \begin{bmatrix} u = \ln \operatorname{arctg} x & u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} & v = \operatorname{arctg} x \end{bmatrix} = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, \; x \in (0;\infty), \; c \in R.$$

Riešené príklady – 074, 075

$$\int \ln x \, \mathrm{d}x = x \ln x - x + c$$

$$= \begin{bmatrix} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{bmatrix} = x \ln x - \int \frac{x\,\mathrm{d}x}{x} = x \ln x - \int \mathrm{d}x = x \ln x - x + c,$$

$$x \in (0; \infty), \ c \in R.$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, \mathrm{d}x = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \begin{bmatrix} \text{Subst. } t = \operatorname{arctg} x & x \in (0; \infty) \\ \mathrm{d}t = \frac{\mathrm{d}x}{x^2+1} & t \in (0; \frac{\pi}{2}) \end{bmatrix} = \int \ln t \, \mathrm{d}t = \begin{bmatrix} u = \ln t & u' = \frac{1}{t} \\ v' = 1 & v = t \end{bmatrix} = t \ln t - \int \mathrm{d}t$$

$$= \begin{bmatrix} u = \ln \operatorname{arctg} x & u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} & v = \operatorname{arctg} x \end{bmatrix} = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{\mathrm{d}x}{x^2+1}$$

$$= \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, \ x \in (0; \infty), \ c \in R.$$

Riešené príklady – 074, 075

$$\int \ln x \, dx = x\ln x - x + c$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=1 & v=x \end{array}\right] = x\ln x - \int \frac{x\,dx}{x} = x\ln x - \int dx = x\ln x - x + c,$$

$$x\in(0;\infty),\ c\in R.$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1}\,dx = \operatorname{arctg} x\cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=\operatorname{arctg} x & x\in(0;\infty) \\ dt=\frac{dx}{x^2+1} & t\in(0;\frac{\pi}{2}) \end{array}\right] = \int \ln t\,dt = \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=1 & v=t \end{array}\right] = t\ln t - \int dt$$

$$= t\ln t - t + c$$

$$= \left[\begin{array}{l|l} u=\ln \operatorname{arctg} x & u'=\frac{1}{\operatorname{arctg} x}\cdot\frac{1}{x^2+1} \\ v'=\frac{1}{x^2+1} & v=\operatorname{arctg} x \end{array}\right] = \operatorname{arctg} x\cdot\ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} x\cdot\ln \operatorname{arctg} x - \operatorname{arctg} x + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 074, 075

$$\int \ln x \, dx = x \ln x - x + c$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{array} \right] = x \ln x - \int \frac{x \, dx}{x} = x \ln x - \int dx = x \ln x - x + c,$$

$$x \in (0; \infty), \ c \in R.$$

$$\int \frac{\ln \operatorname{arctg} x}{x^2+1} \, dx = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \operatorname{arctg} x & x \in (0; \infty) \\ dt = \frac{dx}{x^2+1} & t \in (0; \frac{\pi}{2}) \end{array} \right] = \int \ln t \, dt = \left[\begin{array}{l|l} u = \ln t & u' = \frac{1}{t} \\ v' = 1 & v = t \end{array} \right] = t \ln t - \int dt$$

$$= t \ln t - t + c = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, \ x \in (0; \infty), \ c \in R.$$

$$= \left[\begin{array}{l|l} u = \ln \operatorname{arctg} x & u' = \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{x^2+1} \\ v' = \frac{1}{x^2+1} & v = \operatorname{arctg} x \end{array} \right] = \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \int \frac{dx}{x^2+1}$$

$$= \operatorname{arctg} x \cdot \ln \operatorname{arctg} x - \operatorname{arctg} x + c, \ x \in (0; \infty), \ c \in R.$$

Riešené príklady – 076

$$\int \ln (x-1)^5 \, dx = 5\int \ln (x-1) \, dx$$

Riešené príklady – 076

$$\int \ln(x-1)^5\,dx = 5\int \ln(x-1)\,dx$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x \end{array}\right]$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x-1 \end{array}\right]$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(1;\infty) \\ dt=dx & t\in(0;\infty) \end{array}\right]$$

Riešené príklady – 076

$$\int \ln (x-1)^5 \, dx = 5\int \ln (x-1) \, dx$$

$$= \left[\begin{array}{l|l} u = \ln (x-1) & u' = \frac{1}{x-1} \\ v' = 1 & v = x \end{array} \right] = 5x \ln (x-1) - 5\int \frac{x \, dx}{x-1}$$

$$= \left[\begin{array}{l|l} u = \ln (x-1) & u' = \frac{1}{x-1} \\ v' = 1 & v = x-1 \end{array} \right] = 5(x-1) \ln (x-1) - 5\int \frac{(x-1) \, dx}{x-1}$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x \in (1;\infty) \\ dt = dx & t \in (0;\infty) \end{array} \right] = 5\int \ln t \, dt$$

Riešené príklady – 076

$$\int \ln (x-1)^5\,dx = 5\int \ln (x-1)\,dx$$

$$= \left[\begin{array}{l|l} u = \ln (x-1) & u' = \frac{1}{x-1} \\ v' = 1 & v = x \end{array}\right] = 5x \ln (x-1) - 5\int \frac{x\,dx}{x-1} = 5x \ln (x-1) - 5\int \frac{(x-1+1)\,dx}{x-1}$$

$$= \left[\begin{array}{l|l} u = \ln (x-1) & u' = \frac{1}{x-1} \\ v' = 1 & v = x-1 \end{array}\right] = 5(x-1) \ln (x-1) - 5\int \frac{(x-1)\,dx}{x-1}$$

$$= 5(x-1) \ln (x-1) - 5\int dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = x-1 & x \in (1;\infty) \\ dt = dx & t \in (0;\infty) \end{array}\right] = 5\int \ln t\,dt = \left[\begin{array}{l|l} u = \ln t & u' = \frac{1}{t} \\ v' = 1 & v = t \end{array}\right]$$

Riešené príklady – 076

$$\int \ln (x-1)^5\,dx = 5\int \ln (x-1)\,dx = 5(x-1)\ln (x-1)-5x+c$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x \end{array}\right] = 5x\ln(x-1)-5\int\frac{x\,dx}{x-1} = 5x\ln(x-1)-5\int\frac{(x-1+1)\,dx}{x-1}$$

$$=5x\ln(x-1)-5\int dx-5\int\frac{dx}{x-1}$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x-1 \end{array}\right] = 5(x-1)\ln(x-1)-5\int\frac{(x-1)\,dx}{x-1}$$

$$=5(x-1)\ln(x-1)-5\int dx = 5(x-1)\ln(x-1)-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(1;\infty) \\ dt=dx & t\in(0;\infty) \end{array}\right] = 5\int \ln t\,dt = \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=1 & v=t \end{array}\right] = 5t\ln t-5\int dt$$

Riešené príklady – 076

$$\int \ln (x-1)^5\,dx = 5\int \ln (x-1)\,dx = 5(x-1)\ln (x-1)-5x+c$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x \end{array}\right] = 5x\ln(x-1)-5\int\frac{x\,dx}{x-1} = 5x\ln(x-1)-5\int\frac{(x-1+1)\,dx}{x-1}$$

$$= 5x\ln(x-1)-5\int dx-5\int\frac{dx}{x-1} = 5x\ln(x-1)-5x-5\ln(x-1)+c$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x-1 \end{array}\right] = 5(x-1)\ln(x-1)-5\int\frac{(x-1)\,dx}{x-1}$$

$$= 5(x-1)\ln(x-1)-5\int dx = 5(x-1)\ln(x-1)-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(1;\infty) \\ dt=dx & t\in(0;\infty) \end{array}\right] = 5\int \ln t\,dt = \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=1 & v=t \end{array}\right] = 5t\ln t-5\int dt$$

$$= 5t\ln t-5t+c_1$$

Riešené príklady – 076

$$\int \ln (x-1)^5\,dx = 5\int \ln (x-1)\,dx = 5(x-1)\ln (x-1)-5x+c$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x \end{array}\right] = 5x\ln(x-1)-5\int\frac{x\,dx}{x-1} = 5x\ln(x-1)-5\int\frac{(x-1+1)\,dx}{x-1}$$

$$= 5x\ln(x-1)-5\int dx-5\int\frac{dx}{x-1} = 5x\ln(x-1)-5x-5\ln(x-1)+c$$

$$= 5(x-1)\ln(x-1)-5x+c = (x-1)\ln(x-1)^5-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x-1 \end{array}\right] = 5(x-1)\ln(x-1)-5\int\frac{(x-1)\,dx}{x-1}$$

$$= 5(x-1)\ln(x-1)-5\int dx = 5(x-1)\ln(x-1)-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(1;\infty) \\ dt=dx & t\in(0;\infty) \end{array}\right] = 5\int \ln t\,dt = \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=1 & v=t \end{array}\right] = 5t\ln t-5\int dt$$

$$= 5t\ln t-5t+c_1 = 5(x-1)\ln(x-1)-5(x-1)+c_1$$

Riešené príklady – 076

$$\int \ln (x-1)^5 \,dx = 5\int \ln (x-1)\,dx = 5(x-1)\ln (x-1)-5x+c$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x \end{array}\right] = 5x\ln(x-1)-5\int\frac{x\,dx}{x-1} = 5x\ln(x-1)-5\int\frac{(x-1+1)\,dx}{x-1}$$

$$= 5x\ln(x-1)-5\int dx-5\int\frac{dx}{x-1} = 5x\ln(x-1)-5x-5\ln(x-1)+c$$

$$= 5(x-1)\ln(x-1)-5x+c = (x-1)\ln(x-1)^5-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x-1 \end{array}\right] = 5(x-1)\ln(x-1)-5\int\frac{(x-1)\,dx}{x-1}$$

$$= 5(x-1)\ln(x-1)-5\int dx = 5(x-1)\ln(x-1)-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(1;\infty) \\ dt=dx & t\in(0;\infty) \end{array}\right] = 5\int \ln t\,dt = \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=1 & v=t \end{array}\right] = 5t\ln t-5\int dt$$

$$= 5t\ln t-5t+c_1 = 5(x-1)\ln(x-1)-5(x-1)+c_1 = \left[\begin{array}{c} c=c_1+5 \\ c\in R,\ c_1\in R \end{array}\right]$$

Riešené príklady – 076

$$\int \ln (x-1)^5\,dx = 5\int \ln (x-1)\,dx = 5(x-1)\ln(x-1)-5x+c$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x \end{array}\right] = 5x\ln(x-1)-5\int\frac{x\,dx}{x-1} = 5x\ln(x-1)-5\int\frac{(x-1+1)\,dx}{x-1}$$

$$=5x\ln(x-1)-5\int dx-5\int\frac{dx}{x-1}=5x\ln(x-1)-5x-5\ln(x-1)+c$$

$$=5(x-1)\ln(x-1)-5x+c=(x-1)\ln(x-1)^5-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{l|l} u=\ln(x-1) & u'=\frac{1}{x-1} \\ v'=1 & v=x-1 \end{array}\right] = 5(x-1)\ln(x-1)-5\int\frac{(x-1)\,dx}{x-1}$$

$$=5(x-1)\ln(x-1)-5\int dx=5(x-1)\ln(x-1)-5x+c,\ x\in(1;\infty),\ c\in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t=x-1 & x\in(1;\infty) \\ dt=dx & t\in(0;\infty) \end{array}\right] = 5\int \ln t\,dt = \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=1 & v=t \end{array}\right] = 5t\ln t-5\int dt$$

$$=5t\ln t-5t+c_1=5(x-1)\ln(x-1)-5(x-1)+c_1=\left[\begin{array}{c} c=c_1+5 \\ c\in R,\ c_1\in R \end{array}\right]$$

$$=5(x-1)\ln(x-1)-5x+c,\ x\in(1;\infty),\ c\in R.$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x}\, dx$$

$$\int \frac{dx}{x \ln x}$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x}\, dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \ln x & x \in (0; \infty) \\ dt = \frac{dx}{x} & t \in R \end{array} \right]$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = \frac{1}{x} & v = \ln x \end{array} \right]$$

$$\int \frac{dx}{x \ln x}$$

$$= \int \frac{\frac{1}{x}}{\ln x}\, dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \ln x & x \in (0; 1) \cup (1; \infty) \\ dt = \frac{dx}{x} & t \in (-\infty; 0) \cup (0; \infty) \end{array} \right]$$

Riešené príklady – 077, 078

$$I=\int \frac{\ln x}{x}\,dx$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=\ln x & x\in(0;\infty) \\ dt=\frac{dx}{x} & t\in R \end{array}\right]=\int dt$$

$$=\left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=\frac{1}{x} & v=\ln x \end{array}\right]=\ln^2 x-\int\frac{\ln x}{x}\,dx$$

$$\int \frac{dx}{x\ln x}$$

$$=\int \frac{\frac{1}{x}}{\ln x}\,dx=\int \frac{[\ln x]'}{\ln x}\,dx$$

$$=\left[\begin{array}{r|l} \text{Subst. } t=\ln x & x\in(0;1)\cup(1;\infty) \\ dt=\frac{dx}{x} & t\in(-\infty;0)\cup(0;\infty) \end{array}\right]=\int\frac{dt}{t}$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x}\,dx$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \ln x & x \in (0;\infty) \\ dt = \frac{dx}{x} & t \in R \end{array}\right] = \int dt = \frac{t^2}{2} + c$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = \frac{1}{x} & v = \ln x \end{array}\right] = \ln^2 x - \int \frac{\ln x}{x}\,dx = \ln^2 x - I$$

$$\int \frac{dx}{x \ln x} = \ln|\ln x| + c$$

$$= \int \frac{\frac{1}{x}}{\ln x}\,dx = \int \frac{[\ln x]'}{\ln x}\,dx = \ln|\ln x| + c,\ x \in (0;1)\cup(1;\infty),\ c \in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \ln x & x \in (0;1)\cup(1;\infty) \\ dt = \frac{dx}{x} & t \in (-\infty;0)\cup(0;\infty) \end{array}\right] = \int \frac{dt}{t} = \ln|t| + c$$

Riešené príklady – 077, 078

$$I = \int \frac{\ln x}{x}\,dx = \frac{\ln^2 x}{2} + c$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \ln x & x \in (0;\infty) \\ dt = \frac{dx}{x} & t \in R \end{array}\right] = \int dt = \frac{t^2}{2} + c = \frac{\ln^2 x}{2} + c,\ x \in (0;\infty),\ c \in R.$$

$$I = \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = \frac{1}{x} & v = \ln x \end{array}\right] = \ln^2 x - \int \frac{\ln x}{x}\,dx = \ln^2 x - I = \left[\text{Rovnica } \ I = \ln^2 x - I\right]$$

$$\int \frac{dx}{x \ln x} = \ln|\ln x| + c$$

$$= \int \frac{\frac{1}{x}}{\ln x}\,dx = \int \frac{[\ln x]'}{\ln x}\,dx = \ln|\ln x| + c,\ x \in (0;1)\cup(1;\infty),\ c \in R.$$

$$= \left[\begin{array}{r|l} \text{Subst. } t = \ln x & x \in (0;1)\cup(1;\infty) \\ dt = \frac{dx}{x} & t \in (-\infty;0)\cup(0;\infty) \end{array}\right] = \int \frac{dt}{t} = \ln|t| + c = \ln|\ln x| + c,$$

$$x \in (0;1)\cup(1;\infty),\ c \in R.$$

Riešené príklady – 077, 078

$$I=\int \frac{\ln x}{x}\,dx = \frac{\ln^2 x}{2}+c$$

$$=\left[\begin{array}{c|c}\text{Subst. } t=\ln x & x\in(0;\infty)\\ dt=\frac{dx}{x} & t\in R\end{array}\right]=\int dt=\frac{t^2}{2}+c=\frac{\ln^2 x}{2}+c,\ x\in(0;\infty),\ c\in R.$$

$$=\left[\begin{array}{c|c} u=\ln x & u'=\frac{1}{x}\\ v'=\frac{1}{x} & v=\ln x\end{array}\right]=\ln^2 x-\int\frac{\ln x}{x}\,dx=\ln^2 x-I=\left[\text{Rovnica } I=\ln^2 x-I\right]=\frac{\ln^2 x}{2}+c,$$

$$x\in(0;\infty),\ c\in R.$$

$$\int\frac{dx}{x\ln x}=\ln|\ln x|+c$$

$$=\int\frac{\frac{1}{x}}{\ln x}\,dx=\int\frac{[\ln x]'}{\ln x}\,dx=\ln|\ln x|+c,\ x\in(0;1)\cup(1;\infty),\ c\in R.$$

$$=\left[\begin{array}{c|c}\text{Subst. } t=\ln x & x\in(0;1)\cup(1;\infty)\\ dt=\frac{dx}{x} & t\in(-\infty;0)\cup(0;\infty)\end{array}\right]=\int\frac{dt}{t}=\ln|t|+c=\ln|\ln x|+c,$$

$$x\in(0;1)\cup(1;\infty),\ c\in R.$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x}\, dx = \int x^2 \ln (1-x)^{\frac{1}{2}}\, dx$$

$$\int \frac{\ln x}{\sqrt{x}}\, dx = \int x^{-\frac{1}{2}} \ln x\, dx$$

Riešené príklady – 079, 080

$$\int x^2 \ln \sqrt{1-x}\,dx = \int x^2 \ln (1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2} \ln (1-x)\,dx$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}} \ln x\,dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} & v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array}\right]$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u=\ln(1-x) & u'=\frac{-1}{1-x}=\frac{1}{x-1} \\ v'=\frac{x^2}{2} & v=\frac{x^3}{2\cdot 3}=\frac{x^3}{6} \end{array}\right]$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x^{-\frac{1}{2}} & v=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}} \end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u=\ln(1-x) & u'=\frac{-1}{1-x}=\frac{1}{x-1} \\ v'=\frac{x^2}{2} & v=\frac{x^3}{2\cdot 3}=\frac{x^3}{6} \end{array}\right] = \frac{x^3\ln(1-x)}{6} - \int \frac{x^3\,dx}{6(x-1)}$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x^{-\frac{1}{2}} & v=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}} \end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\cdot\ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u=\ln(1-x) & u'=\frac{-1}{1-x}=\frac{1}{x-1} \\ v'=\frac{x^2}{2} & v=\frac{x^3}{2\cdot 3}=\frac{x^3}{6}\end{array}\right] = \frac{x^3\ln(1-x)}{6} - \int\frac{x^3\,dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int\frac{(x^3-1+1)\,dx}{6(x-1)}$$

$$\int\frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx = 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x^{-\frac{1}{2}} & v=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}}\end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\cdot\ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u=\ln(1-x) & u'=\frac{-1}{1-x}=\frac{1}{x-1} \\ v'=\frac{x^2}{2} & v=\frac{x^3}{2\cdot 3}=\frac{x^3}{6}\end{array}\right] = \frac{x^3\ln(1-x)}{6} - \int\frac{x^3\,dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int\frac{(x^3-1+1)\,dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \int\frac{(x^3-1)\,dx}{6(x-1)} - \int\frac{dx}{6(x-1)}$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx = 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x^{-\frac{1}{2}} & v=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}}\end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\cdot\ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u=\ln(1-x) & u'=\frac{-1}{1-x}=\frac{1}{x-1} \\ v'=\frac{x^2}{2} & v=\frac{x^3}{2\cdot 3}=\frac{x^3}{6}\end{array}\right] = \frac{x^3\ln(1-x)}{6} - \int \frac{x^3\,dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int \frac{(x^3-1+1)\,dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \int \frac{(x^3-1)\,dx}{6(x-1)} - \int \frac{dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int \frac{(x^2+x+1)\,dx}{6} - \int \frac{dx}{6(x-1)}$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx = 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x^{-\frac{1}{2}} & v=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}}\end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\cdot\ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u = \ln(1-x) & u' = \frac{-1}{1-x} = \frac{1}{x-1} \\ v' = \frac{x^2}{2} & v = \frac{x^3}{2\cdot 3} = \frac{x^3}{6} \end{array}\right] = \frac{x^3\ln(1-x)}{6} - \int \frac{x^3\,dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int \frac{(x^3-1+1)\,dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \int \frac{(x^3-1)\,dx}{6(x-1)} - \int \frac{dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int \frac{(x^2+x+1)\,dx}{6} - \int \frac{dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \frac{\frac{x^3}{3}+\frac{x^2}{2}+x}{6} - \frac{\ln|x-1|}{6} + c$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx = 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^{-\frac{1}{2}} & v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} \end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\cdot\ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u=\ln(1-x) & u'=\frac{-1}{1-x}=\frac{1}{x-1} \\ v'=\frac{x^2}{2} & v=\frac{x^3}{2\cdot 3}=\frac{x^3}{6}\end{array}\right] = \frac{x^3\ln(1-x)}{6} - \int \frac{x^3\,dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int \frac{(x^3-1+1)\,dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \int \frac{(x^3-1)\,dx}{6(x-1)} - \int \frac{dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int \frac{(x^2+x+1)\,dx}{6} - \int \frac{dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \frac{\frac{x^3}{3}+\frac{x^2}{2}+x}{6} - \frac{\ln|x-1|}{6} + c$$

$$= \frac{x^3\ln(1-x)}{6} - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} - \frac{\ln(1-x)}{6} + c$$

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx = 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x^{-\frac{1}{2}} & v=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}}\end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\cdot\ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 079, 080

$$\int x^2 \ln\sqrt{1-x}\,dx = \int x^2 \ln(1-x)^{\frac{1}{2}}\,dx = (x^3-1)\ln\sqrt[6]{1-x} - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$$

$$= \int \frac{x^2}{2}\ln(1-x)\,dx = \left[\begin{array}{l|l} u=\ln(1-x) & u'=\frac{-1}{1-x}=\frac{1}{x-1} \\ v'=\frac{x^2}{2} & v=\frac{x^3}{2\cdot 3}=\frac{x^3}{6}\end{array}\right] = \frac{x^3\ln(1-x)}{6} - \int\frac{x^3\,dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int\frac{(x^3-1+1)\,dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \int\frac{(x^3-1)\,dx}{6(x-1)} - \int\frac{dx}{6(x-1)}$$

$$= \frac{x^3\ln(1-x)}{6} - \int\frac{(x^2+x+1)\,dx}{6} - \int\frac{dx}{6(x-1)} = \frac{x^3\ln(1-x)}{6} - \frac{\frac{x^3}{3}+\frac{x^2}{2}+x}{6} - \frac{\ln|x-1|}{6} + c$$

$$= \frac{x^3\ln(1-x)}{6} - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} - \frac{\ln(1-x)}{6} + c = \frac{x^3-1}{6}\ln(1-x) - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c,$$

$$x\in(-\infty;1),\ c\in R.$$

$$\int\frac{\ln x}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\ln x\,dx = 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c$$

$$= \left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x^{-\frac{1}{2}} & v=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}=2x^{\frac{1}{2}}\end{array}\right] = 2x^{\frac{1}{2}}\ln x - 2\int x^{-\frac{1}{2}}\,dx = 2\sqrt{x}\cdot\ln x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x}\cdot\ln x - 4\sqrt{x} + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx$$

$$\int x^2 \ln x \, dx$$

$$\int x^n \ln x \, dx$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{x^2}{2} \end{array} \right]$$

$$\int x^2 \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{array} \right]$$

$$\int x^n \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^n & v = \frac{x^{n+1}}{n+1} \end{array} \right]$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{x^2}{2} \end{array} \right] = \frac{x^2}{2} \ln x - \int \frac{x \, dx}{2}$$

$$\int x^2 \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{array} \right] = \frac{x^3}{3} \ln x - \int \frac{x^2 \, dx}{3}$$

$$\int x^n \ln x \, dx$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^n & v = \frac{x^{n+1}}{n+1} \end{array} \right] = \frac{x^{n+1}}{n+1} \ln x - \int \frac{x^n \, dx}{n+1}$$

Riešené príklady – 081, 082, 083

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{x^2}{2} \end{array} \right] = \frac{x^2}{2} \ln x - \int \frac{x \, dx}{2} = \frac{x^2}{2} \ln x - \frac{x^2}{2 \cdot 2} \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c, \quad x \in (0; \infty), \; c \in R.$$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + c$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^2 & v = \frac{x^3}{3} \end{array} \right] = \frac{x^3}{3} \ln x - \int \frac{x^2 \, dx}{3} = \frac{x^3}{3} \ln x - \frac{x^3}{3 \cdot 3} \, dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + c, \quad x \in (0; \infty), \; c \in R.$$

$$\int x^n \ln x \, dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + c$$

$$= \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x^n & v = \frac{x^{n+1}}{n+1} \end{array} \right] = \frac{x^{n+1}}{n+1} \ln x - \int \frac{x^n \, dx}{n+1} = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + c, \quad x \in (0; \infty), \; c \in R.$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5 \int (x+1)^2 \ln (x-1) \, dx$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5 \int (x+1)^2 \ln (x-1) \, dx = \begin{bmatrix} \text{Subst. } t = x-1 & x = t+1 & x \in (1; \infty) \\ dt = dx & x+1 = t+2 & t \in (0; \infty) \end{bmatrix}$$

Riešené príklady – 084

$\int (x+1)^2 \ln (x-1)^5 \, dx$

$$= 5\int (x+1)^2 \ln (x-1) \, dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{array} \right] = 5\int (t+2)^2 \ln t \, dt$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5\int (x+1)^2 \ln (x-1) \, dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{array} \right] = 5\int (t+2)^2 \ln t \, dt$$

$$= \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{array} \right]$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \,dx$$

$$= 5\int (x+1)^2 \ln (x-1)\,dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{array} \right] = 5\int (t+2)^2 \ln t \,dt$$

$$= \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{array} \right] = \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{(t+2)^3\,dt}{t}$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5\int (x+1)^2 \ln (x-1)\, dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{array}\right] = 5\int (t+2)^2 \ln t \, dt$$

$$= \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{array}\right] = \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{(t+2)^3\, dt}{t}$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{t^3+6t^2+12t+8}{t}\, dt$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5\int (x+1)^2 \ln (x-1) \, dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{array} \right] = 5\int (t+2)^2 \ln t \, dt$$

$$= \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{array} \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3}\int \frac{(t+2)^3 \, dt}{t}$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3}\int \frac{t^3+6t^2+12t+8}{t} \, dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3}\int \left[t^2 + 6t + 12 + \frac{8}{t} \right] dt$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5\int (x+1)^2 \ln (x-1)\, dx = \begin{bmatrix} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{bmatrix} = 5\int (t+2)^2 \ln t \, dt$$

$$= \begin{bmatrix} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{bmatrix} = \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{(t+2)^3\, dt}{t}$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{t^3+6t^2+12t+8}{t}\, dt$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \left[t^2+6t+12+\frac{8}{t}\right] dt$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\left[\frac{t^3}{3}+\frac{6t^2}{2}+12t+8\ln t\right] + c$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5\int (x+1)^2 \ln (x-1)\, dx = \begin{bmatrix} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{bmatrix} = 5\int (t+2)^2 \ln t \, dt$$

$$= \begin{bmatrix} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{bmatrix} = \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{(t+2)^3\, dt}{t}$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{t^3+6t^2+12t+8}{t}\, dt$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \left[t^2+6t+12+\frac{8}{t}\right] dt$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\left[\frac{t^3}{3}+\frac{6t^2}{2}+12t+8\ln t\right] + c$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5t^3}{9} - 5t^2 - 20t - \frac{40}{3}\ln t + c$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx$$

$$= 5\int (x+1)^2 \ln (x-1) \, dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{array} \right] = 5\int (t+2)^2 \ln t \, dt$$

$$= \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{array} \right] = \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{(t+2)^3\,dt}{t}$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \frac{t^3+6t^2+12t+8}{t}\,dt$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\int \left[t^2+6t+12+\frac{8}{t}\right] dt$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5}{3}\left[\frac{t^3}{3}+\frac{6t^2}{2}+12t+8\ln t\right] + c$$

$$= \frac{5(t+2)^3}{3}\ln t - \frac{5t^3}{9} - 5t^2 - 20t - \frac{40}{3}\ln t + c$$

$$= \frac{5(t+2)^3-40}{3}\ln t - \frac{5t^3}{9} - 5t^2 - 20t + c$$

Riešené príklady – 084

$$\int (x+1)^2 \ln (x-1)^5 \, dx = \frac{5(x+1)^3-40}{3} \ln (x-1) - \frac{5(x-1)^3}{9} - 5(x-1)^2 - 20(x-1) + c$$

$$= 5\int (x+1)^2 \ln (x-1) \, dx = \left[\begin{array}{r|l|l} \text{Subst. } t=x-1 & x=t+1 & x\in(1;\infty) \\ dt=dx & x+1=t+2 & t\in(0;\infty) \end{array} \right] = 5\int (t+2)^2 \ln t \, dt$$

$$= \left[\begin{array}{l|l} u=\ln t & u'=\frac{1}{t} \\ v'=(t+2)^2 & v=\frac{(t+2)^3}{3} \end{array} \right] = \frac{5(t+2)^3}{3} \ln t - \frac{5}{3}\int \frac{(t+2)^3 \, dt}{t}$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3}\int \frac{t^3+6t^2+12t+8}{t} \, dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3}\int \left[t^2 + 6t + 12 + \frac{8}{t}\right] dt$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5}{3}\left[\frac{t^3}{3} + \frac{6t^2}{2} + 12t + 8\ln t\right] + c$$

$$= \frac{5(t+2)^3}{3} \ln t - \frac{5t^3}{9} - 5t^2 - 20t - \frac{40}{3}\ln t + c$$

$$= \frac{5(t+2)^3-40}{3} \ln t - \frac{5t^3}{9} - 5t^2 - 20t + c$$

$$= \frac{5(x+1)^3-40}{3} \ln (x-1) - \frac{5(x-1)^3}{9} - 5(x-1)^2 - 20(x-1) + c, \; x\in(1;\infty), \; c\in R.$$

Riešené príklady – 085, 086

$$\int x^x (\ln x + 1)\, dx$$

$$\int x \ln^2 x\, dx$$

Riešené príklady – 085, 086

$\int x^x(\ln x+1)\,dx$

$$=\left[\begin{array}{l|l} u=x^x & u'=\left(e^{\ln x^x}\right)'=(e^{x\ln x})'=e^{x\ln x}(\ln x+\frac{x}{x})=x^x(\ln x+1)\\ v'=\ln x+1 & v=\dots\ \text{nie je nutné, pretože } u'=[x^x]'=x^x(\ln x+1)\end{array}\right]$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst.}\quad t=x^x & dt=x^x(\ln x+1)\,dx & x^x \text{ má min pre } x=\frac{1}{e},\text{ t. j. } \ln x+1=0 & x\in(0;\frac{1}{e}\rangle & x\in\langle\frac{1}{e};\infty) \\ & (x^x)'=\left(e^{\ln x^x}\right)'=(e^{x\ln x})'=e^{x\ln x}(\ln x+\frac{x}{x})=x^x(\ln x+1) & & t\in\langle e^{-\frac{1}{e}};1) & t\in\langle e^{-\frac{1}{e}};\infty)\end{array}\right]$$

$\int x\ln^2 x\,dx$

$$=\left[\begin{array}{l|l} u=\ln^2 x & u'=\frac{2\ln x}{x}\\ v'=x & v=\frac{x^2}{2}\end{array}\right]$$

Riešené príklady – 085, 086

$$\int x^x(\ln x+1)\,dx = x^x + c$$

$$= \left[\begin{array}{l|l} u = x^x & u' = \left(e^{\ln x^x}\right)' = (e^{x\ln x})' = e^{x\ln x}(\ln x + \frac{x}{x}) = x^x(\ln x+1) \\ v' = \ln x + 1 & v = \ldots \text{ nie je nutné, pretože } u' = [x^x]' = x^x(\ln x+1) \end{array}\right] = x^x + c,\ x\in(0;\infty),\ c\in R.$$

$$= \left[\begin{array}{l|l|l|l|l} \text{Subst. } t = x^x & dt = x^x(\ln x+1)\,dx & x^x \text{ má min pre } x=\frac{1}{e}, \text{ t. j. } \ln x+1=0 & x\in(0;\frac{1}{e}\rangle & x\in\langle\frac{1}{e};\infty) \\ & (x^x)' = \left(e^{\ln x^x}\right)' = (e^{x\ln x})' = e^{x\ln x}(\ln x+\frac{x}{x}) = x^x(\ln x+1) & & t\in\langle e^{-\frac{1}{e}};1) & t\in\langle e^{-\frac{1}{e}};\infty) \end{array}\right] = \int dt$$

$$\int x\ln^2 x\,dx$$

$$= \left[\begin{array}{l|l} u = \ln^2 x & u' = \frac{2\ln x}{x} \\ v' = x & v = \frac{x^2}{2} \end{array}\right] = \frac{x^2\ln^2 x}{2} - \int x\ln x\,dx$$

Riešené príklady – 085, 086

$\int x^x(\ln x+1)\,dx = x^x + c$

$$= \left[\begin{array}{l|l} u = x^x & u' = \left(e^{\ln x^x}\right)' = (e^{x\ln x})' = e^{x\ln x}(\ln x + \frac{x}{x}) = x^x(\ln x+1) \\ v' = \ln x+1 & v = \ldots \text{ nie je nutné, pretože } u' = [x^x]' = x^x(\ln x+1) \end{array}\right] = x^x + c,\ x\in(0;\infty),\ c\in R.$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } t = x^x & dt = x^x(\ln x+1)\,dx & x^x \text{ má min pre } x = \frac{1}{e}, \text{ t. j. } \ln x+1=0 & x\in(0;\frac{1}{e}\rangle & x\in\langle\frac{1}{e};\infty) \\ & (x^x)' = \left(e^{\ln x^x}\right)' = (e^{x\ln x})' = e^{x\ln x}(\ln x+\frac{x}{x}) = x^x(\ln x+1) & t\in\langle e^{-\frac{1}{e}};1) & t\in\langle e^{-\frac{1}{e}};\infty) \end{array}\right] = \int dt$$

$$= t + c$$

$\int x\ln^2 x\,dx$

$$= \left[\begin{array}{l|l} u = \ln^2 x & u' = \frac{2\ln x}{x} \\ v' = x & v = \frac{x^2}{2} \end{array}\right] = \frac{x^2\ln^2 x}{2} - \int x\ln x\,dx = \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{x^2}{2} \end{array}\right]$$

Riešené príklady – 085, 086

$\int x^x(\ln x+1)\,dx = x^x+c$

$$=\left[\begin{array}{l|l} u=x^x & u'=\left(e^{\ln x^x}\right)'=(e^{x\ln x})'=e^{x\ln x}(\ln x+\frac{x}{x})=x^x(\ln x+1) \\ v'=\ln x+1 & v=\ldots \text{ nie je nutné, pretože } u'=[x^x]'=x^x(\ln x+1)\end{array}\right]=x^x+c,\; x\in(0;\infty),\; c\in R.$$

$$=\left[\begin{array}{l|l|l|l|l} \text{Subst. } t=x^x & dt=x^x(\ln x+1)\,dx & x^x \text{ má min pre } x=\frac{1}{e}, \text{ t. j. } \ln x+1=0 & x\in(0;\frac{1}{e}\rangle & x\in\langle\frac{1}{e};\infty) \\ & \multicolumn{2}{l|}{(x^x)'=\left(e^{\ln x^x}\right)'=(e^{x\ln x})'=e^{x\ln x}(\ln x+\frac{x}{x})=x^x(\ln x+1)} & t\in\langle e^{-\frac{1}{e}};1) & t\in\langle e^{-\frac{1}{e}};\infty)\end{array}\right]=\int dt$$

$$=t+c=x^x+c,\; x\in(0;\infty),\; c\in R.$$

$\int x\ln^2 x\,dx$

$$=\left[\begin{array}{l|l} u=\ln^2 x & u'=\frac{2\ln x}{x} \\ v'=x & v=\frac{x^2}{2}\end{array}\right]=\frac{x^2\ln^2 x}{2}-\int x\ln x\,dx=\left[\begin{array}{l|l} u=\ln x & u'=\frac{1}{x} \\ v'=x & v=\frac{x^2}{2}\end{array}\right]$$

$$=\frac{x^2\ln^2 x}{2}-\left[\frac{x^2\ln x}{2}-\int\frac{x\,dx}{2}\right]$$

Riešené príklady – 085, 086

$$\int x^x(\ln x+1)\,dx = x^x + c$$

$$= \left[\begin{array}{l|l} u = x^x & u' = \left(e^{\ln x^x}\right)' = (e^{x\ln x})' = e^{x\ln x}(\ln x + \frac{x}{x}) = x^x(\ln x+1) \\ v' = \ln x + 1 & v = \ldots \text{ nie je nutné, pretože } u' = [x^x]' = x^x(\ln x+1) \end{array}\right] = x^x + c,\ x\in(0;\infty),\ c\in R.$$

$$= \left[\begin{array}{l|l|l|l|l} \text{Subst. } t = x^x & dt = x^x(\ln x+1)\,dx & x^x \text{ má min pre } x=\frac{1}{e}, \text{ t. j. } \ln x+1=0 & x\in(0;\frac{1}{e}\rangle & x\in\langle\frac{1}{e};\infty) \\ & \multicolumn{2}{l|}{(x^x)' = \left(e^{\ln x^x}\right)' = (e^{x\ln x})' = e^{x\ln x}(\ln x+\frac{x}{x}) = x^x(\ln x+1)} & t\in\langle e^{-\frac{1}{e}};1) & t\in\langle e^{-\frac{1}{e}};\infty) \end{array}\right] = \int dt$$

$$= t + c = x^x + c,\ x\in(0;\infty),\ c\in R.$$

$$\int x\ln^2 x\,dx = \frac{x^2\ln^2 x}{2} - \frac{x^2\ln x}{2} + \frac{x^2}{4} + c$$

$$= \left[\begin{array}{l|l} u = \ln^2 x & u' = \frac{2\ln x}{x} \\ v' = x & v = \frac{x^2}{2} \end{array}\right] = \frac{x^2\ln^2 x}{2} - \int x\ln x\,dx = \left[\begin{array}{l|l} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{x^2}{2} \end{array}\right]$$

$$= \frac{x^2\ln^2 x}{2} - \left[\frac{x^2\ln x}{2} - \int\frac{x\,dx}{2}\right] = \frac{x^2\ln^2 x}{2} - \frac{x^2\ln x}{2} + \frac{x^2}{4} + c,\ x\in(0;\infty),\ c\in R.$$

Riešené príklady – 087, 088

$$\int \ln\left(x^2+1\right) dx$$

$$\int \ln\left(\sqrt{1+x}+\sqrt{1-x}\right) dx$$

Riešené príklady – 087, 088

$$\int \ln\left(x^2+1\right) dx$$

$$= \left[\begin{array}{l|l} u = \ln\left(x^2+1\right) & u' = \frac{2x}{x^2+1} \\ v' = 1 & v = x \end{array}\right]$$

$$\int \ln\left(\sqrt{1+x}+\sqrt{1-x}\right) dx$$

$$= \left[\begin{array}{l|l} u = \ln\left(\sqrt{1+x}+\sqrt{1-x}\right) & u' = \frac{1}{2x} - \frac{1}{2x\sqrt{1-x^2}} \\ v' = 1 & v = x \end{array}\right]$$

$$\Big[\left[\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)\right]' = \left[\ln\left((1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}\right)\right]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}}-\frac{1}{2\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\frac{\sqrt{1-x}-\sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1+x}+\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}} \cdot \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}-\sqrt{1+x}} \cdot \frac{1}{2\sqrt{1-x^2}} = \frac{1-x-2\sqrt{1-x}\sqrt{1+x}+1+x}{[(1-x)-(1+x)]\cdot 2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x\cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}} \Big]$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1)\,dx$$

$$= \left[\begin{array}{l|l} u=\ln(x^2+1) & u'=\frac{2x}{x^2+1} \\ v'=1 & v=x \end{array}\right] = x\ln(x^2+1) - \int \frac{2x^2\,dx}{x^2+1}$$

$$\int \ln\left(\sqrt{1+x}+\sqrt{1-x}\right)dx$$

$$= \left[\begin{array}{l|l} u=\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) & u'=\frac{1}{2x}-\frac{1}{2x\sqrt{1-x^2}} \\ v'=1 & v=x \end{array}\right] = x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) - \int\left[\frac{1}{2}-\frac{1}{2\sqrt{1-x^2}}\right]dx$$

$$\left[\left[\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)\right]' = \left[\ln\left((1+x)^{\frac12}+(1-x)^{\frac12}\right)\right]' = \frac{\frac12(1+x)^{-\frac12}-\frac12(1-x)^{-\frac12}}{(1+x)^{\frac12}+(1-x)^{\frac12}} = \frac{\frac{1}{2\sqrt{1+x}}-\frac{1}{2\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\frac{\sqrt{1-x}-\sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}} \right.$$

$$\left. = \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1+x}+\sqrt{1-x}}\cdot\frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\cdot\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}-\sqrt{1+x}}\cdot\frac{1}{2\sqrt{1-x^2}} = \frac{1-x-2\sqrt{1-x}\sqrt{1+x}+1+x}{[(1-x)-(1+x)]\cdot 2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x\cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}}\right]$$

Riešené príklady – 087, 088

$$\int \ln(x^2+1)\,dx$$

$$= \left[\begin{array}{l|l} u = \ln(x^2+1) & u' = \frac{2x}{x^2+1} \\ v' = 1 & v = x \end{array} \right] = x\ln(x^2+1) - \int \frac{2x^2\,dx}{x^2+1} = x\ln(x^2+1) - 2\int \frac{x^2+1-1}{x^2+1}\,dx$$

$$\int \ln\left(\sqrt{1+x}+\sqrt{1-x}\right)dx = x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) - \frac{x}{2} + \frac{\arcsin x}{2} + c$$

$$= \left[\begin{array}{l|l} u = \ln\left(\sqrt{1+x}+\sqrt{1-x}\right) & u' = \frac{1}{2x} - \frac{1}{2x\sqrt{1-x^2}} \\ v' = 1 & v = x \end{array} \right] = x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) - \int \left[\frac{1}{2} - \frac{1}{2\sqrt{1-x^2}}\right] dx$$

$$\Bigg[\left[\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)\right]' = \left[\ln\left((1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}\right)\right]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\frac{\sqrt{1-x}-\sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$= \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1+x}+\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}} \cdot \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}-\sqrt{1+x}} \cdot \frac{1}{2\sqrt{1-x^2}} = \frac{1-x-2\sqrt{1-x}\sqrt{1+x}+1+x}{[(1-x)-(1+x)]\cdot 2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x\cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}} \Bigg]$$

$$= x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) - \frac{x}{2} + \frac{\arcsin x}{2} + c,\ x \in (-1;0)\cup(0;1),\ c \in R.$$

Riešené príklady – 087, 088

$\int \ln(x^2+1)\,dx$

$= \left[\begin{array}{l|l} u=\ln(x^2+1) & u'=\frac{2x}{x^2+1} \\ v'=1 & v=x \end{array}\right] = x\ln(x^2+1) - \int \frac{2x^2\,dx}{x^2+1} = x\ln(x^2+1) - 2\int \frac{x^2+1-1}{x^2+1}\,dx$

$= x\ln(x^2+1) - 2\int dx + \int \frac{2\,dx}{x^2+1}$

$\int \ln\left(\sqrt{1+x}+\sqrt{1-x}\right)dx = x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) - \frac{x}{2} + \frac{\arcsin x}{2} + c$

$= \left[\begin{array}{l|l} u=\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) & u'=\frac{1}{2x}-\frac{1}{2x\sqrt{1-x^2}} \\ v'=1 & v=x \end{array}\right] = x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) - \int\left[\frac{1}{2}-\frac{1}{2\sqrt{1-x^2}}\right]dx$

$\left[\left[\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)\right]' = \left[\ln\left((1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}\right)\right]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}}-\frac{1}{2\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\frac{\sqrt{1-x}-\sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}}\right.$

$\left. = \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1+x}+\sqrt{1-x}}\cdot\frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\cdot\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}-\sqrt{1+x}}\cdot\frac{1}{2\sqrt{1-x^2}} = \frac{1-x-2\sqrt{1-x}\sqrt{1+x}+1+x}{[(1-x)-(1+x)]\cdot 2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x\cdot 2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}}\right]$

$= x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) - \frac{x}{2} + \frac{\arcsin x}{2} + c,\ x\in(-1;0)\cup(0;1),\ c\in R.$

Riešené príklady – 087, 088

$$\int \ln(x^2+1)\,dx = x\ln(x^2+1)-2x+2\,\mathrm{arctg}\,x+c$$

$$= \left[\begin{array}{l|l} u=\ln(x^2+1) & u'=\frac{2x}{x^2+1} \\ v'=1 & v=x \end{array}\right] = x\ln(x^2+1)-\int\frac{2x^2\,dx}{x^2+1} = x\ln(x^2+1)-2\int\frac{x^2+1-1}{x^2+1}\,dx$$

$$= x\ln(x^2+1)-2\int dx+\int\frac{2\,dx}{x^2+1} = x\ln(x^2+1)-2x+2\,\mathrm{arctg}\,x+c,\ x\in R,\ c\in R.$$

$$\int \ln\left(\sqrt{1+x}+\sqrt{1-x}\right)dx = x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)-\frac{x}{2}+\frac{\arcsin x}{2}+c$$

$$= \left[\begin{array}{l|l} u=\ln\left(\sqrt{1+x}+\sqrt{1-x}\right) & u'=\frac{1}{2x}-\frac{1}{2x\sqrt{1-x^2}} \\ v'=1 & v=x \end{array}\right] = x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)-\int\left[\frac{1}{2}-\frac{1}{2\sqrt{1-x^2}}\right]dx$$

$$\left[\left[\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)\right]' = \left[\ln\left((1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}\right)\right]' = \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{2}(1-x)^{-\frac{1}{2}}}{(1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{1+x}}-\frac{1}{2\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\frac{\sqrt{1-x}-\sqrt{1+x}}{2\sqrt{1+x}\sqrt{1-x}}}{\sqrt{1+x}+\sqrt{1-x}}\right.$$

$$\left.= \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1+x}+\sqrt{1-x}}\cdot\frac{1}{2\sqrt{1+x}\sqrt{1-x}} = \frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\cdot\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}-\sqrt{1+x}}\cdot\frac{1}{2\sqrt{1-x^2}} = \frac{1-x-2\sqrt{1-x}\sqrt{1+x}+1+x}{[(1-x)-(1+x)]\cdot2\sqrt{1-x^2}} = \frac{2-2\sqrt{1-x^2}}{-2x\cdot2\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}-1}{2x\sqrt{1-x^2}}\right]$$

$$= x\ln\left(\sqrt{1+x}+\sqrt{1-x}\right)-\frac{x}{2}+\frac{\arcsin x}{2}+c,\ x\in(-1;0)\cup(0;1),\ c\in R.$$

Riešené príklady – 089, 090

$\int \ln\left(x+\sqrt{x^2+1}\right)\,dx$

$\int x(x-a)(x-b)\,dx \qquad a,b \in R$

Riešené príklady – 089, 090

$$\int \ln\left(x+\sqrt{x^2+1}\right) dx$$

$$= \left[\begin{array}{l|l} u = \ln\left(x+\sqrt{x^2+1}\right) & u' = \left[\ln\left(x+(x^2+1)^{\frac{1}{2}}\right)\right]' = \frac{1+\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x}{x+\sqrt{x^2+1}} = \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}} \\ v' = 1 & v = x \end{array} \right]$$

$$\int x(x-a)(x-b)\, dx \qquad a,b \in R$$

$$= \int \left[x(x^2-ax-bx+ab\right] dx$$

Riešené príklady – 089, 090

$$\int \ln\left(x+\sqrt{x^2+1}\right) dx$$

$$= \left[\begin{array}{l|l} u=\ln\left(x+\sqrt{x^2+1}\right) & u'=\left[\ln\left(x+(x^2+1)^{\frac{1}{2}}\right)\right]' = \frac{1+\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x}{x+\sqrt{x^2+1}} = \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}} \\ v'=1 & v=x \end{array} \right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x\,dx}{\sqrt{x^2+1}}$$

$$\int x(x-a)(x-b)\,dx \qquad a,b\in R$$

$$= \int \left[x(x^2-ax-bx+ab\right] dx = \int \left[x^3-(a+b)x^2+abx\right] dx$$

Riešené príklady – 089, 090

$$\int \ln\left(x+\sqrt{x^2+1}\right) dx$$

$$= \left[\begin{array}{l|l} u=\ln\left(x+\sqrt{x^2+1}\right) & u'=\left[\ln\left(x+(x^2+1)^{\frac{1}{2}}\right)\right]' = \frac{1+\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x}{x+\sqrt{x^2+1}} = \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}} \\ v'=1 & v=x \end{array} \right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{l|l|l} \text{Subst. } t=x^2+1 & x\in(-\infty;0\rangle & x\in\langle 0;\infty) \\ dt=2x\,dx & t\in\langle 1;\infty) & t\in\langle 1;\infty) \end{array} \right]$$

$$\int x(x-a)(x-b)\,dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c \qquad a,b\in R$$

$$= \int \left[x(x^2-ax-bx+ab\right]dx = \int \left[x^3-(a+b)x^2+abx\right]dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 089, 090

$$\int \ln\left(x+\sqrt{x^2+1}\right) dx$$

$$= \left[\begin{array}{l|l} u = \ln\left(x+\sqrt{x^2+1}\right) & u' = \left[\ln\left(x+(x^2+1)^{\frac{1}{2}}\right)\right]' = \frac{1+\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x}{x+\sqrt{x^2+1}} = \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}} \\ v' = 1 & v = x \end{array} \right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{ll|l|l} \text{Subst.} & t=x^2+1 & x\in(-\infty;0\rangle & x\in\langle 0;\infty) \\ & dt=2x\,dx & t\in\langle 1;\infty) & t\in\langle 1;\infty) \end{array} \right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\int \frac{dt}{\sqrt{t}} = x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$\int x(x-a)(x-b)\,dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c \qquad a,b\in R$$

$$= \int \left[x(x^2-ax-bx+ab\right]dx = \int \left[x^3-(a+b)x^2+abx\right]dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 089, 090

$$\int \ln\left(x+\sqrt{x^2+1}\right)\,dx$$

$$= \left[\begin{array}{l|l} u=\ln\left(x+\sqrt{x^2+1}\right) & u'=\left[\ln\left(x+(x^2+1)^{\frac{1}{2}}\right)\right]' = \frac{1+\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x}{x+\sqrt{x^2+1}} = \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}} \\ v'=1 & v=x \end{array}\right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{l|l|l} \text{Subst. } t=x^2+1 & x\in(-\infty;0\rangle & x\in\langle 0;\infty) \\ dt=2x\,dx & t\in\langle 1;\infty) & t\in\langle 1;\infty) \end{array}\right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\int \frac{dt}{\sqrt{t}} = x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\int t^{-\frac{1}{2}}\,dt$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\int x(x-a)(x-b)\,dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c \qquad a,b\in R$$

$$= \int \left[x(x^2-ax-bx+ab\right]\,dx = \int \left[x^3-(a+b)x^2+abx\right]\,dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 089, 090

$$\int \ln\left(x+\sqrt{x^2+1}\right) dx = x\ln\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1} + c$$

$$= \left[\begin{array}{l|l} u=\ln\left(x+\sqrt{x^2+1}\right) & u'=\left[\ln\left(x+(x^2+1)^{\frac12}\right)\right]' = \frac{1+\frac12(x^2+1)^{-\frac12}\cdot 2x}{x+\sqrt{x^2+1}} = \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}} \\ v'=1 & v=x \end{array}\right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{x\,dx}{\sqrt{x^2+1}} = \left[\begin{array}{l|l|l} \text{Subst. } t=x^2+1 & x\in(-\infty;0) & x\in\langle 0;\infty) \\ dt=2x\,dx & t\in\langle 1;\infty) & t\in\langle 1;\infty) \end{array}\right]$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \frac12\int\frac{dt}{\sqrt t} = x\ln\left(x+\sqrt{x^2+1}\right) - \frac12\int t^{-\frac12}\,dt$$

$$= x\ln\left(x+\sqrt{x^2+1}\right) - \frac12\frac{t^{\frac12}}{\frac12} + c = x\ln\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1} + c,\ x\in R,\ c\in R.$$

$$\int x(x-a)(x-b)\,dx = \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c \qquad a,b\in R$$

$$= \int \left[x(x^2-ax-bx+ab\right]dx = \int\left[x^3-(a+b)x^2+abx\right]dx$$

$$= \frac{x^4}{4} - \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 091, 092

$\int |x| \, dx$

$\int \min\limits_{x\in(0;\infty)} \left\{1, \frac{1}{x}\right\} dx$ $\qquad x \in (0; 1\rangle$ $\qquad x \in \langle 1; \infty)$

Riešené príklady – 091, 092

$$\int |x| \, dx$$

$x \geq 0$ $= \int x \, dx$

$x \leq 0$ $= \int (-x) \, dx$

$$\int \min_{x\in(0;\infty)} \left\{1, \frac{1}{x}\right\} dx \qquad x\in(0;1\rangle \qquad x\in\langle 1;\infty)$$

$x\in(0;1\rangle$ $= \int dx$

$x\in\langle 1;\infty)$ $= \int \frac{dx}{x}$

Riešené príklady – 091, 092

$$\int |x|\, dx$$

$x \geq 0$: $= \int x\, dx = \frac{x^2}{2} + c$

$x \leq 0$: $= \int (-x)\, dx = -\frac{x^2}{2} + c$

$$\int \min_{x \in (0;\infty)} \left\{1, \frac{1}{x}\right\} dx \qquad x \in (0; 1\rangle \qquad x \in \langle 1; \infty)$$

$x \in (0; 1\rangle$: $= \int dx = x + c$

$x \in \langle 1; \infty)$: $= \int \frac{dx}{x} = \ln x + c_1$

Riešené príklady – 091, 092

$\int |x|\,dx$

$x\geq 0$ $\quad = \int x\,dx = \frac{x^2}{2} + c = \frac{x\cdot x}{2} + c$

$x\leq 0$ $\quad = \int (-x)\,dx = -\frac{x^2}{2} + c = \frac{x\cdot(-x)}{2} + c$

$\int \min\limits_{x\in(0;\infty)} \left\{1, \frac{1}{x}\right\} dx$ $\quad x\in(0;1\rangle$ $\quad x\in\langle 1;\infty)$

$x\in(0;1\rangle$ $\quad = \int dx = x + c$

$x\in\langle 1;\infty)$ $\quad = \int \frac{dx}{x} = \ln x + c_1$

[Neurčitý integrál na intervale tvoria spojité primitívne funkcie.]

Riešené príklady – 091, 092

$$\int |x|\,dx = \frac{x\cdot|x|}{2}+c$$

$x\geq 0$ $$= \int x\,dx = \frac{x^2}{2}+c = \frac{x\cdot x}{2}+c = \frac{x\cdot|x|}{2}+c,\ x\in\langle 0;\infty),\ c\in R.$$

$x\leq 0$ $$= \int (-x)\,dx = -\frac{x^2}{2}+c = \frac{x\cdot(-x)}{2}+c = \frac{x\cdot|x|}{2}+c,\ x\in(-\infty;0\rangle,\ c\in R.$$

$$\int \min_{x\in(0;\infty)} \left\{1,\frac{1}{x}\right\} dx \qquad x\in(0;1\rangle \qquad x\in\langle 1;\infty)$$

$x\in(0;1\rangle$ $$= \int dx = x+c$$

$x\in\langle 1;\infty)$ $$= \int \frac{dx}{x} = \ln x + c_1$$

[Neurčitý integrál na intervale tvoria spojité primitívne funkcie. Aby uvedené funkcie tvorili neurčitý integrál danej funkcie, musia byť spojité na celom $(0;\infty)$, t. j. aj v bode $x=1$.]

Riešené príklady – 091, 092

$$\int |x|\,dx = \frac{x\cdot|x|}{2}+c$$

$x\geq 0$: $$= \int x\,dx = \frac{x^2}{2}+c = \frac{x\cdot x}{2}+c = \frac{x\cdot|x|}{2}+c,\ x\in\langle 0;\infty),\ c\in R.$$

$x\leq 0$: $$= \int (-x)\,dx = -\frac{x^2}{2}+c = \frac{x\cdot(-x)}{2}+c = \frac{x\cdot|x|}{2}+c,\ x\in(-\infty;0\rangle,\ c\in R.$$

$$\int \min_{x\in(0;\infty)} \left\{1,\frac{1}{x}\right\} dx = x+c \text{ pre } x\in(0;1\rangle, \text{ resp. } 1+\ln x+c \text{ pre } x\in\langle 1;\infty).$$

$x\in(0;1\rangle$: $$= \int dx = x+c = x+c,\ x\in(0;1\rangle,\ c\in R.$$

$x\in\langle 1;\infty)$: $$= \int \frac{dx}{x} = \ln x+c_1 = 1+\ln x+c,\ x\in\langle 1;\infty),\ c,c_1\in R.$$

[Neurčitý integrál na intervale tvoria spojité primitívne funkcie. Aby uvedené funkcie tvorili neurčitý integrál danej funkcie, musia byť spojité na celom $(0;\infty)$, t. j. aj v bode $x=1$. $\Rightarrow$ Musí platiť $1+c=\ln 1+c_1$ pre všetky $c,c_1\in R$, t. j. $c_1=1+c$.]

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right]$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right]$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$=\left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right]=\int\frac{dt}{t(5+4t)}=\int\frac{dt}{4t^2+5t}$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$=\left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right]=\int\frac{dt}{t\sqrt{5+4t}}=\int\frac{4\,dt}{4t\sqrt{5+4t}}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst.}\ u=\sqrt{5+4t}=\sqrt{5+4\,e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array}\right]$$

Riešené príklady – 093, 094

$$\int \frac{\mathrm{d}x}{5+4\,\mathrm{e}^x}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\mathrm{e}^x & x\in R \\ x=\ln t, & \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\mathrm{d}t}{t(5+4t)} = \int \frac{\mathrm{d}t}{4t^2+5t} = \frac{1}{4}\int \frac{\mathrm{d}t}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{\mathrm{d}t}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{\mathrm{d}t}{(t+\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$\int \frac{\mathrm{d}x}{\sqrt{5+4\,\mathrm{e}^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\mathrm{e}^x & x\in R \\ x=\ln t, & \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\mathrm{d}t}{t\sqrt{5+4t}} = \int \frac{4\,\mathrm{d}t}{4t\sqrt{5+4t}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4\,\mathrm{e}^x} & t\in(0;\infty) \\ u^2=5+4t, \quad 2u\,\mathrm{d}u=4\,\mathrm{d}t & u\in\left(\sqrt{5};\infty\right) \end{array}\right]$$

$$= \int \frac{2u\,\mathrm{d}u}{(u^2-5)u}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{rl|l} \text{Subst.} & u=t+\frac{5}{8} & t\in(0;\infty) \\ & du=dt & u\in(\frac{5}{8};\infty) \end{array} \right]$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{rl|l} \text{Subst.} & u=\sqrt{5+4t}=\sqrt{5+4\,e^x} & t\in(0;\infty) \\ & u^2=5+4t, \quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array} \right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5}$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{rl|l} \text{Subst.} & u=t+\frac{5}{8} & t\in(0;\infty) \\ & du=dt & u\in(\frac{5}{8};\infty) \end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2}$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{rl|l} \text{Subst.} & u=\sqrt{5+4t}=\sqrt{5+4\,e^x} & t\in(0;\infty) \\ & u^2=5+4t, \quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right| + c$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{rl|l} \text{Subst.} & u=t+\frac{5}{8} & t\in(0;\infty) \\ & du=dt & u\in(\frac{5}{8};\infty) \end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}} \ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}} + c_1$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right| + c$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{rl|l} \text{Subst.} & u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ & u^2=5+4t, \quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right| + c = \frac{1}{\sqrt{5}} \ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right| + c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{r|l} \text{Subst. } u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty)\end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4\,e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty)\end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t(5+4t)} = \int\frac{dt}{4t^2+5t} = \frac{1}{4}\int\frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int\frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int\frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{r|l} \text{Subst. } u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty)\end{array}\right] = \frac{1}{4}\int\frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1$$

$$\int\frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t\sqrt{5+4t}} = \int\frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty)\end{array}\right]$$

$$= \int\frac{2u\,du}{(u^2-5)u} = \int\frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x}$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{r|l} \text{Subst. } u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty) \end{array} \right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}} \ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}} + c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}} + c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}} + c_1 = \frac{1}{5}\ln\frac{4t}{4t+5} + c_1$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}} \ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right| + c$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{r|l} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ u^2=5+4t, \quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array} \right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}} \ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right| + c = \frac{1}{\sqrt{5}} \ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right| + c, \ x\in R, \ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{ll|l} \text{Subst.} & u=t+\frac{5}{8} & t\in(0;\infty) \\ & du=dt & u\in(\frac{5}{8};\infty)\end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ \ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{ll|l} \text{Subst.} & u=\sqrt{5+4t}=\sqrt{5+4\,e^x} & t\in(0;\infty) \\ & u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty)\end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t(5+4t)} = \int\frac{dt}{4t^2+5t} = \frac{1}{4}\int\frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int\frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int\frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{l|l} \text{Subst.}\ u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty)\end{array}\right] = \frac{1}{4}\int\frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$= \left[c=\frac{\ln 4}{5}+c_1,\ c\in R,\ c_1\in R\right]$$

$$\int\frac{dx}{\sqrt{5+4\,e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t\sqrt{5+4t}} = \int\frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst.}\ u=\sqrt{5+4t}=\sqrt{5+4\,e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty)\end{array}\right]$$

$$= \int\frac{2u\,du}{(u^2-5)u} = \int\frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5}\ln(4e^x+5)+c$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{c|c} \text{Subst.} \ u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty) \end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$= \left[c=\frac{\ln 4}{5}+c_1,\ c\in R,\ c_1\in R\right] = \frac{x}{5}-\frac{1}{5}\ln(4e^x+5)+c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{c|c} \text{Subst.} \ u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ u^2=5+4t, \quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5}\ln(4e^x+5)+c$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int\frac{dt}{t(5+4t)} = \int\frac{dt}{4t^2+5t} = \frac{1}{4}\int\frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int\frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int\frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{c|c} \text{Subst.}\ u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty) \end{array}\right] = \frac{1}{4}\int\frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$= \left[c=\frac{\ln 4}{5}+c_1,\ c\in R,\ c_1\in R\right] = \frac{x}{5}-\frac{1}{5}\ln(4e^x+5)+c,\ x\in R,\ c\in R.$$

$$\int\frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int\frac{dt}{t\sqrt{5+4t}} = \int\frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{c|c} \text{Subst.}\ u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array}\right]$$

$$= \int\frac{2u\,du}{(u^2-5)u} = \int\frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5}\ln(4e^x+5)+c$$

$$= \left[\begin{array}{c|c} \text{Subst. } \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{c|c} \text{Subst. } u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty) \end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ \ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$= \left[c=\frac{\ln 4}{5}+c_1,\ c\in R,\ c_1\in R\right] = \frac{x}{5}-\frac{1}{5}\ln(4e^x+5)+c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt5}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt5}{\sqrt{5+4e^x}+\sqrt5}\right|+c$$

$$= \left[\begin{array}{c|c} \text{Subst. } \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{c|c} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt5;\infty) \end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt5}\ln\left|\frac{u-\sqrt5}{u+\sqrt5}\right|+c = \frac{1}{\sqrt5}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt5}{\sqrt{5+4e^x}+\sqrt5}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4\,e^x} = \frac{x}{5} - \frac{1}{5}\ln(4\,e^x+5)+c$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t(5+4t)} = \int\frac{dt}{4t^2+5t} = \frac{1}{4}\int\frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int\frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int\frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{rl|l} \text{Subst.} & u=t+\frac{5}{8} & t\in(0;\infty) \\ & du=dt & u\in(\frac{5}{8};\infty)\end{array}\right] = \frac{1}{4}\int\frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$= \left[c=\frac{\ln 4}{5}+c_1,\ c\in R,\ c_1\in R\right] = \frac{x}{5}-\frac{1}{5}\ln(4\,e^x+5)+c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{5+4\,e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{rl|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int\frac{dt}{t\sqrt{5+4t}} = \int\frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{rl|l} \text{Subst.} & u=\sqrt{5+4t}=\sqrt{5+4\,e^x} & t\in(0;\infty) \\ & u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty)\end{array}\right]$$

$$= \int\frac{2u\,du}{(u^2-5)u} = \int\frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4\,e^x}-\sqrt{5}}{\sqrt{5+4\,e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5}\ln(4e^x+5)+c$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{c|c} \text{Subst. } u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty)\end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$= \left[c=\frac{\ln 4}{5}+c_1,\ c\in R,\ c_1\in R\right] = \frac{x}{5}-\frac{1}{5}\ln(4e^x+5)+c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{c|c} \text{Subst. } t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty)\end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{c|c} \text{Subst. } u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty)\end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 093, 094

$$\int \frac{dx}{5+4e^x} = \frac{x}{5} - \frac{1}{5}\ln(4e^x+5)+c$$

$$= \left[\begin{array}{c|c} \text{Subst.}\quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t(5+4t)} = \int \frac{dt}{4t^2+5t} = \frac{1}{4}\int \frac{dt}{t^2+\frac{5t}{4}} = \frac{1}{4}\int \frac{dt}{t^2+2\cdot\frac{5t}{8}+(\frac{5}{8})^2-(\frac{5}{8})^2}$$

$$= \frac{1}{4}\int \frac{dt}{(t+\frac{5}{8})^2-(\frac{5}{8})^2} = \left[\begin{array}{c|c} \text{Subst.}\ u=t+\frac{5}{8} & t\in(0;\infty) \\ du=dt & u\in(\frac{5}{8};\infty) \end{array}\right] = \frac{1}{4}\int \frac{du}{u^2-(\frac{5}{8})^2} = \frac{1}{4}\cdot\frac{1}{2\cdot\frac{5}{8}}\ln\frac{u-\frac{5}{8}}{u+\frac{5}{8}}+c_1$$

$$= \frac{1}{5}\ln\frac{t+\frac{5}{8}-\frac{5}{8}}{t+\frac{5}{8}+\frac{5}{8}}+c_1 = \frac{1}{5}\ln\frac{t}{t+\frac{5}{4}}+c_1 = \frac{1}{5}\ln\frac{4t}{4t+5}+c_1 = \frac{\ln 4}{5}+\frac{\ln t}{5}-\frac{1}{5}\ln(4t+5)+c_1$$

$$= \left[c=\frac{\ln 4}{5}+c_1,\ c\in R,\ c_1\in R \right] = \frac{x}{5}-\frac{1}{5}\ln(4e^x+5)+c,\ x\in R,\ c\in R.$$

$$\int \frac{dx}{\sqrt{5+4e^x}} = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c$$

$$= \left[\begin{array}{c|c} \text{Subst.}\quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \int \frac{4\,dt}{4t\sqrt{5+4t}} = \left[\begin{array}{c|c} \text{Subst.}\ u=\sqrt{5+4t}=\sqrt{5+4e^x} & t\in(0;\infty) \\ u^2=5+4t,\quad 2u\,du=4\,dt & u\in(\sqrt{5};\infty) \end{array}\right]$$

$$= \int \frac{2u\,du}{(u^2-5)u} = \int \frac{2\,du}{u^2-5} = \frac{2}{2\sqrt{5}}\ln\left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right|+c = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5+4e^x}-\sqrt{5}}{\sqrt{5+4e^x}+\sqrt{5}}\right|+c,\ x\in R,\ c\in R.$$

Riešené príklady – 094

$$\int \frac{\mathrm{d}x}{\sqrt{5+4\,\mathrm{e}^{x}}}$$

Riešené príklady – 094

$$\int \frac{\mathrm{d}x}{\sqrt{5+4\,\mathrm{e}^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\mathrm{e}^x & x\in R \\ x=\ln t, & \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array}\right]$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t = e^x & x \in R \\ x = \ln t, & dx = \frac{dt}{t} & t \in (0;\infty) \end{array} \right] = \int \frac{dt}{t\sqrt{5+4t}}$$

Riešené príklady – 094

$$\int \frac{\mathrm{d}x}{\sqrt{5+4\,\mathrm{e}^x}}$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=\mathrm{e}^x & x\in R \\ x=\ln t,\ \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\mathrm{d}t}{t\sqrt{5+4t}} = \left[\begin{array}{c|c} \text{Subst.}\ u^2=4t & t\in(0;\infty) \\ 2u\,\mathrm{d}u=4\,\mathrm{d}t & u\in(0;\infty) \end{array}\right]$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^{x}}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^{x} & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst. } u^{2}=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^{2}}{4}\sqrt{5+u^{2}}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{r|l} \text{Subst. } u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t = e^x & x \in R \\ x = \ln t, & dx = \frac{dt}{t} & t \in (0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst. } u^2 = 4t & t \in (0;\infty) \\ 2u\,du = 4\,dt & u \in (0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } v = \frac{1}{u},\ u = \frac{1}{v} = v^{-1} & \sqrt{5+u^2} = \sqrt{5+\frac{1}{v^2}} = \frac{\sqrt{5v^2+1}}{v} & u \in (0;\infty) \\ du = -v^{-2}\,dv = -\frac{dv}{v^2} & & v \in (0;\infty) \end{array}\right]$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst.}\ u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst.}\ u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}}$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst. } u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}}\ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{ll|l} \text{Subst.} & u^2=4t & t\in(0;\infty) \\ & 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}}\ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1 = -\frac{2}{\sqrt{5}}\ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right]+c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst. } u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}}\ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1 = -\frac{2}{\sqrt{5}}\ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right]+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u}+c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst. } u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } v=\frac{1}{u},\; u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}} \int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1 = -\frac{2}{\sqrt{5}} \ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right]+c_1$$

$$= -\frac{2}{\sqrt{5}} \ln\frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u}+c_1 = -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+u^2}\right) + \frac{2}{\sqrt{5}} \ln\sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst.}\ u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}}\ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1 = -\frac{2}{\sqrt{5}}\ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right]+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u}+c_1 = -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+u^2}\right)+\frac{2}{\sqrt{5}}\ln\sqrt{5}+\frac{2}{\sqrt{5}}\ln u+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+u^2}\right)+\frac{\ln 5}{\sqrt{5}}+\frac{\ln u^2}{\sqrt{5}}+c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{l|l} \text{Subst.}\quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst.}\ u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}}\ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1 = -\frac{2}{\sqrt{5}}\ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right]+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u}+c_1 = -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+u^2}\right)+\frac{2}{\sqrt{5}}\ln\sqrt{5}+\frac{2}{\sqrt{5}}\ln u+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+u^2}\right)+\frac{\ln 5}{\sqrt{5}}+\frac{\ln u^2}{\sqrt{5}}+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+4t}\right)+\frac{\ln 5}{\sqrt{5}}+\frac{\ln(4t)}{\sqrt{5}}+c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst.}\ u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}}\ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1 = -\frac{2}{\sqrt{5}}\ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right]+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u}+c_1 = -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+u^2}\right)+\frac{2}{\sqrt{5}}\ln\sqrt{5}+\frac{2}{\sqrt{5}}\ln u+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+u^2}\right)+\frac{\ln 5}{\sqrt{5}}+\frac{\ln u^2}{\sqrt{5}}+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+4t}\right)+\frac{\ln 5}{\sqrt{5}}+\frac{\ln(4t)}{\sqrt{5}}+c_1$$

$$= -\frac{2}{\sqrt{5}}\ln\left(\sqrt{5}+\sqrt{5+4t}\right)+\frac{\ln 5}{\sqrt{5}}+\frac{\ln 4}{\sqrt{5}}+\frac{\ln t}{\sqrt{5}}+c_1$$

Riešené príklady – 094

$$\int \frac{dx}{\sqrt{5+4\,e^x}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{dt}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst. } u^2=4t & t\in(0;\infty) \\ 2u\,du=4\,dt & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,du}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,du}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ du=-v^{-2}\,dv=-\frac{dv}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,dv}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,dv}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{dv}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln\left[v+\sqrt{v^2+\frac{1}{5}}\right]+c_1 = -\frac{2}{\sqrt{5}} \ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right]+c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u}+c_1 = -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+u^2}\right) + \frac{2}{\sqrt{5}}\ln\sqrt{5} + \frac{2}{\sqrt{5}}\ln u+c_1$$

$$= -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+u^2}\right) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln u^2}{\sqrt{5}}+c_1$$

$$= -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+4t}\right) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln(4t)}{\sqrt{5}}+c_1$$

$$= -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+4t}\right) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 4}{\sqrt{5}} + \frac{\ln t}{\sqrt{5}}+c_1 = \left[c=\frac{\ln 5}{\sqrt{5}}+\frac{\ln 4}{\sqrt{5}}+c_1,\ c\in R,\ c_1\in R\right]$$

Riešené príklady – 094

$$\int \frac{\mathrm{d}x}{\sqrt{5+4\,\mathrm{e}^x}} = \frac{x}{\sqrt{5}} - \frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+4\,\mathrm{e}^x}\right) + c$$

$$= \left[\begin{array}{l|l} \text{Subst. } t=\mathrm{e}^x & x\in R \\ x=\ln t,\ \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\mathrm{d}t}{t\sqrt{5+4t}} = \left[\begin{array}{l|l} \text{Subst. } u^2=4t & t\in(0;\infty) \\ 2u\,\mathrm{d}u=4\,\mathrm{d}t & u\in(0;\infty) \end{array}\right] = \int \frac{\frac{u\,\mathrm{d}u}{2}}{\frac{u^2}{4}\sqrt{5+u^2}} = \int \frac{2\,\mathrm{d}u}{u\sqrt{5+u^2}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } v=\frac{1}{u},\ u=\frac{1}{v}=v^{-1} & \sqrt{5+u^2}=\sqrt{5+\frac{1}{v^2}}=\frac{\sqrt{5v^2+1}}{v} & u\in(0;\infty) \\ \mathrm{d}u=-v^{-2}\,\mathrm{d}v=-\frac{\mathrm{d}v}{v^2} & \sqrt{v^2+\frac{1}{5}}=\sqrt{\frac{1}{u^2}+\frac{1}{5}}=\frac{\sqrt{5+u^2}}{\sqrt{5}u} & v\in(0;\infty) \end{array}\right] = \int \frac{-\frac{2\,\mathrm{d}v}{v^2}}{\frac{1}{v}\,\frac{\sqrt{5v^2+1}}{v}} = \int \frac{-2\,\mathrm{d}v}{\sqrt{5v^2+1}}$$

$$= -\frac{2}{\sqrt{5}}\int \frac{\mathrm{d}v}{\sqrt{v^2+\frac{1}{5}}} = -\frac{2}{\sqrt{5}} \ln\left[v+\sqrt{v^2+\frac{1}{5}}\right] + c_1 = -\frac{2}{\sqrt{5}} \ln\left[\frac{1}{u}+\frac{\sqrt{5+u^2}}{\sqrt{5}u}\right] + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln \frac{\sqrt{5}+\sqrt{5+u^2}}{\sqrt{5}u} + c_1 = -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+u^2}\right) + \frac{2}{\sqrt{5}} \ln\sqrt{5} + \frac{2}{\sqrt{5}} \ln u + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+u^2}\right) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln u^2}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+4t}\right) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln(4t)}{\sqrt{5}} + c_1$$

$$= -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+4t}\right) + \frac{\ln 5}{\sqrt{5}} + \frac{\ln 4}{\sqrt{5}} + \frac{\ln t}{\sqrt{5}} + c_1 = \left[c=\frac{\ln 5}{\sqrt{5}}+\frac{\ln 4}{\sqrt{5}}+c_1,\ c\in R,\ c_1\in R\right]$$

$$= -\frac{2}{\sqrt{5}} \ln\left(\sqrt{5}+\sqrt{5+4\,\mathrm{e}^x}\right) + \frac{x}{\sqrt{5}} + c,\ x\in R,\ c\in R.$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x} + e^x + 1}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t = e^x & x \in R \\ x = \ln t, & dx = \frac{dt}{t} & t \in (0; \infty) \end{array} \right]$$

Riešené príklady – 095

$$\int \frac{\mathrm{d}x}{\sqrt{\mathrm{e}^{2x}+\mathrm{e}^{x}+1}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=\mathrm{e}^{x} & x\in R \\ x=\ln t, & \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{\mathrm{d}t}{t}}{\sqrt{t^2+t+1}} = \int \frac{\mathrm{d}t}{t\sqrt{t^2+t+1}}$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^{x}+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^{x} & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & & u\in(0;\infty) \end{array} \right]$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^{x}+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^{x} & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & & u\in(0;\infty) \end{array} \right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}}$$

Riešené príklady – 095

$$\int \frac{\mathrm{d}x}{\sqrt{\mathrm{e}^{2x}+\mathrm{e}^{x}+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=\mathrm{e}^{x} & x\in R \\ x=\ln t,\ \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{\mathrm{d}t}{t}}{\sqrt{t^2+t+1}} = \int \frac{\mathrm{d}t}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ \mathrm{d}t=-u^{-2}\,\mathrm{d}u=-\frac{\mathrm{d}u}{u^2} & & u\in(0;\infty) \end{array} \right] = \int \frac{-\frac{\mathrm{d}u}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,\mathrm{d}u}{\sqrt{1+u+u^2}}$$

Riešené príklady – 095

$$\int \frac{\mathrm{d}x}{\sqrt{\mathrm{e}^{2x}+\mathrm{e}^{x}+1}}$$

$$= \left[\begin{array}{c|c} \text{Subst.} \quad t=\mathrm{e}^{x} & x\in R \\ x=\ln t,\ \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\frac{\mathrm{d}t}{t}}{\sqrt{t^2+t+1}} = \int \frac{\mathrm{d}t}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{c|c|c} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ \mathrm{d}t=-u^{-2}\,\mathrm{d}u=-\frac{\mathrm{d}u}{u^2} & & u\in(0;\infty) \end{array}\right] = \int \frac{-\frac{\mathrm{d}u}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,\mathrm{d}u}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{\mathrm{d}u}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{\mathrm{d}u}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}}$$

Riešené príklady – 095

$$\int \frac{\mathrm{d}x}{\sqrt{e^{2x}+e^{x}+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ \mathrm{d}x=\frac{\mathrm{d}t}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{\mathrm{d}t}{t}}{\sqrt{t^2+t+1}} = \int \frac{\mathrm{d}t}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ \mathrm{d}t=-u^{-2}\,\mathrm{d}u=-\frac{\mathrm{d}u}{u^2} & & u\in(0;\infty) \end{array} \right] = \int \frac{-\frac{\mathrm{d}u}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,\mathrm{d}u}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{\mathrm{d}u}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{\mathrm{d}u}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{l|l} \text{Subst.}\quad v=u+\frac{1}{2} & u\in(0;\infty) \\ \mathrm{d}v=\mathrm{d}u & v\in(\frac{1}{2};\infty) \end{array} \right]$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & & u\in(0;\infty) \end{array} \right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{ll|l} \text{Subst.} & v=u+\frac{1}{2} & u\in(0;\infty) \\ & dv=du & v\in\left(\frac{1}{2};\infty\right) \end{array} \right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & & u\in(0;\infty) \end{array}\right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{l|l} \text{Subst.}\quad v=u+\frac{1}{2} & u\in(0;\infty) \\ dv=du & v\in(\frac{1}{2};\infty) \end{array}\right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right) + c_1$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^{x}+1}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^{x} & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & & u\in(0;\infty) \end{array} \right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{ll|l} \text{Subst.} & v=u+\frac{1}{2} & u\in(0;\infty) \\ & dv=du & v\in(\frac{1}{2};\infty) \end{array} \right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right)+c_1 = -\ln\left(u+\frac{1}{2}+\sqrt{\left(u+\frac{1}{2}\right)^2+\frac{3}{4}}\right)+c_1$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & & u\in(0;\infty) \end{array} \right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{l|l} \text{Subst.} \quad v=u+\frac{1}{2} & u\in(0;\infty) \\ dv=du & v\in(\frac{1}{2};\infty) \end{array} \right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right)+c_1 = -\ln\left(u+\frac{1}{2}+\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}\right)+c_1$$

$$= -\ln\left(u+\frac{1}{2}+\sqrt{1+u+u^2}\right)+c_1$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & \sqrt{1+u+u^2}=\sqrt{1+\frac{1}{t}+\frac{1}{t^2}}=\frac{\sqrt{t^2+t+1}}{t} & u\in(0;\infty) \end{array}\right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{l|l} \text{Subst.} \quad v=u+\frac{1}{2} & u\in(0;\infty) \\ dv=du & v\in(\frac{1}{2};\infty) \end{array}\right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right)+c_1 = -\ln\left(u+\frac{1}{2}+\sqrt{\left(u+\frac{1}{2}\right)^2+\frac{3}{4}}\right)+c_1$$

$$= -\ln\left(u+\frac{1}{2}+\sqrt{1+u+u^2}\right)+c_1 = -\ln\left(\frac{1}{t}+\frac{1}{2}+\frac{\sqrt{t^2+t+1}}{t}\right)+c_1$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$$

$$= \left[\begin{array}{ll|l} \text{Subst.} & t=e^x & x\in R \\ x=\ln t, & dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst. } u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & \sqrt{1+u+u^2}=\sqrt{1+\frac{1}{t}+\frac{1}{t^2}}=\frac{\sqrt{t^2+t+1}}{t} & u\in(0;\infty) \end{array}\right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{ll|l} \text{Subst.} & v=u+\frac{1}{2} & u\in(0;\infty) \\ & dv=du & v\in(\frac{1}{2};\infty) \end{array}\right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right)+c_1 = -\ln\left(u+\frac{1}{2}+\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}\right)+c_1$$

$$= -\ln\left(u+\frac{1}{2}+\sqrt{1+u+u^2}\right)+c_1 = -\ln\left(\frac{1}{t}+\frac{1}{2}+\frac{\sqrt{t^2+t+1}}{t}\right)+c_1$$

$$= -\ln\frac{2+t+2\sqrt{t^2+t+1}}{2t}+c_1$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^{x}+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^{x} & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array} \right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & \sqrt{1+u+u^2}=\sqrt{1+\frac{1}{t}+\frac{1}{t^2}}=\frac{\sqrt{t^2+t+1}}{t} & u\in(0;\infty) \end{array} \right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{l|l} \text{Subst.} \quad v=u+\frac{1}{2} & u\in(0;\infty) \\ dv=du & v\in(\frac{1}{2};\infty) \end{array} \right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right)+c_1 = -\ln\left(u+\frac{1}{2}+\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}\right)+c_1$$

$$= -\ln\left(u+\frac{1}{2}+\sqrt{1+u+u^2}\right)+c_1 = -\ln\left(\frac{1}{t}+\frac{1}{2}+\frac{\sqrt{t^2+t+1}}{t}\right)+c_1$$

$$= -\ln\frac{2+t+2\sqrt{t^2+t+1}}{2t}+c_1 = -\ln\left(2+t+2\sqrt{t^2+t+1}\right)+\ln 2+\ln t+c_1$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^x+1}}$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^x & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & \sqrt{1+u+u^2}=\sqrt{1+\frac{1}{t}+\frac{1}{t^2}}=\frac{\sqrt{t^2+t+1}}{t} & u\in(0;\infty) \end{array}\right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{l|l} \text{Subst.} \quad v=u+\frac{1}{2} & u\in(0;\infty) \\ dv=du & v\in\left(\frac{1}{2};\infty\right) \end{array}\right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right)+c_1 = -\ln\left(u+\frac{1}{2}+\sqrt{\left(u+\frac{1}{2}\right)^2+\frac{3}{4}}\right)+c_1$$

$$= -\ln\left(u+\frac{1}{2}+\sqrt{1+u+u^2}\right)+c_1 = -\ln\left(\frac{1}{t}+\frac{1}{2}+\frac{\sqrt{t^2+t+1}}{t}\right)+c_1$$

$$= -\ln\frac{2+t+2\sqrt{t^2+t+1}}{2t}+c_1 = -\ln\left(2+t+2\sqrt{t^2+t+1}\right)+\ln 2+\ln t+c_1$$

$$= \left[\begin{array}{l} c=c_1+\ln 2 \\ c\in R,\ c_1\in R \end{array}\right]$$

Riešené príklady – 095

$$\int \frac{dx}{\sqrt{e^{2x}+e^{x}+1}} = x - \ln\left(2+e^{x}+2\sqrt{e^{2x}+e^{x}+1}\right) + c$$

$$= \left[\begin{array}{l|l} \text{Subst.} \quad t=e^{x} & x\in R \\ x=\ln t,\ dx=\frac{dt}{t} & t\in(0;\infty) \end{array}\right] = \int \frac{\frac{dt}{t}}{\sqrt{t^2+t+1}} = \int \frac{dt}{t\sqrt{t^2+t+1}}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ u=\frac{1}{t},\ t=\frac{1}{u}=u^{-1} & \sqrt{t^2+t+1}=\sqrt{\frac{1}{u^2}+\frac{1}{u}+1}=\frac{\sqrt{1+u+u^2}}{u} & t\in(0;\infty) \\ dt=-u^{-2}\,du=-\frac{du}{u^2} & \sqrt{1+u+u^2}=\sqrt{1+\frac{1}{t}+\frac{1}{t^2}}=\frac{\sqrt{t^2+t+1}}{t} & u\in(0;\infty) \end{array}\right] = \int \frac{-\frac{du}{u^2}}{\frac{1}{u}\,\frac{\sqrt{1+u+u^2}}{u}} = \int \frac{-\,du}{\sqrt{1+u+u^2}}$$

$$= -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+1-\frac{1}{4}}} = -\int \frac{du}{\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}} = \left[\begin{array}{l|l} \text{Subst.} \quad v=u+\frac{1}{2} & u\in(0;\infty) \\ dv=du & v\in\left(\frac{1}{2};\infty\right) \end{array}\right] = -\int \frac{dv}{\sqrt{v^2+\frac{3}{4}}}$$

$$= -\ln\left(v+\sqrt{v^2+\frac{3}{4}}\right) + c_1 = -\ln\left(u+\frac{1}{2}+\sqrt{(u+\frac{1}{2})^2+\frac{3}{4}}\right) + c_1$$

$$= -\ln\left(u+\frac{1}{2}+\sqrt{1+u+u^2}\right) + c_1 = -\ln\left(\frac{1}{t}+\frac{1}{2}+\frac{\sqrt{t^2+t+1}}{t}\right) + c_1$$

$$= -\ln\frac{2+t+2\sqrt{t^2+t+1}}{2t} + c_1 = -\ln\left(2+t+2\sqrt{t^2+t+1}\right) + \ln 2 + \ln t + c_1$$

$$= \left[\begin{array}{l} c=c_1+\ln 2 \\ c\in R,\ c_1\in R \end{array}\right] = x - \ln\left(2+e^{x}+2\sqrt{e^{2x}+e^{x}+1}\right) + c,\ x\in R,\ c\in R.$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\, dx$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \qquad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array} \right]$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array}\right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & & u\in\langle 0;1) \end{array} \right]$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & & u\in\langle 0;1) \end{array} \right] = \int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array}\right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ \quad dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & & u\in\langle 0;1) \end{array}\right] = \int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$= \int \frac{-4u^2\,du}{(1+u^2)(1-u^2)} = \int \frac{4u^2\,du}{(1+u^2)(u^2-1)}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{ll|l|l} \text{Subst.} & t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t, & dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\ \ u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ \ u^2+u^2t=1-t,\ \ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & & u\in\langle 0;1) \end{array} \right] = \int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$= \int \frac{-4u^2\,du}{(1+u^2)(1-u^2)} = \int \frac{4u^2\,du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2)\,du}{(u^2-1)(u^2+1)}$$

Riešené príklady – 096

$$\int\sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$=\left[\begin{array}{l|l|l} \text{Subst.}\quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array}\right]=\int\sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t}=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$=\left[\begin{array}{l|l|l} \text{Subst.}\quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & & u\in\langle 0;1) \end{array}\right]=\int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$=\int\frac{-4u^2\,du}{(1+u^2)(1-u^2)}=\int\frac{4u^2\,du}{(1+u^2)(u^2-1)}=\int\frac{(2u^2-2+2u^2+2)\,du}{(u^2-1)(u^2+1)}=\int\left[\frac{2}{u^2+1}+\frac{2}{u^2-1}\right]du$$

Riešené príklady – 096

$$\int\sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$=\left[\begin{array}{l|l|l}\text{Subst.}\quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle\end{array}\right]=\int\sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t}=\int\frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$=\left[\begin{array}{l|l|l}\text{Subst.}\quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & & u\in\langle 0;1)\end{array}\right]=\int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$=\int\frac{-4u^2\,du}{(1+u^2)(1-u^2)}=\int\frac{4u^2\,du}{(1+u^2)(u^2-1)}=\int\frac{(2u^2-2+2u^2+2)\,du}{(u^2-1)(u^2+1)}=\int\left[\frac{2}{u^2+1}+\frac{2}{u^2-1}\right]du$$

$$=2\,\mathrm{arctg}\,u+\frac{2}{2}\ln\left|\frac{u-1}{u+1}\right|+c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\, dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array} \right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & 1\pm u=1\pm\frac{\sqrt{1-t}}{\sqrt{1+t}}=\frac{\sqrt{1+t}\pm\sqrt{1-t}}{\sqrt{1+t}}>0 & u\in\langle 0;1) \\ \multicolumn{3}{l}{\frac{1-u}{1+u}=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}\cdot\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}-\sqrt{1-t}}=\frac{1+t-2\sqrt{1-t^2}+1-t}{1+t-(1-t)}=\frac{2-2\sqrt{1-t^2}}{2t}=\frac{1-\sqrt{1-t^2}}{t}>0} \end{array} \right] = \int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$= \int \frac{-4u^2\,du}{(1+u^2)(1-u^2)} = \int \frac{4u^2\,du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2)\,du}{(u^2-1)(u^2+1)} = \int \left[\frac{2}{u^2+1}+\frac{2}{u^2-1}\right] du$$

$$= 2\,\mathrm{arctg}\, u + \frac{2}{2}\ln\left|\frac{u-1}{u+1}\right| + c = 2\,\mathrm{arctg}\sqrt{\frac{1-t}{1+t}} + \ln\frac{1-\sqrt{1-t^2}}{t} + c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array}\right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.}\quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & 1\pm u=1\pm\frac{\sqrt{1-t}}{\sqrt{1+t}}=\frac{\sqrt{1+t}\pm\sqrt{1-t}}{\sqrt{1+t}}>0 & u\in\langle 0;1) \\ \multicolumn{3}{l}{\frac{1-u}{1+u}=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}\cdot\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}-\sqrt{1-t}}=\frac{1+t-2\sqrt{1-t^2}+1-t}{1+t-(1-t)}=\frac{2-2\sqrt{1-t^2}}{2t}=\frac{1-\sqrt{1-t^2}}{t}>0} \end{array}\right] = \int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$=\int\frac{-4u^2\,du}{(1+u^2)(1-u^2)}=\int\frac{4u^2\,du}{(1+u^2)(u^2-1)}=\int\frac{(2u^2-2+2u^2+2)\,du}{(u^2-1)(u^2+1)}=\int\left[\frac{2}{u^2+1}+\frac{2}{u^2-1}\right]du$$

$$=2\,\text{arctg}\,u+\frac{2}{2}\ln\left|\frac{u-1}{u+1}\right|+c=2\,\text{arctg}\sqrt{\frac{1-t}{1+t}}+\ln\frac{1-\sqrt{1-t^2}}{t}+c$$

$$=2\,\text{arctg}\sqrt{\frac{1-t}{1+t}}+\ln\left(1-\sqrt{1-t^2}\right)-\ln t+c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx = 2\,\mathrm{arctg}\sqrt{\frac{1-e^x}{1+e^x}} + \ln\left(1-\sqrt{1-e^{2x}}\right) - x + c$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad t=e^x & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ x=\ln t,\ dx=\frac{dt}{t} & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in(0;1\rangle \end{array}\right] = \int \sqrt{\frac{1-t}{1+t}}\,\frac{dt}{t} = \int \frac{\sqrt{1-t}}{\sqrt{1+t}}\,\frac{dt}{t}$$

$$= \left[\begin{array}{l|l|l} \text{Subst.} \quad u=\sqrt{\frac{1-t}{1+t}}=\sqrt{\frac{2-1-t}{1+t}}=\sqrt{\frac{2}{1+t}-1} & u^2=\frac{1-t}{1+t},\ u^2+u^2t=1-t,\ t=\frac{1-u^2}{1+u^2} & t\in(0;1\rangle \\ \qquad dt=\frac{-2u(1+u^2)-2u(1-u^2)}{(1+u^2)^2}\,du=\frac{-4u\,du}{(1+u^2)^2} & 1\pm u=1\pm\frac{\sqrt{1-t}}{\sqrt{1+t}}=\frac{\sqrt{1+t}\pm\sqrt{1-t}}{\sqrt{1+t}}>0 & u\in\langle 0;1) \\ \multicolumn{3}{l}{\frac{1-u}{1+u}=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}=\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}\cdot\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}-\sqrt{1-t}}=\frac{1+t-2\sqrt{1-t^2}+1-t}{1+t-(1-t)}=\frac{2-2\sqrt{1-t^2}}{2t}=\frac{1-\sqrt{1-t^2}}{t}>0} \end{array}\right] = \int u\cdot\frac{\frac{-4u\,du}{(1+u^2)^2}}{\frac{1-u^2}{1+u^2}}$$

$$= \int \frac{-4u^2\,du}{(1+u^2)(1-u^2)} = \int \frac{4u^2\,du}{(1+u^2)(u^2-1)} = \int \frac{(2u^2-2+2u^2+2)\,du}{(u^2-1)(u^2+1)} = \int \left[\frac{2}{u^2+1}+\frac{2}{u^2-1}\right]du$$

$$= 2\,\mathrm{arctg}\,u + \frac{2}{2}\ln\left|\frac{u-1}{u+1}\right| + c = 2\,\mathrm{arctg}\sqrt{\frac{1-t}{1+t}} + \ln\frac{1-\sqrt{1-t^2}}{t} + c$$

$$= 2\,\mathrm{arctg}\sqrt{\frac{1-t}{1+t}} + \ln\left(1-\sqrt{1-t^2}\right) - \ln t + c$$

$$= 2\,\mathrm{arctg}\sqrt{\frac{1-e^x}{1+e^x}} + \ln\left(1-\sqrt{1-e^{2x}}\right) - x + c,\ x\in(-\infty;0\rangle,\ c\in R.$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\, dx$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{r|l|l|l} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array} \right]$$

Riešené príklady – 096

$$\int\sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array}\right]$$

$$= \int\left[\frac{2t^2}{t^2-1}-\frac{2t^2}{t^2+1}\right]dt$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{r|l|l|l} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1}-\frac{2t^2}{t^2+1}\right]dt = \int \left[\frac{2t^2-2+2}{t^2-1}-\frac{2t^2+2-2}{t^2+1}\right]dt$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt$$

Riešené príklady – 096

$$\int\sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{c|c|c|c} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array}\right]$$

$$=\int\left[\frac{2t^2}{t^2-1}-\frac{2t^2}{t^2+1}\right]dt=\int\left[\frac{2t^2-2+2}{t^2-1}-\frac{2t^2+2-2}{t^2+1}\right]dt$$

$$=\int\left[2+\frac{2}{t^2-1}-2+\frac{2}{t^2+1}\right]dt=2\int\frac{dt}{t^2-1}+2\int\frac{dt}{t^2+1}$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\, dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \ \Rightarrow\ e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1}\right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1}\right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1}\right] dt = 2\int \frac{dt}{t^2-1} + 2\int \frac{dt}{t^2+1}$$

$$= \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + 2\,\text{arctg}\, t + c = \frac{2}{2}\ln\frac{1-t}{1+t} + 2\,\text{arctg}\, t + c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$= \left[\begin{array}{l|l|l|l} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1}\right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1}\right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1}\right] dt = 2\int \frac{dt}{t^2-1} + 2\int \frac{dt}{t^2+1}$$

$$= \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + 2\,\text{arctg}\, t + c = \frac{2}{2}\ln\frac{1-t}{1+t} + 2\,\text{arctg}\, t + c$$

$$= \left[\frac{1-t}{1+t} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{1-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{1+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}} = \frac{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}} \cdot \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}} = \frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)} = \frac{2-2\sqrt{1-e^{2x}}}{2\,e^x} \right]$$

Riešené príklady – 096

$$\int\sqrt{\frac{1-e^x}{1+e^x}}\,dx$$

$$=\left[\begin{array}{l|l|l|l} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array}\right]$$

$$=\int\left[\frac{2t^2}{t^2-1}-\frac{2t^2}{t^2+1}\right]dt=\int\left[\frac{2t^2-2+2}{t^2-1}-\frac{2t^2+2-2}{t^2+1}\right]dt$$

$$=\int\left[2+\frac{2}{t^2-1}-2+\frac{2}{t^2+1}\right]dt=2\int\frac{dt}{t^2-1}+2\int\frac{dt}{t^2+1}$$

$$=\frac{2}{2}\ln\left|\frac{t-1}{t+1}\right|+2\operatorname{arctg} t+c=\frac{2}{2}\ln\frac{1-t}{1+t}+2\operatorname{arctg} t+c$$

$$=\left[\frac{1-t}{1+t}=\frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}}=\frac{1-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{1+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}=\frac{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}=\frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}}\cdot\frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}}=\frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)}=\frac{2-2\sqrt{1-e^{2x}}}{2\,e^x}\right]$$

$$=\ln\frac{1-1\sqrt{1-e^{2x}}}{e^x}+2\operatorname{arctg}\sqrt{\frac{1-e^x}{1+e^x}}+c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\, dx$$

$$= \left[\begin{array}{c|c|c|c} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0\rangle \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1} \right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1} \right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1} \right] dt = 2\int \frac{dt}{t^2-1} + 2\int \frac{dt}{t^2+1}$$

$$= \frac{2}{2} \ln \left| \frac{t-1}{t+1} \right| + 2 \operatorname{arctg} t + c = \frac{2}{2} \ln \frac{1-t}{1+t} + 2 \operatorname{arctg} t + c$$

$$= \left[\frac{1-t}{1+t} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{1-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{1+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}} = \frac{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}} \cdot \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}} = \frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)} = \frac{2-2\sqrt{1-e^{2x}}}{2\,e^x} \right]$$

$$= \ln \frac{1-1\sqrt{1-e^{2x}}}{e^x} + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c$$

$$= \ln\left(1-\sqrt{1-e^{2x}}\right) - \ln e^x + 2 \operatorname{arctg} \sqrt{\frac{1-e^x}{1+e^x}} + c$$

Riešené príklady – 096

$$\int \sqrt{\frac{1-e^x}{1+e^x}}\,dx = \ln\left(1-\sqrt{1-e^{2x}}\right) - x + 2\,\mathrm{arctg}\sqrt{\frac{1-e^x}{1+e^x}} + c$$

$$= \left[\begin{array}{c|c|c|c} \text{Subst. } t=\sqrt{\frac{1-e^x}{1+e^x}}=\sqrt{\frac{2-1-e^x}{1+e^x}}=\sqrt{\frac{2}{1+e^x}-1} & t^2(1+e^x)=1-e^x \;\Rightarrow\; e^x=\frac{1-t^2}{1+t^2} & 1+e^x>0 \text{ pre } x\in R & x\in(-\infty;0) \\ dx=\left[\frac{-2t}{1-t^2}-\frac{2t}{1+t^2}\right]dt=\left[\frac{2t}{t^2-1}-\frac{2t}{t^2+1}\right]dt & x=\ln\frac{1-t^2}{1+t^2}=\ln(1-t^2)-\ln(1+t^2) & 1-e^x\geq 0 \text{ pre } x\leq 0 & t\in\langle 0;1) \end{array} \right]$$

$$= \int \left[\frac{2t^2}{t^2-1} - \frac{2t^2}{t^2+1}\right] dt = \int \left[\frac{2t^2-2+2}{t^2-1} - \frac{2t^2+2-2}{t^2+1}\right] dt$$

$$= \int \left[2 + \frac{2}{t^2-1} - 2 + \frac{2}{t^2+1}\right] dt = 2\int \frac{dt}{t^2-1} + 2\int \frac{dt}{t^2+1}$$

$$= \frac{2}{2}\ln\left|\frac{t-1}{t+1}\right| + 2\,\mathrm{arctg}\,t + c = \frac{2}{2}\ln\frac{1-t}{1+t} + 2\,\mathrm{arctg}\,t + c$$

$$= \left[\frac{1-t}{1+t} = \frac{1-\sqrt{\frac{1-e^x}{1+e^x}}}{1+\sqrt{\frac{1-e^x}{1+e^x}}} = \frac{1-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{1+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}} = \frac{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}-\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}}{\frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}+\frac{\sqrt{1-e^x}}{\sqrt{1+e^x}}} = \frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}+\sqrt{1-e^x}}\cdot\frac{\sqrt{1+e^x}-\sqrt{1-e^x}}{\sqrt{1+e^x}-\sqrt{1-e^x}} = \frac{(1+e^x)-2\sqrt{1+e^x}\sqrt{1-e^x}+(1-e^x)}{(1+e^x)-(1-e^x)} = \frac{2-2\sqrt{1-e^{2x}}}{2\,e^x} \right]$$

$$= \ln\frac{1-1\sqrt{1-e^{2x}}}{e^x} + 2\,\mathrm{arctg}\sqrt{\frac{1-e^x}{1+e^x}} + c$$

$$= \ln\left(1-\sqrt{1-e^{2x}}\right) - \ln e^x + 2\,\mathrm{arctg}\sqrt{\frac{1-e^x}{1+e^x}} + c$$

$$= \ln\left(1-\sqrt{1-e^{2x}}\right) - x + 2\,\mathrm{arctg}\sqrt{\frac{1-e^x}{1+e^x}} + c,\; x\in(-\infty;0),\; c\in R.$$

Riešené príklady – 097, 098

$$\int (x+1)\,e^{x}\,dx$$

$$I_2 = \int x^2\,e^{ax}\,dx \qquad a \in R,\ a \neq 0$$

Riešené príklady – 097, 098

$$\int (x+1)\, e^x \, dx$$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = e^x & v = e^x \end{array}\right]$$

$$I_2 = \int x^2\, e^{ax}\, dx \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right]$$

[Odhad riešenia metóda neurčitých koeficientov] $I_2 = (\alpha + \beta x + \gamma x^2)\, e^{ax} + c$

Riešené príklady – 097, 098

$$\int (x+1)\,\mathrm{e}^x\,\mathrm{d}x$$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = \mathrm{e}^x & v = \mathrm{e}^x \end{array}\right] = (x+1)\,\mathrm{e}^x - \int \mathrm{e}^x\,\mathrm{d}x$$

$$I_2 = \int x^2\,\mathrm{e}^{ax}\,\mathrm{d}x \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \mathrm{e}^{ax} & v = \frac{\mathrm{e}^{ax}}{a} \end{array}\right] = \frac{x^2\,\mathrm{e}^{ax}}{a} - \int \frac{2x\,\mathrm{e}^{ax}\,\mathrm{d}x}{a}$$

$\left[\begin{array}{c}\text{Odhad riešenia} \\ \text{metóda neurčitých koeficientov}\end{array}\right]$ $I_2 = (\alpha + \beta x + \gamma x^2)\,\mathrm{e}^{ax} + c$

$\left[\begin{array}{c}\text{Derivácia rovnosti} \\ I_2 = (\alpha+\beta x+\gamma x^2)\,\mathrm{e}^{ax}+c\end{array}\right]$ $x^2\,\mathrm{e}^{ax} = (\beta + 2\gamma x)\,\mathrm{e}^{ax} + (\alpha + \beta x + \gamma x^2)\,a\,\mathrm{e}^{ax}$

Riešené príklady – 097, 098

$\int (x+1)\, e^x \, dx$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = e^x & v = e^x \end{array}\right] = (x+1)\, e^x - \int e^x dx = (x+1)\, e^x - e^x + c$$

$I_2 = \int x^2 e^{ax} \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right] = \frac{x^2 e^{ax}}{a} - \int \frac{2x\, e^{ax} \, dx}{a} = \left[\begin{array}{l|l} u = \frac{2x}{a} & u' = \frac{2}{a} \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right]$$

[Odhad riešenia metóda neurčitých koeficientov] $I_2 = (\alpha + \beta x + \gamma x^2)\, e^{ax} + c$

[Derivácia rovnosti $I_2 = (\alpha+\beta x+\gamma x^2)\, e^{ax} + c$] $\quad x^2 e^{ax} = (\beta + 2\gamma x)\, e^{ax} + (\alpha + \beta x + \gamma x^2)\, a\, e^{ax}$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] e^{ax} = \qquad = \left[(\beta + \alpha a) + (2\gamma + \beta a) x + \gamma a x^2\right] e^{ax}.$$

Riešené príklady – 097, 098

$$\int (x+1)\,\mathrm{e}^x\,\mathrm{d}x = x\,\mathrm{e}^x + c$$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = \mathrm{e}^x & v = \mathrm{e}^x \end{array}\right] = (x+1)\,\mathrm{e}^x - \int \mathrm{e}^x\,\mathrm{d}x = (x+1)\,\mathrm{e}^x - \mathrm{e}^x + c = x\,\mathrm{e}^x + c,\ x \in R,\ c \in R.$$

$$I_2 = \int x^2\,\mathrm{e}^{ax}\,\mathrm{d}x \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \mathrm{e}^{ax} & v = \frac{\mathrm{e}^{ax}}{a} \end{array}\right] = \frac{x^2\,\mathrm{e}^{ax}}{a} - \int \frac{2x\,\mathrm{e}^{ax}\,\mathrm{d}x}{a} = \left[\begin{array}{l|l} u = \frac{2x}{a} & u' = \frac{2}{a} \\ v' = \mathrm{e}^{ax} & v = \frac{\mathrm{e}^{ax}}{a} \end{array}\right] = \frac{x^2\,\mathrm{e}^{ax}}{a} - \left[\frac{2x\,\mathrm{e}^{ax}}{a^2} - \int \frac{2\,\mathrm{e}^{ax}\,\mathrm{d}x}{a^2}\right]$$

[Odhad riešenia / metóda neurčitých koeficientov] $I_2 = (\alpha + \beta x + \gamma x^2)\,\mathrm{e}^{ax} + c$

[Derivácia rovnosti / $I_2 = (\alpha+\beta x+\gamma x^2)\,\mathrm{e}^{ax} + c$] $x^2\,\mathrm{e}^{ax} = (\beta + 2\gamma x)\,\mathrm{e}^{ax} + (\alpha + \beta x + \gamma x^2)\,a\,\mathrm{e}^{ax}$

$$\left[0 + 0\cdot x + 1\cdot x^2\right]\mathrm{e}^{ax} = \qquad = \left[(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2\right]\mathrm{e}^{ax}.$$

[3 rovnice / 3 neznáme] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 1 = \gamma a$

Riešené príklady – 097, 098

$$\int (x+1)\,e^x\,dx = x\,e^x + c$$

$$= \left[\begin{array}{l|l} u=x+1 & u'=1 \\ v'=e^x & v=e^x \end{array}\right] = (x+1)\,e^x - \int e^x dx = (x+1)\,e^x - e^x + c = x\,e^x + c,\ x\in R,\ c\in R.$$

$$I_2 = \int x^2\,e^{ax}\,dx \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u=x^2 & u'=2x \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \int \frac{2x\,e^{ax}\,dx}{a} = \left[\begin{array}{l|l} u=\frac{2x}{a} & u'=\frac{2}{a} \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \left[\frac{2x\,e^{ax}}{a^2} - \int \frac{2\,e^{ax}\,dx}{a^2}\right]$$

$$= \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a\cdot a^2} + c$$

[Odhad riešenia metóda neurčitých koeficientov] $I_2 = (\alpha+\beta x+\gamma x^2)\,e^{ax} + c$

[Derivácia rovnosti $I_2=(\alpha+\beta x+\gamma x^2)\,e^{ax}+c$] $x^2\,e^{ax} = (\beta+2\gamma x)\,e^{ax} + (\alpha+\beta x+\gamma x^2)\,a\,e^{ax}$

$$[0+0\cdot x+1\cdot x^2]\,e^{ax} = \qquad = [(\beta+\alpha a)+(2\gamma+\beta a)x+\gamma a x^2]\,e^{ax}.$$

[3 rovnice 3 neznáme] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 1=\gamma a$

Riešené príklady – 097, 098

$$\int (x+1)\,e^x\,dx = x\,e^x + c$$

$$= \left[\begin{array}{l|l} u=x+1 & u'=1 \\ v'=e^x & v=e^x \end{array}\right] = (x+1)\,e^x - \int e^x dx = (x+1)\,e^x - e^x + c = x\,e^x + c,\ x\in R,\ c\in R.$$

$$I_2 = \int x^2\,e^{ax}\,dx = \frac{x^2 e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u=x^2 & u'=2x \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2 e^{ax}}{a} - \int \frac{2x\,e^{ax}\,dx}{a} = \left[\begin{array}{l|l} u=\frac{2x}{a} & u'=\frac{2}{a} \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2 e^{ax}}{a} - \left[\frac{2x\,e^{ax}}{a^2} - \int \frac{2\,e^{ax}\,dx}{a^2}\right]$$

$$= \frac{x^2 e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a\cdot a^2} + c = \frac{x^2 e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c,\ x\in R,\ c\in R.$$

[Odhad riešenia / metóda neurčitých koeficientov] $I_2 = (\alpha+\beta x+\gamma x^2)\,e^{ax} + c$

[Derivácia rovnosti $I_2=(\alpha+\beta x+\gamma x^2)\,e^{ax}+c$] $x^2 e^{ax} = (\beta+2\gamma x)\,e^{ax} + (\alpha+\beta x+\gamma x^2)\,a\,e^{ax}$

$$[0+0\cdot x+1\cdot x^2]\,e^{ax} = \qquad = [(\beta+\alpha a)+(2\gamma+\beta a)x+\gamma a x^2]\,e^{ax}.$$

[3 rovnice / 3 neznáme] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 1=\gamma a$

Riešené príklady – 097, 098

$$\int (x+1)\,e^x\,dx = x\,e^x + c$$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = e^x & v = e^x \end{array}\right] = (x+1)\,e^x - \int e^x dx = (x+1)\,e^x - e^x + c = x\,e^x + c,\ x \in R,\ c \in R.$$

$$I_2 = \int x^2\,e^{ax}\,dx = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \int \frac{2x\,e^{ax}\,dx}{a} = \left[\begin{array}{l|l} u = \frac{2x}{a} & u' = \frac{2}{a} \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \left[\frac{2x\,e^{ax}}{a^2} - \int \frac{2\,e^{ax}\,dx}{a^2}\right]$$

$$= \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a \cdot a^2} + c = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c,\ x \in R,\ c \in R.$$

[Odhad riešenia / metóda neurčitých koeficientov] $I_2 = (\alpha + \beta x + \gamma x^2)\,e^{ax} + c$

[Derivácia rovnosti / $I_2 = (\alpha+\beta x+\gamma x^2)\,e^{ax} + c$] $x^2\,e^{ax} = (\beta + 2\gamma x)\,e^{ax} + (\alpha + \beta x + \gamma x^2)\,a\,e^{ax}$

$$\left[0 + 0 \cdot x + 1 \cdot x^2\right] e^{ax} = \qquad = \left[(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2\right] e^{ax}.$$

[3 rovnice / 3 neznáme] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 1 = \gamma a$

Riešené príklady – 097, 098

$\int (x+1)\,e^x\,dx = x\,e^x + c$

$$= \left[\begin{array}{l|l} u=x+1 & u'=1 \\ v'=e^x & v=e^x \end{array}\right] = (x+1)\,e^x - \int e^x dx = (x+1)\,e^x - e^x + c = x\,e^x + c,\ x\in R,\ c\in R.$$

$I_2 = \int x^2\,e^{ax}\,dx = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c$ $\quad a\in R,\ a\neq 0$

$$= \left[\begin{array}{l|l} u=x^2 & u'=2x \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \int \frac{2x\,e^{ax}\,dx}{a} = \left[\begin{array}{l|l} u=\frac{2x}{a} & u'=\frac{2}{a} \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \left[\frac{2x\,e^{ax}}{a^2} - \int \frac{2\,e^{ax}\,dx}{a^2}\right]$$
$$= \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a\cdot a^2} + c = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c,\ x\in R,\ c\in R.$$

[Odhad riešenia metóda neurčitých koeficientov] $I_2 = (\alpha+\beta x+\gamma x^2)\,e^{ax} + c$

[Derivácia rovnosti $I_2=(\alpha+\beta x+\gamma x^2)\,e^{ax}+c$] $x^2\,e^{ax} = (\beta+2\gamma x)\,e^{ax} + (\alpha+\beta x+\gamma x^2)\,a\,e^{ax}$

$$[0+0\cdot x+1\cdot x^2]\,e^{ax} = \quad = [(\beta+\alpha a)+(2\gamma+\beta a)x+\gamma a x^2]\,e^{ax}.$$

[3 rovnice 3 neznáme] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 1=\gamma a \ \Rightarrow\ \gamma=\frac{1}{a},$

Riešené príklady – 097, 098

$$\int (x+1)\,e^x\,dx = x\,e^x + c$$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = e^x & v = e^x \end{array}\right] = (x+1)\,e^x - \int e^x dx = (x+1)\,e^x - e^x + c = x\,e^x + c,\ x \in R,\ c \in R.$$

$$I_2 = \int x^2\,e^{ax}\,dx = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \int \frac{2x\,e^{ax}\,dx}{a} = \left[\begin{array}{l|l} u = \frac{2x}{a} & u' = \frac{2}{a} \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \left[\frac{2x\,e^{ax}}{a^2} - \int \frac{2\,e^{ax}\,dx}{a^2}\right]$$

$$= \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a \cdot a^2} + c = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c,\ x \in R,\ c \in R.$$

[Odhad riešenia / metóda neurčitých koeficientov] $I_2 = (\alpha + \beta x + \gamma x^2)\,e^{ax} + c$

[Derivácia rovnosti / $I_2 = (\alpha+\beta x+\gamma x^2)\,e^{ax} + c$] $x^2\,e^{ax} = (\beta + 2\gamma x)\,e^{ax} + (\alpha + \beta x + \gamma x^2)\,a\,e^{ax}$

$$[0 + 0\cdot x + 1\cdot x^2]\,e^{ax} = \qquad = [(\beta + \alpha a) + (2\gamma + \beta a)x + \gamma a x^2]\,e^{ax}.$$

[3 rovnice / 3 neznáme] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 1 = \gamma a \ \Rightarrow\ \gamma = \frac{1}{a},\ \beta = \frac{2\gamma}{-a} = -\frac{2}{a^2},$

Riešené príklady – 097, 098

$$\int (x+1)\,e^x\,dx = x\,e^x + c$$

$$= \left[\begin{array}{l|l} u=x+1 & u'=1 \\ v'=e^x & v=e^x \end{array}\right] = (x+1)\,e^x - \int e^x dx = (x+1)\,e^x - e^x + c = x\,e^x + c,\ x\in R,\ c\in R.$$

$$I_2 = \int x^2\,e^{ax}\,dx = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u=x^2 & u'=2x \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \int \frac{2x\,e^{ax}\,dx}{a} = \left[\begin{array}{l|l} u=\frac{2x}{a} & u'=\frac{2}{a} \\ v'=e^{ax} & v=\frac{e^{ax}}{a} \end{array}\right] = \frac{x^2\,e^{ax}}{a} - \left[\frac{2x\,e^{ax}}{a^2} - \int \frac{2\,e^{ax}\,dx}{a^2}\right]$$

$$= \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a\cdot a^2} + c = \frac{x^2\,e^{ax}}{a} - \frac{2x\,e^{ax}}{a^2} + \frac{2\,e^{ax}}{a^3} + c,\ x\in R,\ c\in R.$$

[Odhad riešenia, metóda neurčitých koeficientov] $I_2 = (\alpha+\beta x+\gamma x^2)\,e^{ax} + c$

[Derivácia rovnosti $I_2=(\alpha+\beta x+\gamma x^2)\,e^{ax}+c$] $x^2\,e^{ax} = (\beta+2\gamma x)\,e^{ax} + (\alpha+\beta x+\gamma x^2)\,a\,e^{ax}$

$$\left[0+0\cdot x+1\cdot x^2\right]e^{ax} = \qquad = \left[(\beta+\alpha a)+(2\gamma+\beta a)x+\gamma a x^2\right]e^{ax}.$$

[3 rovnice, 3 neznáme] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 1=\gamma a \Rightarrow \gamma=\frac{1}{a},\ \beta=\frac{2\gamma}{-a}=-\frac{2}{a^2},\ \alpha=\frac{\beta}{-a}=\frac{2}{a^3}.$

Riešené príklady – 097, 098

$$\int (x+1)\,\mathrm{e}^x\,dx = x\,\mathrm{e}^x + c$$

$$= \left[\begin{array}{l|l} u = x+1 & u' = 1 \\ v' = \mathrm{e}^x & v = \mathrm{e}^x \end{array}\right] = (x+1)\,\mathrm{e}^x - \int \mathrm{e}^x dx = (x+1)\,\mathrm{e}^x - \mathrm{e}^x + c = x\,\mathrm{e}^x + c,\ x\in R,\ c\in R.$$

$$I_2 = \int x^2\,\mathrm{e}^{ax}\,dx = \frac{x^2\,\mathrm{e}^{ax}}{a} - \frac{2x\,\mathrm{e}^{ax}}{a^2} + \frac{2\,\mathrm{e}^{ax}}{a^3} + c \qquad a\in R,\ a\neq 0$$

$$= \left[\begin{array}{l|l} u = x^2 & u' = 2x \\ v' = \mathrm{e}^{ax} & v = \frac{\mathrm{e}^{ax}}{a} \end{array}\right] = \frac{x^2\,\mathrm{e}^{ax}}{a} - \int \frac{2x\,\mathrm{e}^{ax}\,dx}{a} = \left[\begin{array}{l|l} u = \frac{2x}{a} & u' = \frac{2}{a} \\ v' = \mathrm{e}^{ax} & v = \frac{\mathrm{e}^{ax}}{a} \end{array}\right] = \frac{x^2\,\mathrm{e}^{ax}}{a} - \left[\frac{2x\,\mathrm{e}^{ax}}{a^2} - \int \frac{2\,\mathrm{e}^{ax}\,dx}{a^2}\right]$$

$$= \frac{x^2\,\mathrm{e}^{ax}}{a} - \frac{2x\,\mathrm{e}^{ax}}{a^2} + \frac{2\,\mathrm{e}^{ax}}{a\cdot a^2} + c = \frac{x^2\,\mathrm{e}^{ax}}{a} - \frac{2x\,\mathrm{e}^{ax}}{a^2} + \frac{2\,\mathrm{e}^{ax}}{a^3} + c,\ x\in R,\ c\in R.$$

[Odhad riešenia metóda neurčitých koeficientov]
$$I_2 = (\alpha+\beta x+\gamma x^2)\,\mathrm{e}^{ax} + c = \left(\frac{2}{a^3} - \frac{2x}{a^2} + \frac{x^2}{a}\right)\mathrm{e}^{ax} + c,\ x\in R,\ c\in R.$$

[Derivácia rovnosti $I_2 = (\alpha+\beta x+\gamma x^2)\,\mathrm{e}^{ax} + c$]
$$x^2\,\mathrm{e}^{ax} = (\beta+2\gamma x)\,\mathrm{e}^{ax} + (\alpha+\beta x+\gamma x^2)\,a\,\mathrm{e}^{ax}$$

$$\left[0+0\cdot x+1\cdot x^2\right]\mathrm{e}^{ax} = \qquad = \left[(\beta+\alpha a)+(2\gamma+\beta a)x+\gamma a x^2\right]\mathrm{e}^{ax}.$$

[3 rovnice 3 neznáme]
$$0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 1=\gamma a \ \Rightarrow\ \gamma=\frac{1}{a},\ \beta=\frac{2\gamma}{-a}=-\frac{2}{a^2},\ \alpha=\frac{\beta}{-a}=\frac{2}{a^3}.$$

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\qquad a \in R,\ a \neq 0$

Riešené príklady – 099

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[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$$

Riešené príklady – 099

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[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, \mathrm{e}^{ax} + c$]

$$x^8\, \mathrm{e}^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7)\, \mathrm{e}^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, a\, \mathrm{e}^{ax}.$$

Riešené príklady – 099

$$I_8 = \int x^8\, e^{ax}\, dx \qquad a \in R,\ a \neq 0$$

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$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$]

$$x^8\, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7)\, e^{ax}$$
$$+ (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, a\, e^{ax}.$$

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\qquad a \in R, \; a \neq 0$

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Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

$$x^8 \, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) \, e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,$

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\qquad a \in R, \ a \neq 0$

$$= \left[\begin{array}{c|c} u = x^8 & u' = 8x^7 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 \, e^{ax}}{a} - \frac{8}{a} \int x^7 \, e^{ax} \, dx = \left[\begin{array}{c|c} u = x^7 & u' = 7x^6 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

$$x^8 \, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) \, e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a, \ 0 = 2\gamma + \beta a,$

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\qquad a \in R, \; a \neq 0$

$$= \left[\begin{array}{c|c} u = x^8 & u' = 8x^7 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 \, e^{ax}}{a} - \frac{8}{a} \int x^7 \, e^{ax} \, dx = \left[\begin{array}{c|c} u = x^7 & u' = 7x^6 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \cdots$$

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$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

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[9 rovníc a 9 neznámych] $0 = \beta + \alpha a, \; 0 = 2\gamma + \beta a, \; 0 = 3\delta + \gamma a,$

Riešené príklady – 099

$$I_8 = \int x^8\, e^{ax}\, dx \qquad a \in R,\ a \neq 0$$

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[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$]

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[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 0 = 3\delta + \gamma a,\ 0 = 4\psi + \delta a,$

Riešené príklady – 099

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[Odhad riešenia metódou neurčitých koeficientov]

$$I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$$

[Derivácia rovnosti $I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$]

$$x^8\,\mathrm{e}^{ax}=(\beta+2\gamma x+3\delta x^2+4\psi x^3+5\varphi x^4+6\mu x^5+7\nu x^6+8\theta x^7)\,\mathrm{e}^{ax}$$
$$+(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,a\,\mathrm{e}^{ax}.$$

[9 rovníc a 9 neznámych] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 0=3\delta+\gamma a,\ 0=4\psi+\delta a,$

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[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 0 = 3\delta + \gamma a,\ 0 = 4\psi + \delta a,$

$0 = 5\varphi + \psi a,\ 0 = 6\mu + \varphi a,$

Riešené príklady – 099

$$I_8 = \int x^8\, e^{ax}\, dx \qquad a \in R,\ a \neq 0$$

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Riešené príklady – 099

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$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

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[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 0 = 3\delta + \gamma a,\ 0 = 4\psi + \delta a,$

$0 = 5\varphi + \psi a,\ 0 = 6\mu + \varphi a,\ 0 = 7\nu + \mu a,\ 0 = 8\theta + \nu a,$

Riešené príklady – 099

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$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

$$x^8 \, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) \, e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a$, $0 = 2\gamma + \beta a$, $0 = 3\delta + \gamma a$, $0 = 4\psi + \delta a$,
$0 = 5\varphi + \psi a$, $0 = 6\mu + \varphi a$, $0 = 7\nu + \mu a$, $0 = 8\theta + \nu a$, $1 = \theta a$.

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\qquad a \in R, \; a \neq 0$

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[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

$$+ \frac{8! x^8}{8! a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

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$$+ (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a$, $0 = 2\gamma + \beta a$, $0 = 3\delta + \gamma a$, $0 = 4\psi + \delta a$,
$0 = 5\varphi + \psi a$, $0 = 6\mu + \varphi a$, $0 = 7\nu + \mu a$, $0 = 8\theta + \nu a$, $1 = \theta a$.

$\Rightarrow \; \theta = \frac{1}{a},$

Riešené príklady – 099

$I_8 = \int x^8\, e^{ax}\, dx$ $\qquad a \in R,\ a \neq 0$

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$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$$

$$-\frac{8!x^7}{7!a^2} + \frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$]

$$x^8\, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7)\, e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, a\, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 0 = 3\delta + \gamma a,\ 0 = 4\psi + \delta a,$
$0 = 5\varphi + \psi a,\ 0 = 6\mu + \varphi a,\ 0 = 7\nu + \mu a,\ 0 = 8\theta + \nu a,\ 1 = \theta a.$

$$\Rightarrow \theta = \frac{1}{a},\ \nu = \frac{8\theta}{-a} = -\frac{8}{a^2},$$

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{c|c} u = x^8 & u' = 8x^7 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 \, e^{ax}}{a} - \frac{8}{a} \int x^7 \, e^{ax} \, dx = \left[\begin{array}{c|c} u = x^7 & u' = 7x^6 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \cdots$$

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[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

$$+ \frac{8! x^6}{6! a^3} - \frac{8! x^7}{7! a^2} + \frac{8! x^8}{8! a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

$$x^8 \, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) \, e^{ax}$$
$$+ (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 0 = 3\delta + \gamma a,\ 0 = 4\psi + \delta a,$

$$0 = 5\varphi + \psi a,\ 0 = 6\mu + \varphi a,\ 0 = 7\nu + \mu a,\ 0 = 8\theta + \nu a,\ 1 = \theta a.$$

$$\Rightarrow \theta = \frac{1}{a},\ \nu = \frac{8\theta}{-a} = -\frac{8}{a^2},\ \mu = \frac{7\nu}{-a} = \frac{8 \cdot 6}{a^3},$$

Riešené príklady – 099

$$I_8=\int x^8\,\mathrm{e}^{ax}\,\mathrm{d}x \qquad a\in R,\ a\neq 0$$

$$=\left[\begin{array}{l|l} u=x^8 & u'=8x^7 \\ v'=\mathrm{e}^{ax} & v=\frac{\mathrm{e}^{ax}}{a}\end{array}\right]=\frac{x^8\,\mathrm{e}^{ax}}{a}-\frac{8}{a}\int x^7\,\mathrm{e}^{ax}\,\mathrm{d}x=\left[\begin{array}{l|l} u=x^7 & u'=7x^6 \\ v'=\mathrm{e}^{ax} & v=\frac{\mathrm{e}^{ax}}{a}\end{array}\right]=\cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$$

$$-\frac{8!x^5}{5!a^4}+\frac{8!x^6}{6!a^3}-\frac{8!x^7}{7!a^2}+\frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$]

$$x^8\,\mathrm{e}^{ax}=(\beta+2\gamma x+3\delta x^2+4\psi x^3+5\varphi x^4+6\mu x^5+7\nu x^6+8\theta x^7)\,\mathrm{e}^{ax}$$
$$+(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,a\,\mathrm{e}^{ax}.$$

[9 rovníc a 9 neznámych] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 0=3\delta+\gamma a,\ 0=4\psi+\delta a,$

$$0=5\varphi+\psi a,\ 0=6\mu+\varphi a,\ 0=7\nu+\mu a,\ 0=8\theta+\nu a,\ 1=\theta a.$$

$$\Rightarrow\ \theta=\frac{1}{a},\ \nu=\frac{8\theta}{-a}=-\frac{8}{a^2},\ \mu=\frac{7\nu}{-a}=\frac{8\cdot 6}{a^3},\ \varphi=\frac{6\mu}{-a}=-\frac{8\cdot 7\cdot 5}{a^4},$$

Riešené príklady – 099

$$I_8 = \int x^8 \, e^{ax} \, dx \qquad a \in R, \; a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^8 & u' = 8x^7 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 \, e^{ax}}{a} - \frac{8}{a} \int x^7 \, e^{ax} \, dx = \left[\begin{array}{l|l} u = x^7 & u' = 7x^6 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$
$$+ \frac{8! x^4}{4! a^5} - \frac{8! x^5}{5! a^4} + \frac{8! x^6}{6! a^3} - \frac{8! x^7}{7! a^2} + \frac{8! x^8}{8! a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

$$x^8 \, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) \, e^{ax}$$
$$+ (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a$, $0 = 2\gamma + \beta a$, $0 = 3\delta + \gamma a$, $0 = 4\psi + \delta a$,

$0 = 5\varphi + \psi a$, $0 = 6\mu + \varphi a$, $0 = 7\nu + \mu a$, $0 = 8\theta + \nu a$, $1 = \theta a$.

$$\Rightarrow \theta = \frac{1}{a}, \; \nu = \frac{8\theta}{-a} = -\frac{8}{a^2}, \; \mu = \frac{7\nu}{-a} = \frac{8 \cdot 6}{a^3}, \; \varphi = \frac{6\mu}{-a} = -\frac{8 \cdot 7 \cdot 5}{a^4}, \; \psi = \frac{5\varphi}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{a^5},$$

Riešené príklady – 099

$$I_8 = \int x^8\, e^{ax}\, dx \qquad a \in R,\ a \neq 0$$

$$= \left[\begin{array}{l|l} u = x^8 & u' = 8x^7 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right] = \frac{x^8\, e^{ax}}{a} - \frac{8}{a}\int x^7\, e^{ax}\, dx = \left[\begin{array}{l|l} u = x^7 & u' = 7x^6 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array}\right] = \cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$$

$$-\frac{8!x^3}{3!a^6} + \frac{8!x^4}{4!a^5} - \frac{8!x^5}{5!a^4} + \frac{8!x^6}{6!a^3} - \frac{8!x^7}{7!a^2} + \frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, e^{ax} + c$]

$$x^8\, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7)\, e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8)\, a\, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 0 = 3\delta + \gamma a,\ 0 = 4\psi + \delta a,$
$0 = 5\varphi + \psi a,\ 0 = 6\mu + \varphi a,\ 0 = 7\nu + \mu a,\ 0 = 8\theta + \nu a,\ 1 = \theta a.$

$$\Rightarrow \theta = \frac{1}{a},\ \nu = \frac{8\theta}{-a} = -\frac{8}{a^2},\ \mu = \frac{7\nu}{-a} = \frac{8\cdot 7}{a^3},\ \varphi = \frac{6\mu}{-a} = -\frac{8\cdot 7\cdot 6}{a^4},\ \psi = \frac{5\varphi}{-a} = \frac{8\cdot 7\cdot 6\cdot 5}{a^5},$$

$$\delta = \frac{4\psi}{-a} = -\frac{8\cdot 7\cdot 6\cdot 5\cdot 4}{a^6},$$

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\quad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^8 & u' = 8x^7 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 \, e^{ax}}{a} - \frac{8}{a} \int x^7 \, e^{ax} \, dx = \left[\begin{array}{l|l} u = x^7 & u' = 7x^6 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

$$+ \frac{8!x^2}{2!a^7} - \frac{8!x^3}{3!a^6} + \frac{8!x^4}{4!a^5} - \frac{8!x^5}{5!a^4} + \frac{8!x^6}{6!a^3} - \frac{8!x^7}{7!a^2} + \frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

$$x^8 \, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) \, e^{ax}$$
$$+ (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a,\ 0 = 2\gamma + \beta a,\ 0 = 3\delta + \gamma a,\ 0 = 4\psi + \delta a,$

$$0 = 5\varphi + \psi a,\ 0 = 6\mu + \varphi a,\ 0 = 7\nu + \mu a,\ 0 = 8\theta + \nu a,\ 1 = \theta a.$$

$$\Rightarrow \theta = \frac{1}{a},\ \nu = \frac{8\theta}{-a} = -\frac{8}{a^2},\ \mu = \frac{7\nu}{-a} = \frac{8 \cdot 6}{a^3},\ \varphi = \frac{6\mu}{-a} = -\frac{8 \cdot 7 \cdot 5}{a^4},\ \psi = \frac{5\varphi}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{a^5},$$

$$\delta = \frac{4\psi}{-a} = -\frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{a^6},\ \gamma = \frac{3\delta}{-a} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{a^7},$$

Riešené príklady – 099

$I_8=\int x^8\,\mathrm{e}^{ax}\,\mathrm{d}x$ $\qquad a\in R,\ a\neq 0$

$$=\left[\begin{array}{c|c} u=x^8 & u'=8x^7 \\ v'=\mathrm{e}^{ax} & v=\frac{\mathrm{e}^{ax}}{a}\end{array}\right]=\frac{x^8\,\mathrm{e}^{ax}}{a}-\frac{8}{a}\int x^7\,\mathrm{e}^{ax}\,\mathrm{d}x=\left[\begin{array}{c|c} u=x^7 & u'=7x^6 \\ v'=\mathrm{e}^{ax} & v=\frac{\mathrm{e}^{ax}}{a}\end{array}\right]=\cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$$

$$-\frac{8!x}{1!a^8}+\frac{8!x^2}{2!a^7}-\frac{8!x^3}{3!a^6}+\frac{8!x^4}{4!a^5}-\frac{8!x^5}{5!a^4}+\frac{8!x^6}{6!a^3}-\frac{8!x^7}{7!a^2}+\frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$]

$$x^8\,\mathrm{e}^{ax}=(\beta+2\gamma x+3\delta x^2+4\psi x^3+5\varphi x^4+6\mu x^5+7\nu x^6+8\theta x^7)\,\mathrm{e}^{ax}$$
$$+(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,a\,\mathrm{e}^{ax}.$$

[9 rovníc a 9 neznámych] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 0=3\delta+\gamma a,\ 0=4\psi+\delta a,$

$$0=5\varphi+\psi a,\ 0=6\mu+\varphi a,\ 0=7\nu+\mu a,\ 0=8\theta+\nu a,\ 1=\theta a.$$

$$\Rightarrow\ \theta=\frac{1}{a},\ \nu=\frac{8\theta}{-a}=-\frac{8}{a^2},\ \mu=\frac{7\nu}{-a}=\frac{8\cdot7\cdot6}{a^3},\ \varphi=\frac{6\mu}{-a}=-\frac{8\cdot7\cdot5}{a^4},\ \psi=\frac{5\varphi}{-a}=\frac{8\cdot7\cdot6\cdot5}{a^5},$$

$$\delta=\frac{4\psi}{-a}=-\frac{8\cdot7\cdot6\cdot5\cdot4}{a^6},\ \gamma=\frac{3\delta}{-a}=\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3}{a^7},\ \beta=\frac{2\gamma}{-a}=-\frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2}{a^8},$$

Riešené príklady – 099

$I_8 = \int x^8 \, e^{ax} \, dx$ $\qquad a \in R,\ a \neq 0$

$$= \left[\begin{array}{l|l} u = x^8 & u' = 8x^7 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \frac{x^8 \, e^{ax}}{a} - \frac{8}{a} \int x^7 \, e^{ax} \, dx = \left[\begin{array}{l|l} u = x^7 & u' = 7x^6 \\ v' = e^{ax} & v = \frac{e^{ax}}{a} \end{array} \right] = \cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$$

$$\frac{8!}{0!a^9} - \frac{8!x}{1!a^8} + \frac{8!x^2}{2!a^7} - \frac{8!x^3}{3!a^6} + \frac{8!x^4}{4!a^5} - \frac{8!x^5}{5!a^4} + \frac{8!x^6}{6!a^3} - \frac{8!x^7}{7!a^2} + \frac{8!x^8}{8!a}$$

[Derivácia rovnosti $I_8 = (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) \, e^{ax} + c$]

$$x^8 \, e^{ax} = (\beta + 2\gamma x + 3\delta x^2 + 4\psi x^3 + 5\varphi x^4 + 6\mu x^5 + 7\nu x^6 + 8\theta x^7) \, e^{ax} + (\alpha + \beta x + \gamma x^2 + \delta x^3 + \psi x^4 + \varphi x^5 + \mu x^6 + \nu x^7 + \theta x^8) a \, e^{ax}.$$

[9 rovníc a 9 neznámych] $0 = \beta + \alpha a$, $0 = 2\gamma + \beta a$, $0 = 3\delta + \gamma a$, $0 = 4\psi + \delta a$,
$0 = 5\varphi + \psi a$, $0 = 6\mu + \varphi a$, $0 = 7\nu + \mu a$, $0 = 8\theta + \nu a$, $1 = \theta a$.

$$\Rightarrow \theta = \frac{1}{a},\ \nu = \frac{8\theta}{-a} = -\frac{8}{a^2},\ \mu = \frac{7\nu}{-a} = \frac{8\cdot 6}{a^3},\ \varphi = \frac{6\mu}{-a} = -\frac{8\cdot 7\cdot 5}{a^4},\ \psi = \frac{5\varphi}{-a} = \frac{8\cdot 7\cdot 6\cdot 5}{a^5},$$

$$\delta = \frac{4\psi}{-a} = -\frac{8\cdot 7\cdot 6\cdot 5\cdot 4}{a^6},\ \gamma = \frac{3\delta}{-a} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3}{a^7},\ \beta = \frac{2\gamma}{-a} = -\frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}{a^8},\ \alpha = \frac{\beta}{-a} = \frac{8!}{a^9}.$$

Riešené príklady – 099

$$I_8=\int x^8\,\mathrm{e}^{ax}\,\mathrm{d}x=\sum_{i=0}^{8}\frac{(-1)^i 8!\cdot x^i\,\mathrm{e}^{ax}}{i!\cdot a^{9-i}}+c=8!\,\mathrm{e}^{ax}\sum_{i=0}^{8}\frac{(-x)^i}{i!\cdot a^{9-i}}+c \qquad a\in R,\ a\neq 0$$

$$=\left[\begin{array}{c|c} u=x^8 & u'=8x^7 \\ v'=\mathrm{e}^{ax} & v=\frac{\mathrm{e}^{ax}}{a}\end{array}\right]=\frac{x^8\,\mathrm{e}^{ax}}{a}-\frac{8}{a}\int x^7\,\mathrm{e}^{ax}\,\mathrm{d}x=\left[\begin{array}{c|c} u=x^7 & u'=7x^6 \\ v'=\mathrm{e}^{ax} & v=\frac{\mathrm{e}^{ax}}{a}\end{array}\right]=\cdots$$

Ako vidíme, týmto smerom pre normálneho smrteľníka cesta riešenia nevedie. Je potrebné vykonať 8-krát per partes.

[Odhad riešenia metódou neurčitých koeficientov]

$$I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$$

$$=\left(\frac{8!}{0!a^9}-\frac{8!x}{1!a^8}+\frac{8!x^2}{2!a^7}-\frac{8!x^3}{3!a^6}+\frac{8!x^4}{4!a^5}-\frac{8!x^5}{5!a^4}+\frac{8!x^6}{6!a^3}-\frac{8!x^7}{7!a^2}+\frac{8!x^8}{8!a}\right)\mathrm{e}^{ax}+c,\ x\in R,\ c\in R.$$

[Derivácia rovnosti $I_8=(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)\,\mathrm{e}^{ax}+c$]

$$x^8\,\mathrm{e}^{ax}=(\beta+2\gamma x+3\delta x^2+4\psi x^3+5\varphi x^4+6\mu x^5+7\nu x^6+8\theta x^7)\,\mathrm{e}^{ax}$$
$$+(\alpha+\beta x+\gamma x^2+\delta x^3+\psi x^4+\varphi x^5+\mu x^6+\nu x^7+\theta x^8)a\,\mathrm{e}^{ax}.$$

[9 rovníc a 9 neznámych] $0=\beta+\alpha a,\ 0=2\gamma+\beta a,\ 0=3\delta+\gamma a,\ 0=4\psi+\delta a,$

$$0=5\varphi+\psi a,\ 0=6\mu+\varphi a,\ 0=7\nu+\mu a,\ 0=8\theta+\nu a,\ 1=\theta a.$$

$$\Rightarrow\ \theta=\frac{1}{a},\ \nu=\frac{8\theta}{-a}=-\frac{8}{a^2},\ \mu=\frac{7\nu}{-a}=\frac{8\cdot 6}{a^3},\ \varphi=\frac{6\mu}{-a}=-\frac{8\cdot 7\cdot 5}{a^4},\ \psi=\frac{5\varphi}{-a}=\frac{8\cdot 7\cdot 6\cdot 5}{a^5},$$

$$\delta=\frac{4\psi}{-a}=-\frac{8\cdot 7\cdot 6\cdot 5\cdot 4}{a^6},\ \gamma=\frac{3\delta}{-a}=\frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3}{a^7},\ \beta=\frac{2\gamma}{-a}=-\frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}{a^8},\ \alpha=\frac{\beta}{-a}=\frac{8!}{a^9}.$$

Riešené príklady – 100

$I_n = \int x^n \, e^x \, dx$ $\qquad n \in N$

Riešené príklady – 100

$I_n = \int x^n \, e^x \, dx$ $\qquad n \in N$

$$= \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = e^x & v = e^x \end{array} \right]$$

$$I_1 = \int x \, e^x \, dx = x \, e^x - 1 I_0 = x \, e^x - e^x + c$$

Riešené príklady – 100

$$I_n = \int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx \qquad n \in N$$

$$= \left[\begin{array}{c|c} u = x^n & u' = nx^{n-1} \\ v' = e^x & v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x \, dx = x^n e^x - nI_{n-1}, \; x \in R, \; c \in R.$$

$$I_1 = \int x e^x \, dx = x e^x - 1 I_0 = x e^x - e^x + c,$$

pričom $I_0 = \int e^x \, dx = e^x + c.$

Riešené príklady – 100

$$I_n = \int x^n e^x \,dx = x^n e^x - n\int x^{n-1} e^x \,dx \qquad n\in N$$

$$= \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = e^x & v = e^x \end{array}\right] = x^n e^x - n\int x^{n-1} e^x \,dx = x^n e^x - nI_{n-1},\ x\in R,\ c\in R.$$

$$I_1 = \int x e^x \,dx = x e^x - 1I_0 = x e^x - e^x + c,$$

pričom $I_0 = \int e^x \,dx = e^x + c.$

$$I_2 = \int x^2 e^x \,dx = x^2 e^x - 2I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2\cdot 1 e^x + c.$$

Riešené príklady – 100

$$I_n = \int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx \qquad n \in N$$

$$= \left[\begin{array}{c|c} u = x^n & u' = nx^{n-1} \\ v' = e^x & v = e^x \end{array}\right] = x^n e^x - n\int x^{n-1} e^x\,dx = x^n e^x - nI_{n-1},\ x\in R,\ c\in R.$$

$$I_1 = \int x e^x\,dx = x e^x - 1I_0 = x e^x - e^x + c,$$

pričom $I_0 = \int e^x\,dx = e^x + c.$

$$I_2 = \int x^2 e^x\,dx = x^2 e^x - 2I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2\cdot 1 e^x + c.$$

$$I_3 = \int x^3 e^x\,dx = x^3 e^x - 3I_2 = x^3 e^x - 3(x^2 e^x - 2x e^x + 2\cdot 1 e^x) + c$$

$$= x^3 e^x - 3x^2 e^x + 3\cdot 2x e^x - 3\cdot 2\cdot 1 e^x + c.$$

Riešené príklady – 100

$$I_n = \int x^n e^x\, dx = x^n e^x - n \int x^{n-1} e^x\, dx \qquad n \in N$$

$$= \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = e^x & v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x\, dx = x^n e^x - nI_{n-1},\ x \in R,\ c \in R.$$

$$I_1 = \int x e^x\, dx = x e^x - 1 I_0 = x e^x - e^x + c, \qquad \text{pričom } I_0 = \int e^x\, dx = e^x + c.$$

$$I_2 = \int x^2 e^x\, dx = x^2 e^x - 2 I_1 = x^2 e^x - 2(x e^x - e^x) + c = x^2 e^x - 2x e^x + 2\cdot 1 e^x + c.$$

$$I_3 = \int x^3 e^x\, dx = x^3 e^x - 3 I_2 = x^3 e^x - 3(x^2 e^x - 2x e^x + 2\cdot 1 e^x) + c$$
$$= x^3 e^x - 3x^2 e^x + 3\cdot 2x e^x - 3\cdot 2\cdot 1 e^x + c.$$

$$I_4 = \int x^4 e^x\, dx = x^4 e^x - 4 I_3 = x^4 e^x - 4(x^3 e^x - 3x^2 e^x + 3\cdot 2x e^x - 3\cdot 2\cdot 1 e^x) + c$$
$$= x^4 e^x - 4x^3 e^x + 4\cdot 3x^2 e^x - 4\cdot 3\cdot 2x e^x + 4\cdot 3\cdot 2\cdot 1 e^x + c.$$

Riešené príklady – 100

$$I_n = \int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx \qquad n \in N$$

$$= \left[\begin{array}{l|l} u = x^n & u' = nx^{n-1} \\ v' = e^x & v = e^x \end{array} \right] = x^n e^x - n \int x^{n-1} e^x \, dx = x^n e^x - nI_{n-1}, \; x \in R, \; c \in R.$$

$$I_1 = \int x e^x \, dx = x e^x - 1 I_0 = x e^x - e^x + c, \qquad \text{pričom } I_0 = \int e^x \, dx = e^x + c.$$

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$$= x^4 e^x - 4x^3 e^x + 4 \cdot 3x^2 e^x - 4 \cdot 3 \cdot 2x e^x + 4 \cdot 3 \cdot 2 \cdot 1 e^x + c.$$

$\ldots$

$$I_n = \int x^n e^x \, dx = x^n e^x - nI_{n-1} = e^x \sum_{i=0}^{n} (-1)^i n(n-1) \cdots (n-i+1) x^{n-i} + c$$

Riešené príklady – 100

$$I_n=\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x\,dx \qquad n\in N$$

$$=\left[\begin{array}{c|c} u=x^n & u'=nx^{n-1} \\ v'=e^x & v=e^x \end{array}\right]=x^n e^x - n\int x^{n-1} e^x\,dx = x^n e^x - nI_{n-1},\ x\in R,\ c\in R.$$

$$I_1=\int x e^x\,dx = x e^x - 1I_0 = x e^x - e^x + c,$$ pričom $I_0=\int e^x\,dx = e^x + c.$

$$I_2=\int x^2 e^x\,dx = x^2 e^x - 2I_1 = x^2 e^x - 2(x e^x - e^x)+c = x^2 e^x - 2x e^x + 2\cdot 1 e^x + c.$$

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$\ldots$

$$I_n=\int x^n e^x\,dx = x^n e^x - nI_{n-1} = e^x\sum_{i=0}^{n}(-1)^i n(n-1)\cdots(n-i+1)x^{n-i} + c$$
$$=\left[x^n - nx^{n-1} + n(n-1)x^{n-2} - \cdots + (-1)^n n!\right]e^x + c$$

Riešené príklady – 100

$$I_n = \int x^n e^x \, dx = x^n e^x - n\int x^{n-1} e^x \, dx = n!\, e^x \sum_{i=0}^{n} \frac{(-1)^i x^{n-i}}{(n-i)!} + c \qquad n \in N$$

$$= \left[\begin{array}{c|c} u = x^n & u' = nx^{n-1} \\ v' = e^x & v = e^x \end{array}\right] = x^n e^x - n\int x^{n-1} e^x \, dx = x^n e^x - nI_{n-1}, \; x \in R, \; c \in R.$$

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$\ldots$

$$I_n = \int x^n e^x \, dx = x^n e^x - nI_{n-1} = e^x \sum_{i=0}^{n} (-1)^i n(n-1)\cdots(n-i+1) x^{n-i} + c$$
$$= \left[x^n - nx^{n-1} + n(n-1)x^{n-2} - \cdots + (-1)^n n!\right] e^x + c = \sum_{i=0}^{n} \frac{(-1)^i n! x^{n-i} e^x}{(n-i)!} + c.$$